THEORY OF ELASTICITY

N. V. ZVOLINSKII

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Abstract

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N. V. ZVOLINSKII

PLANE EXPLOSIVE WAVES IN AN ELASTO-PLASTIC MEDIUM

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1. Plane one-dimensional motions of an elastoplastic medium are considered, a medium which, under small deformations, behaves as an elastic one, and under large deformations passes into the plastic state either in volumetric deformation, or in change of shape, or in both of these types of deformation together. An attempt is made to consider one of the simplest models of such a medium, so that it will be possible to trace effectively the development in it of the dynamic process of propagation of waves caused by an explosion.

The law of volumetric compression is assumed to be piecewise linear:
for decreasing σ

$$\begin{aligned} &\sigma = q_1\theta, && \theta_s < \theta < 0, \\ &\sigma = q_2(\theta - \theta_s) + \sigma_s, \quad \sigma_s = q_1\theta_s, && \theta < \theta_s, \\ & && 0 < q_1 < q_2; \end{aligned} \tag{1'}$$

for increasing σ after the least value has been reached,
$\sigma_{\min} = q_1\theta_{\min}$ or $\sigma_{\min} = q_2(\theta_{\min} - \theta_s) + \sigma_s$,

$$\begin{aligned} \sigma &= q_1\theta, && \theta_s < \theta_{\min} < 0, \\ \sigma \text{ is undefined}, & && \theta = \theta_{\min},\ \theta_{\min} < \theta_s. \end{aligned} \tag{1''}$$

Here $\sigma = \frac{1}{3}(\sigma_x + \sigma_y + \sigma_z)$; θ is the dilatation. A compressive stress and the corresponding dilatation are taken to be negative.

The plasticity condition is taken in the form

$$J_2 = -k_1\sigma - b_1, \tag{2}$$

where J_2 is the second invariant of the stress deviator.

In what follows, such a particular case is meant in which, for every point of the medium, the directions parallel to the coordinate axes remain throughout the principal directions of both the stress and deformation tensors, and the symmetry condition $\sigma_z = \sigma_y$ is satisfied. In this case condition (2) is simplified:

$$|\tau| \equiv |\sigma_x - \sigma_y| = -k\sigma - b, \quad k > 0,\ b < 0. \tag{3}$$

It is assumed that either $|\tau| = -k\sigma - b$, or $|\tau| < -k\sigma - b$, and in this latter case the shear strains and tangential stresses obey the following laws:
for decreasing τ

$$\begin{aligned} &\tau = 2\mu_1\gamma, && \theta_s < \theta < 0, \\ &\tau = 2\mu_2(\gamma - \gamma_s) + \tau_s, \quad \tau_s = 2\mu_1\gamma_s, && \theta < \theta_s; \end{aligned} \tag{4'}$$

for increasing τ, after it has reached its least value $\tau_{\min}$ (here $\tau_s = (\tau)_{\sigma=\sigma_s}$),

$$\begin{aligned} &\tau = 2\mu_1\gamma, && \gamma_s < \gamma_{\min} < 0, \\ &\tau \text{ is undefined}, \quad \gamma = \gamma_{\min}, && \gamma_{\min} < \gamma_s. \end{aligned} \tag{4''}$$

Here $\tau \equiv \sigma_x - \sigma_y,\ \gamma = \varepsilon_x - \varepsilon_y$.

From the listed conditions it is seen that the value σ_s is the threshold after passing through which the medium becomes fully plastic, and unloading (both volumetric and shear) already occurs at constant deformation. The parameter τ_s is not a characteristic of the medium, but depends ···

from the process of deformation. The model of the medium adopted here is very close to the particular case of the model described in paper (1), and differs from it by the additional assumption that the laws of loading and unloading are different under shear deformation. At any instant a material particle may be in one of the following four states, characterized by the inequalities:

$$\begin{array}{lll} \text{I.} & |\theta| < |\theta_s|, & |\tau| < -k\sigma - b. \\ \text{II.} & |\theta| < |\theta_s|, & |\tau| = -k\sigma - b. \\ \text{III.} & |\theta| > |\theta_s|, & |\tau| < -k\sigma - b. \\ \text{IV.} & |\theta| > |\theta_s|, & |\tau| = -k\sigma - b. \end{array}$$

For the study of one-dimensional plane motions, for which the direction of propagation is the direction of the axis Ox, $\varepsilon_y = 0$, $\varepsilon_x \equiv \varepsilon$, one must extract from conditions (1′), (1″), (3), (4′), (4″) the dependence between σ_x and ε. It turns out that here four types of media are possible, each of which is determined by relations among the parameters

$$q_1,\ q_2,\ \mu_1,\ \mu_2,\ \sigma_s,\ k,\ b.$$

These types are classified according to the order in which regimes change under loading and one-dimensional deformation, namely:

Fig. 1

Figure 1: Fig. 1

First type is determined by the change I → II → IV.
Second type » » I → II → III.
Third type » » I → III.
Fourth type » » I → III → IV.

For each of them the dependence between σ_x and ε can be found. For a medium of the first type it is represented by the formulas

$$\begin{aligned}
\sigma_x &= \left(q_1 + \frac{4}{3}\mu_1\right)\varepsilon, && \tilde{\varepsilon} < \varepsilon < 0,\\
\sigma_x &= q_2\left(1 + \frac{2k}{3}\right)(\varepsilon - \tilde{\varepsilon}) + \tilde{\sigma}_x, && \varepsilon_s < \varepsilon < \tilde{\varepsilon},\\
\sigma_x &= q_2\left(1 + \frac{2k}{3}\right)(\varepsilon - \varepsilon_s) + \sigma_{xs}, && \varepsilon < \varepsilon_s.
\end{aligned} \tag{5}$$

The graph of this dependence is shown in Fig. 1, on which the unloading laws following from the assumptions made above are also indicated. For a medium of the second type the (σ_x, ε) diagram is qualitatively similar to Fig. 1; for the third type it has a simpler form, and for the fourth it is somewhat artificial and bears little resemblance to the regularities observed in real materials.

Fig. 1

2. Suppose that to some plane (we choose the axis Ox perpendicular to this plane) there is applied a compressive stress $\sigma_x = \sigma_0(t)$, which instantaneously assumes a finite value $\sigma_0(0) < 0$ and then decreases monotonically and sufficiently rapidly in absolute value. We assume the impulse of the stress $\sigma_0(t)$ to be finite. Such a character of change is typical for the pressure developing in an explosion chamber; therefore we shall call the motion of the medium arising in this case an explosive wave. Let us set ourselves the goal of studying the propagation of an explosive wave in a medium of the first type. It turns out that the motion splits into several stages successively replacing one another. We assume a medium of the first type.

Let $|\sigma_0(0)| > |\sigma_{xs}|$. Introduce Lagrangian coordinates h, t such that $x(h,t) = h + u(h,t)$, where u is the displacement. At the first stage the plastic-unloading wave occupies the interval $(0, h_*(t))$, and propagates through the undisturbed medium from which it is separated by a shock front. The propagation speed $h_*'(t)$ of the shock front decreases monotonically, as shown by the equation

$$\frac{h_* h_*'}{a_3^2 - h_*'^2} = \frac{-\displaystyle\int_0^t \sigma_0(\tau)\, d\tau}{\sigma_{xs} - \rho_0 \varepsilon_s a_3^2}, \tag{6}$$

where $\rho_0 a_3^2 = q_2(1 + 2k/3)$.

Starting from the time $t = t_1$, when $h_*'(t_1) = a_1 = \sqrt{\dfrac{q_1 + {}^4\!/_3\, \mu_1}{\rho_0}}$, $h_*(t_1) = h_1$, an elastic wave appears ahead of the plastic region, separated from the plastic one, as before, by a shock front. The propagation of the front is described by the equation

$$\frac{h_* h_*'}{a_3^2 - h_*'^2} = \frac{h_1 a_1}{a_3^2 - a_1^2} - \frac{\displaystyle\int_{t_1}^t [\sigma_0(\tau) - \tilde{\sigma}_x]\, d\tau}{\rho_0(a_3^2 - a_2^2)(\tilde{\varepsilon} - \varepsilon_s)}, \tag{7}$$

where $\rho_0 a_2^2 = q_1(1 + 2k/3)$.

Starting from the time $t = t_2$, $h_*(t_2) = h_2$, the region of plastic unloading with constant density of each particle is separated from the elastic wave by a region of "inclined unloading" ($|\varepsilon| < |\varepsilon_s|$ in Fig. 1). The two plastic regions are separated from one another by a contact discontinuity. The velocity of the shock front here proves to be constant and equal to a_2. The intensity of the shock wave continues to decrease, and by the end of this stage of motion the shock wave is completely exhausted. The characteristic functions can be expressed, for example, in terms of the velocity $v(t)$ of the particles in the initial section ($h = 0$). This velocity is found from the differential-functional equation

$$\frac{h_2}{a_1}\left[\alpha f'(\alpha\tau) + \beta f'(\beta\tau)\right] - \alpha f(\alpha\tau) + \beta f(\beta\tau) = -\frac{1}{\rho_0 a_1}\left[\alpha g(\alpha\tau) + \beta g(\beta\tau)\right] + 2a_1\alpha\tilde{\varepsilon}, \tag{8}$$

where $f(t) = v(t + t_2)$, $g(t) = \sigma_0(t + t_2)$, $\tau = t - t_2$, $\alpha = 1 - a_2/a_1$, $\beta = 1 + a_2/a_1$, $\gamma = a_2/a_1$.

This equation admits an exact solution in explicit form, which we do not give here. An approximate solution is easily obtained for $\gamma \ll 1$. Starting from the time $t = t_3$, when the intensity of the shock wave becomes zero, a region of elastic unloading appears between the plastic region and the previously formed elastic wave. The elastic wave, propagating, goes off to infinity, and the motion of each fixed particle decays monotonically, tending asymptotically to the state of rest. At this stage, for example, for the particle velocity in the initial section we have

$$v(t) = v(t_3)e^{-\frac{a_1(t-t_3)}{h_2}} - \frac{1}{\rho_0 h_2}\int_{t_3}^{t} \sigma_0(\tau)e^{-\frac{a_1(t-\tau)}{h_2}}\,d\tau. \tag{9}$$

The present study is a necessary extension and supplement to the works ([2],[3]), in which the propagation of explosive waves was studied without a prior sufficiently complete consideration of the properties of the elastoplastic medium. Questions close to the present work and considered in a general form are contained in the papers ([4],[5]).

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Institute of Physics of the Earth named after O. Yu. Schmidt
Academy of Sciences of the USSR

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