Cybernetics and Control Theory

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1964

SovietRxiv

Abstract

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The Principle of Constructing Invariant Automatic Control Systems with Variable Structure

A principle is considered for constructing one class of invariant automatic control systems with variable structure. It is assumed that the system has a low order of astatism (down to zero) and that disturbing actions are applied to various points of the controlled plant. The control law must ensure complete reproducibility, by the controlled coordinate, of the control action. In ([1]) it was shown that the stated problem can be solved by means of a two-channel invariant linear automatic control system.

In the present work an attempt is made to make use of certain properties of automatic control systems with variable structure in order to ensure complete reproducibility, by the controlled coordinate, of the control action. It is assumed that the disturbing and control actions belong to a sufficiently broad class of functions—the class of polynomials of any, but finite, degree in time t. The control law must be formulated without measuring the disturbances or any internal coordinates of the plant. We shall require that this law satisfy the conditions of physical realizability, and that the conditions of complete reproducibility be insensitive to changes in the parameters of the plant.

Let, in a region G of the n-dimensional phase space $(x_1, \dots, x_n)$, the motion of the automatic control system be described by the system of differential equations

$$\frac{d\mathbf{x}}{dt} = \mathbf{f}(\mathbf{x}, \vec{\psi}, t), \tag{1}$$

where

$$\mathbf{x} = (x_1, \dots, x_n); \qquad \vec{\psi} = (\psi_1, \dots, \psi_{n-1}); \qquad \mathbf{f} = (f_1, \dots, f_n);$$

$$f_i = x_{i+1} \ (i = 1, 2, \dots, n-1); \qquad f_n = -\sum_{i=1}^{n} a_i x_i - \sum_{i=1}^{n-1} \psi_i(\bar{x}) x_i - \Phi(t);$$

$$\psi_i(x) = \begin{cases} \omega_i, & \text{when } \sigma x_i > 0, \\ \lambda_i, & \text{when } \sigma x_i < 0, \end{cases} \qquad (i = 1, 2, \dots, n-1)^*; \tag{2}$$

Fig. 1

Figure 1: Fig. 1

$$\sigma = \sum_{i=1}^{n} c_i x_i;$$

ω_i, λ_i, c_i are constant quantities, $c_n = 1$.

* In the case $\sigma x_i = 0$

$$\psi_i = \omega_i \quad \text{when } \sigma x_i \to +0; \qquad \psi_i = \lambda_i \quad \text{when } \sigma x_i \to -0.$$

Let the object be controlled by an astatic regulator with rigid feedback (Fig. 1); then a_i are quantities linearly dependent on k, the coefficient of rigid feedback of the regulator,

$$\Phi(t) = kG(t) + pG(t), \qquad G(t) = \sum_{i=0}^{m} Q_i(p) g_i(t). \tag{3}$$

Here g_0 is the control action; $g_1, \dots, g_m$ are disturbances applied at m different points of the object; $Q_1, \dots, Q_m$ are polynomials in p, $p = d/dt$.

We note that the function $G(t)$ is related to the coordinates $x_1, \dots, x_n$ and to the output coordinate of the regulator μ by the relation

$$G(t) = \mu + \sum_{i=1}^{n} b_i x_i^*; \tag{4}$$

$b_1, \dots, b_n$ are constant coefficients.

Fig. 1

It is known ([3]) that in such systems, in some region U of the phase space $(x_1, \dots, x_n)$ belonging to the discontinuity boundary S (S is a hyperplane given by the equation $\sigma = 0$), the motion is described by a homogeneous differential equation. An essential feature of this differential equation is that it depends neither on the parameters of the object nor on the function $\Phi(t)$, but is determined only by the coefficients $c_1, \dots, c_n$ of the hyperplane S.

The region U is determined by the following relations ([2]):

$$\mathbf{c}\frac{dx}{dt} > 0 \quad \text{for } \sigma < 0, \qquad \mathbf{c}\frac{dx}{dt} < 0 \quad \text{for } \sigma > 0, \tag{5}$$

where $\mathbf{c} = (c_1, \dots, c_n)$.

According to the equations of motion (1), condition (5) can be written in the form

$$\sum_{i=1}^{n-1} c_i x_{i+1} + \left[-\sum_{i=1}^{n} a_i x_i - \sum_{i=1}^{n-1} \psi_i(x) x_i + \Phi(t) \right] > 0 \quad \text{for } \sigma < 0,$$

$$\sum_{i=1}^{n-1} c_i x_{i+1} + \left[-\sum_{i=1}^{n} a_i x_i - \sum_{i=1}^{n-1} \psi_i(x) x_i + \Phi(t) \right] < 0 \quad \text{for } \sigma > 0. \tag{6}$$

As follows from (6), the boundaries of the region U vary in time with the change in the quantity $\Phi(t)$. Let us choose such a sequence of changes of the system structure for which the region U always coincides with the hyperplane S. For this purpose, according to (3), (4), (6), the switched rigid feedback

* Objects are considered whose transfer function contains no zeros.

the regulator $k(\mu, \mathbf{x})$ must have the form

$$k(\mu, \mathbf{x}) = \begin{cases} k_1, & \text{for } \sigma \left(\mu + \sum_{i=1}^{n} b_i x_i \right) < 0, \\ -k_2, & \text{for } \sigma \left(\mu + \sum_{i=1}^{n} b_i x_i \right) > 0^*, \end{cases} \tag{7}$$

where k_1 and k_2 are constant positive quantities, and the control vector $\vec{\psi}$ must satisfy the conditions indicated in (4):

$$\omega_i > \max_{a_i, a_n} \left(c_{i-1} - a_i - c_{n-1} c_i + a_n c_i \right),$$

$$\lambda_i < \min_{a_i, a_n} \left(c_{i-1} - a_i - c_{n-1} c_i + a_n c_i \right). \qquad (i = 1, 2, \dots, n-1). \tag{8}$$

In this case, after the representative point reaches the hyperplane S, the subsequent motion in the system is determined by the coefficients $c_1, c_2, \dots, c_n$ of the homogeneous differential equation.

Consequently, when conditions (7) and (8) are simultaneously satisfied, the system ensures complete reproducibility, in the regulated coordinate φ, of the control action $g_0(t)$.

In the system under consideration, the motion is independent of the right-hand side $\Phi(t)$ only in a certain region U of the phase space $(x_1, \dots, x_n)$. This feature leads to the fact that the system turns out to be insensitive to changes in the parameters of the controlled object, since the independence conditions are based on inequalities. In conclusion, it should be noted that the required law can be realized with the aid of fairly simple technical means.

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Received
29.XI 1963

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* In the case $\sigma\left(\mu + \sum_{i=1}^{n} b_i x_i\right) = 0$

$$k(\mu, \mathbf{x}) = k_1 \quad \text{for } \sigma\left(\mu + \sum_{i=1}^{n} b_i x_i\right) \to +0,$$

$$k(\mu, \mathbf{x}) = -k_2 \quad \text{for } \sigma\left(\mu + \sum_{i=1}^{n} b_i x_i\right) \to -0.$$

Note: Figure translations are in progress. See original paper for figures.