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Abstract

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THE INFLUENCE OF ELASTIC SCATTERING ON RESONANT CHARGE EXCHANGE IN A SLOW COLLISION

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1. The dependence of the cross section for resonant charge exchange on the collision velocity is determined by the relation ([1,2])

$$\int_0^\infty [\varepsilon_u(R) - \varepsilon_g(R)]\,dt = \frac{11}{40}, \qquad \sigma = \frac{\pi R_0^2}{2}, \tag{1}$$

where $\varepsilon_u - \varepsilon_g$ is the magnitude of the splitting of the electron energy as an ion approaches it, depending on the distance R between the nuclei; R_0 is the impact parameter of the collision for which the given relation is satisfied. We use the system of atomic units $\hbar = m_{\rm el} = e^2 = 1$.

It is usually assumed that the relative motion of the colliding atomic particles at impact parameters making the main contribution to the resonant-charge-exchange cross section is well described by the law of free motion. This is true if the elastic-scattering cross section is much smaller than the resonant-charge-exchange cross section, and gives the dependence of the collision velocity on the resonant-charge-exchange cross section:

$$v = \frac{40}{11}\int_{R_0}^\infty (\varepsilon_u - \varepsilon_g)\frac{R\,dR}{\sqrt{R^2 - R_0^2}} \equiv F(R_0), \qquad \sigma_0 = \frac{\pi R_0^2}{2}. \tag{2}$$

The influence of elastic scattering on the resonant-charge-exchange cross section was investigated in the work of Bates and Boyd ([3]). In the present note a general relation is obtained between the exact cross section for resonant charge exchange and the resonant-charge-exchange cross section without taking elastic scattering into account.

2. Let us calculate the resonant-charge-exchange cross section (1) with allowance for the interaction between the colliding atomic systems. We have ([4])

$$\int_{R_{\min}}^{\infty} (\varepsilon_u - \varepsilon_g) \frac{dR}{v[1 - R_0^2/R^2 - U(R)/E]^{1/2}} = \frac{11}{40}, \tag{3}$$

where $R_{\min}$ is the distance of closest approach for the given collision parameter; $E = \mu v^2/2$ is the kinetic energy of the nuclei of the colliding atoms in the center-of-inertia system; μ is the reduced mass of the nuclei; $U(R)$ is the interaction potential between the atomic particles. The quantity $\varepsilon_u - \varepsilon_g$ has the asymptotic form ([1,5])

$$\varepsilon_u - \varepsilon_g = AR^{2/\gamma - 1} e^{-R\gamma}, \qquad \frac{R\gamma^2}{2} \gg 1, \tag{4}$$

where $\gamma = \sqrt{2\varepsilon}$, ε is the binding energy of the electron undergoing the transition. Since the relation

$$R_{\min} \frac{\gamma^2}{2} \gg 1 \tag{5}$$

is practically always fulfilled, the integral (3) converges rapidly near the point $R = R_{\min}$ because of the sharp damping of the exponential. Expanding the denominator in the neighborhood of the turning point $R_{\min}$

$$1 - \frac{R_0^2}{R_{\min}^2} - \frac{U(R_{\min})}{E} = 0, \tag{6}$$

compute the integral (3). Comparing it with the integral (2), found in the same way, we obtain

$$v' \equiv v \left[\frac{R_0^2}{R_{\min}^2} - \frac{R_{\min} U'(R_{\min})}{2E} \right] = F(R_{\min}). \tag{7}$$

Using relations (6) and (7), we find the relation between the resonant charge-exchange cross section and the cross section σ_0 obtained without taking direct scattering into account:

$$\sigma_{\text{res}} = \frac{\pi R_0^2}{2} = \frac{\pi}{2} \left[R_{\min}^2 - R_{\min}^2 \frac{U(R_{\min})}{E} \right] = \sigma_0(v') \left[1 - \frac{U\left(\sqrt{2\sigma_0(v')/\pi}\right)}{E} \right]. \tag{8}$$

Here σ_0 is the resonant charge-exchange cross section, determined by formula (2), corresponding to the collision velocity

$$v' = v\sqrt{\frac{R_0^2}{R_{\min}^2} - \frac{R_{\min}U'(R_{\min})}{2E}}.$$

If the interaction potential at large distances between the nuclei has the asymptotic form $U = -a/R^n$, then

$$v' = v\left[1 + \frac{(n-2)}{2}\frac{U(R_{\min})}{E}\right]^{1/2}. \tag{9}$$

3. The law of motion $R(t)$ used in (3), valid for infinite motion, is violated if the interaction potential varies more sharply than $1/R^2$. In this case capture of the ion is possible. Capture leads to a strong approach of the particles, so that the probability of charge exchange in such collisions is equal to 1/2. If the values of $R_{\min}$ making the main contribution to the cross section σ_0 satisfy the inequality $R_{\min} \ll R_{\min}^{\text{capt}}$, then the resonant charge-exchange cross section without allowing for elastic scattering, σ_0, should be taken as $\frac{\pi}{2}(R_{\min}^{\text{capt}})^2$, where $R_{\min}^{\text{capt}} = [a(n-2)/2E]^{1/n}$ is the distance of closest approach at which capture is still possible (the interaction potential is $U = -a/R^n$). Taking elastic scattering (6) into account gives

$$\sigma_{\text{res}} = \frac{\sigma_{\text{capt}}}{2} = \frac{\pi}{2}\frac{n}{n-2}\left[\frac{a(n-2)}{2E}\right]^{2/n}, \qquad F(R_{\min}^{\text{capt}}) \ll v. \tag{10}$$

Assuming that relation (8) continuously passes into (10)*, we obtain, in the presence of capture,

$$\sigma = \begin{cases} \sigma_0(v)\left\{1 - \dfrac{U\left(\sqrt{2\sigma_0(v)/\pi}\right)}{E}\right\}, & F(R_{\min}^{\text{capt}}) \gg v, \\ \dfrac{\sigma_{\text{capt}}}{2} = \dfrac{\pi}{2}\dfrac{n}{n-2}\left[\dfrac{a(n-2)}{2E}\right]^{2/n}, & F(R_{\min}^{\text{capt}}) \ll v, \end{cases} \tag{11}$$

$$\sigma_0(v) = \frac{\pi R_0^2}{2}, \qquad \text{where } F(R_0) = v.$$

4. From (8), (11) it is seen that, if attractive forces act between the atomic particles, the resonant charge-exchange cross section increases due to elastic scattering and capture ($U < 0$); in the case of repulsive forces, the resonant charge-exchange cross section decreases. Elastic scattering may be neglected in calculating the resonant charge-exchange cross section if, for impact parameters $\sim R_0$, which make the main contribution to the resonant charge-exchange cross section, the interaction potential between the atomic particles is much less—

* We note that at the point of transition from one dependence to the other in (11), not only the cross section

than the kinetic energy of their nuclei,

$$U(R_0) \ll E.$$

Estimates show that, in the case of resonant charge exchange of singly charged ions on atoms in the ground state ($U = -\alpha/2R^4,\ \alpha$ is the polarizability), this relation can be violated only at thermal velocities.

The effect under consideration becomes substantial in collisions of atomic systems with strong interaction and not very large cross sections for resonant charge exchange (charge exchange of an ion on an ion, of a multiply charged ion on an atom, of an ion on an excited atom). For example, in the case of a collision of a proton with a hydrogen atom in an excited state ($n = 2,\ n_1 \neq n_2$), elastic scattering changes the value of the resonant charge-exchange cross section by an amount of the order of the cross section itself at energies of 0.1 eV.

Thus, relation (11) has been found between the exact resonant charge-exchange cross section and the resonant charge-exchange cross section obtained without taking elastic scattering and capture into account. Elastic scattering may be significant in resonant charge exchange of multiply charged ions, ions on ions, and ions on excited atoms. In determining the cross section for resonant charge exchange of singly charged ions on atoms in the ground state, elastic scattering may be neglected.

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