DISTRIBUTION OF ELECTRIC AND MAGNETIC FIELDS IN A ROTATING THERMODYNAMICALLY EQUILIBRIUM PLASMA

1964

SovietRxiv

Abstract

Full Text

PHYSICS

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DISTRIBUTION OF ELECTRIC AND MAGNETIC FIELDS IN A ROTATING THERMODYNAMICALLY EQUILIBRIUM PLASMA

(Presented by Academician M. A. Leontovich on 29 XI 1963)

1. It is known that, in a state of thermodynamic equilibrium, the motion of a neutral gas is composed of translational motion and rotation about a fixed axis. If external forces that do not depend on the velocity act on the gas, then these forces have a potential, and the density distribution obeys the barometric formula. In this case the temperature does not depend on the coordinates, while the density varies in the direction perpendicular to the mean velocity of the gas ([1]).

If the gas consists of charged particles, then in a state of thermodynamic equilibrium there is a separation of charges caused by the combined action of rotation and of the intrinsic electric and magnetic fields. The conditions for the existence of thermodynamic equilibrium of a plasma coincide with the conditions listed above for the existence of thermodynamic equilibrium for ordinary gases, with the difference that the external forces are replaced by internal forces, which are due to the electric and magnetic fields arising from the centrifugal separation of charges and the convective current.

The force exerted by the magnetic field on a particle is of order $(v_0/c)^2$ in comparison with the force due to the electric field. Since, as was said above, the total force acting on a particle of each species has a potential, the basic equations have the form

$$n = n_0 \exp\left(-\frac{\psi}{\theta}\right), \qquad q\mathbf{E} + m[\mathbf{v}_0,\ \vec{\omega} + \vec{\Omega}] = -\frac{\partial \varphi}{\partial \mathbf{r}}, \tag{1}$$

where n_0 is the particle density at $\psi = 0$; ψ is the generalized potential; q and m are the charge and mass of a particle; $\vec{\Omega}$ is the rotation frequency; $\vec{\omega}$ is the Larmor frequency; $\mathbf{v}_0$ is the mean velocity of the component, $\mathbf{v}_0 = \mathbf{v}' + [\vec{\Omega}, \mathbf{r}]$. Here $\mathbf{v}'$ and $\vec{\Omega}$ are constants.

To these equations, written for each component, one must add Maxwell' s equations and the equations relating the current and charge to the densities and mean velocities of the particles.

We shall restrict ourselves to the case when the generalized potential is small in comparison with the temperature. Then $\exp(-\psi/\theta) \approx 1 - \psi/\theta$, and the equations become linear.

Let us consider a thermodynamically equilibrium plasma consisting of electrons and singly charged ions, rotating in a cylinder of radius ρ_0. In this case all quantities depend only on ρ, the angular velocities of rotation are the same, and $\mathbf{v}' = 0$. Then the system of equations has the form:

$$\sigma = e(n_i - n_e) = e(n_2 - n_1) + \frac{e}{\theta}(n_1\psi_1 - n_2\psi_2); \tag{2}$$

$$j = \Omega\rho\sigma, \qquad \frac{1}{\rho}\frac{d}{d\rho}(\rho E) = 4\pi\sigma, \qquad \frac{dB}{d\rho} = -\frac{4\pi}{c}j, \tag{3}$$

$$\frac{d\psi_1}{d\rho} = eE + \frac{e\Omega}{c}B\rho - m\Omega^2\rho, \qquad \frac{d\psi_2}{d\rho} = -eE - \frac{e\Omega}{c}B\rho - M\Omega^2\rho. \tag{4}$$

Here and below the indices 1 and e refer to electrons, 2 and i to ions; n_1 and n_2 are the particle densities on the axis of the cylinder (the potentials are measured from the axis); $j = j_\varphi$, $E = E_\rho$, $B = B_z$.

Assuming that the total charge per unit length of the cylinder is zero and that external fields are absent, we obtain the following boundary conditions of the problem: on the axis of the cylinder the electric field vanishes and the magnetic field remains finite, while at the edge of the cylinder the electric and magnetic fields are zero.

2. From equations (3) it follows that $\rho E + \frac{c}{\Omega}B = 0$. From (4), neglecting $(v_0/c)^2$, it is easy to obtain

$$\chi\frac{d^2B}{d\chi^2} - bB = -a\chi, \tag{5}$$

where

$$a = \frac{c\Omega}{e}\frac{Mn_2}{n_1 + n_2}, \qquad b = \frac{c}{2\Omega\rho_d}, \qquad \chi = \frac{\Omega\rho^2}{2c\rho_d}, \qquad \rho_d = \sqrt{\frac{\theta}{4\pi e^2(n_1 + n_2)}}$$

is the Debye radius.

The number of particles per unit length of the cylinder is

$$N = \frac{\pi n_1 n_2 \rho_0^2}{n_1 + n_2}\left(2 + \frac{M\Omega^2\rho_0^2}{4\theta}\right). \tag{6}$$

The generalized potential has the order of magnitude of the ion rotation potential. Consequently, the linear approximation of the exponential assumes the smallness of the ion rotation potential in comparison with θ, i.e. $v_0 \ll v_{Ti}$, where v_{Ti} is the thermal velocity of the ions.

The solution of equation (5) satisfying the boundary conditions is

$$B(\chi) = \frac{a}{b}\left[\chi - \sqrt{\chi\chi_0}\,\frac{I_1(2\sqrt{b\chi})}{I_1(2\sqrt{b\chi_0})}\right]. \tag{7}$$

The remaining physical quantities are expressed through $B(\chi)$ as follows:

$$\begin{gathered}
E = -\frac{c}{\Omega\rho}B, \qquad \sigma = \frac{1}{4\pi\rho_d}\frac{dB}{d\chi}, \qquad j = \Omega\rho\sigma, \\
\psi_1 = \frac{\theta}{n_1+n_2}\left[\frac{\sigma + e(n_1-n_2)}{e} - \frac{M\Omega^2 n_2\rho^2}{2\theta}\right], \\
\psi_2 = -\frac{\theta}{n_1+n_2}\left[\frac{\sigma + e(n_1-n_2)}{e} + \frac{M\Omega^2 n_1\rho^2}{2\theta}\right].
\end{gathered} \tag{8}$$

The charge separation at the center of the cylinder is determined by the rotation frequency and the radius of the cylinder. The equation relating these quantities is obtained by comparing formulas (2) and (8), which determine the charge on the axis. It has the form

$$\frac{2M\Omega^2}{m\omega_p^2}\left[1 - \frac{\chi_0}{2I_1(\chi_0)}\right] = \frac{1}{\chi} - \chi, \tag{9}$$

where $\chi = n_2/n_1$, $\chi_0 = \rho_0/\rho_d$, $\omega_p = \sqrt{4\pi n_1 e^2/m}$.

The left-hand side of (9) is always positive, i.e. always $n_1 > n_2$. It is easy to see that for $v_0 \ll v_{Ti}$ the quantity

$$A = \frac{M\Omega^2}{m\omega_p^2}\left[1 - \frac{\chi_0}{2I_1(\chi_0)}\right] \ll 1.$$

Consequently,

$$\chi \approx 1 - A.$$

For $\chi_0 \lesssim 1$

$$\chi = 1 - \frac{M\Omega^2}{m\omega_p^2}\frac{\chi_0^2/8}{1+\chi_0^2/8},$$

whereas for $\chi_0 \gg 1$

$$\chi = 1 - \frac{M\Omega^2}{m\omega_p^2}.$$

Thus, for $\chi_0 \gg 1$ the charge separation on the axis does not depend on the radius of the cylinder.

Using (8), it is easy to find the fields, charge, current, and potentials. Figure 1 shows the behavior of these quantities, normalized to their greatest values. The latter are equal to:

$$B_m = -\frac{\pi}{8}\frac{M\Omega^3 e n_2 \rho_0^4}{c\theta(1+\chi_0^2/8)}, \qquad E_m = \frac{8}{3\sqrt{3}}\frac{cB_m}{\Omega\rho_0}, \qquad \sigma_m = e(n_1 - n_2), \qquad j_m = \Omega\rho_0\sigma_m,$$

$$\psi_{1m} = \frac{c\theta B_m \chi_0^2}{16 e\Omega\rho_0^2(n_1+n_2)}, \qquad \psi_{2m} = \frac{M\Omega^2\rho_0^2}{2}\left[1 - \frac{n_2\chi_0^2}{8(n_1+n_2)(1+\chi_0^2/8)}\right]$$

for $\chi_0 \ll 1$;

$$B_m = B^{(m)}\left(1 - 2\frac{\ln\chi_0}{\chi_0} - 2\frac{1-\ln 2}{\chi_0} + \frac{\ln^2\chi_0}{\chi_0^2}\right), \qquad E_m = \frac{cB^{(m)}}{\Omega\rho_0}\left(1 - \frac{\ln\chi_0}{\chi_0}\right),$$

$$\sigma_m = \frac{cB^{(m)}}{4\pi\rho_d\rho_0\Omega}\left(1 - \frac{2}{\chi_0}\right), \qquad j_m = \Omega\rho_0\sigma_m, \qquad \psi_{1m} = 2\pi e\rho_d\rho_0\sigma_m,$$

$$\psi_{2m} = \frac{n_1\psi_{1m}}{n_2}\frac{1+2n_2/\chi_0 n_1}{1-2/\chi_0}, \qquad B^{(m)} = \frac{M\Omega^3 n_2\rho_0^2}{ce(n_1+n_2)}$$

for $\chi_0 \gg 1$.

Along the abscissa axis is plotted the quantity $\tau = \rho/\rho_0$. In the case $\chi_0 \ll 1$, the behavior of the curves does not depend on χ_0. As χ_0 increases, all extremal points move toward the edge of the cylinder. All calculations were carried out using two terms of the series and one term of the asymptotics of the Bessel functions.

Fig. 1. 1—reduced magnetic field; 2—reduced electric field; 3—charge; 4—current; 5—electron potential; 6—ion potential

Fig. 1

Figure 1: Fig. 1

For $\chi_0 \gg 1$, determination of the extremal points reduces to solving transcendental equations, the asymptotics of whose roots can be obtained by the method of iterations [2]. The behavior of curves 6 for $\chi_0 = 10$ and $\chi_0 = 100$ depends on χ ($\chi = 0.98$ and $\chi = 0.9998$). For $v_0 < v_{Ti}$, the inequality $1 - 6/\chi_0^2 < \chi < 1$ holds for χ.

Received
27 XI 1963

References Cited

1. S. Chapman, T. Cowling, *The Mathematical Theory of Non-Uniform Gases*, IL, 1960, p. 102.

2. N. G. de Bruijn, *Asymptotic Methods in Analysis*, IL, 1961, p. 36.

Note: Figure translations are in progress. See original paper for figures.