Physical Chemistry

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1964

SovietRxiv

Abstract

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A METHOD FOR CALCULATING THE INTEGRAL ABSORBED POWER OF GAMMA RADIATION IN MACROSYSTEMS

(Presented by Academician S. S. Medvedev, 9 IX 1963)

The calculation of the integral absorbed power of γ-radiation in macrosystems (I.A.P.) for a number of cases has been considered in works ([1–5]). In works ([1–3]), only an approximate calculation of the I.A.P. was carried out (without rigorous allowance for multiple scattering) for some source–irradiated-object systems. In works ([4,5]), a semiempirical method was proposed for calculating the I.A.P. in a cylindrical absorbing object from a point or linear source.

In the present work a method is proposed for calculating the I.A.P. (w_a) in arbitrary macrosystems.

Let us denote $w_a/w_0 = \eta$; $w_{\rm p}/w_0 = j_{\rm p}$; $w_{\rm r}/w_0 = j_{\rm r}$; $w_{\rm in}/w_0 = j_{\rm in}$; $w_{\rm op}/w_0 = j_{\rm op}$, where $w_{\rm p}$, $w_{\rm r}$ are, respectively, the energy-flux powers of the primary attenuated and scattered radiation emerging from the surface of the irradiated object; $w_{\rm in}$ is the energy-flux power of γ-radiation entering through the surface of the irradiated object; w_0 is the power of the γ-radiation source; $w_{\rm op}$ is the power of the primary γ-radiation scattered without interaction with the macrosystem; η is the energy efficiency of the macrosystem with respect to γ-radiation (e.c.p.); $j_{\rm p}$ and $j_{\rm r}$ are, respectively, the relative energy-flux powers of the primary attenuated and scattered γ-radiation emerging from the surface of the irradiated object; $j_{\rm in}$ is the fraction of the power of the γ-radiation source reaching the macrosystem; $j_{\rm op}$ is the relative power of the primary γ-radiation scattered without interaction with the macrosystem.

For a given power of the γ-radiation source, the value of the I.A.P. is uniquely determined by the e.c.p. of the macrosystem ([4,5]). We note that

$$\eta = 1 - (j_{\rm op} + j_{\rm r}), \tag{1}$$

where

$$j_{\rm op} = 1 - j_{\rm in} + j_{\rm p}. \tag{2}$$

Fig. 1. Dependence of B_j on b and E

Figure 1: Fig. 1. Dependence of B_j on b and E

Thus, the calculation of the e.c.p. reduces to determining j_p, j_r, and j_in. We note that when an isotropic radiation source is located inside the macrosystem, $j_\text{in} = 1$ and $j_\text{op} = j_\text{p}$.

Calculation of the e.c.p. of a sphere. Consider the case when a point isotropic source of γ-radiation lies at the center of an absorbing sphere of radius R (in centimeters). Obviously, in this case $j_\text{p} = e^{-\mu R}$, where μ is the linear attenuation coefficient of γ-radiation in the substance filling the sphere (in inverse centimeters). To calculate the value of η, we integrate the energy absorbed in an elementary spherical layer of thickness $d\rho$, located at a distance ρ from the center of the sphere, over the entire volume of the sphere:

$$\eta = \int_0^R B(E, \mu\rho)\, e^{-\mu\rho} \mu_a \, d\rho, \tag{3}$$

where $B(E, \mu\rho)$ is the buildup factor of absorbed energy, μ_a is the linear

the absorption coefficient of γ-radiation in the substance filling the sphere (in inverse centimeters).

The quantity $B(E, \mu\rho)$, as is known [6], can be represented in the form:

$$B(E, \mu\rho) = A_1 e^{-\alpha_1 \mu\rho} + A_2 e^{-\alpha_2 \mu\rho}, \tag{4}$$

where A_1, A_2, α_1, α_2 are coefficients depending on E. Substitution of (4) into equation (3) gives

$$\eta = \frac{\mu_a}{\mu} \left\{ \frac{A_1}{1+\alpha_1} \left[1 - e^{-(1+\alpha_1)\mu R}\right] + \frac{A_2}{1+\alpha_2} \left[1 - e^{-(1+\alpha_2)\mu R}\right] \right\} \tag{5}$$

Fig. 1. Dependence of B_j on b and E

It is obvious that for $R \to \infty$, $\eta \to 1$, and, consequently, the equality must hold

$$\eta_{R\to\infty} = \frac{\mu_a}{\mu} \left[\frac{A_1}{1+\alpha_1} + \frac{A_2}{1+\alpha_2} \right] = 1. \tag{5'}$$

Calculations show that when the coefficients A_1, A_2, α_1, and α_2 given in [6] are used, equation (5′) is not satisfied. In this connection, when determining the efficiency of the sphere by (5), it is necessary to use coefficients corrected according to equation (5′). Thus, in what follows, the efficiency of the sphere is calculated from equation (5), taking into account correction of the coefficients by (5′).

As shown in works [4,5], it is convenient to calculate the efficiency by the formula

$$\eta = 1 - B_j j_\mathrm{p}, \tag{6}$$

where B_j is the factor introduced in [4,5] for the build-up of the integral energy current of γ-radiation emerging from the surface of a macrosystem. By definition [4,5]:

$$B_j = 1 + j_\mathrm{p}/j\,. \tag{7}$$

Let us consider the relation between j and j_p, having calculated B_j from equation (6) by substituting into (6) the values of the sphere efficiency obtained from equation (5), taking into account the correction by (5′). This calculation showed that in the energy interval $E = 0.7$-5.0 MeV, for $b = \mu R = 0 \div 4.0$, the values of B_j (see Fig. 1) are described with an accuracy of approximately 5% by the equation

$$B_j = 1 + k(E)b, \tag{8}$$

where it turned out that $k(E) = \mu_s/\mu$, where $\mu = \mu - \mu_a$. From (7) and (8) it follows that

$$j_\mathrm{p} = \frac{\mu_s}{\mu}\, bj\,, \tag{9}$$

i.e., the relative energy current of scattered radiation emerging from the surface of the sphere can be expressed as the product of three dimensionless criteria: μ_s/μ, b, and j .

It follows from (6) and (8) that for a sphere

$$\eta = 1 - \left[1 + \frac{\mu_s}{\mu} b\right] j_\mathrm{p}. \tag{10}$$

General course of calculating the integral absorbed dose in a macrosystem of arbitrary configuration. Suppose that for a macrosystem of specified shape and size one can find an equivalent sphere of radius R_eff, in which the quantities η, j_p, and j_r are equal to the corresponding quantities of the given macrosystem. Consequently, the calculation of the integral absorbed power and η in the irradiated macrosystem is reduced to the corresponding calculation of these quantities in a sphere, as described above.

The attenuation of the primary γ-radiation in an element of solid angle of the macrosystem $d\Omega$ of thickness ρ_Ω is equal to $e^{-\mu\rho_\Omega}$, and over the whole macrosystem the following attenuation $\bar{\xi}$ takes place:

$$\bar{\xi} = \frac{1}{\Omega_k} \int_{\Omega_k} e^{-\mu\rho_\Omega} \, d\Omega, \tag{11}$$

where Ω_k is the solid angle under which the surface of the irradiated system is seen from the location of the radiation source.

It can readily be shown that numerically $\bar{\xi} = j_\text{p}$. On the other hand, under our assumption concerning the properties of the equivalent sphere,

$$\bar{\xi} = e^{-\mu R_\text{eff}}.$$

Consequently,

$$R_\text{eff} = \frac{1}{\mu} \ln \frac{1}{j_\text{p}}. \tag{12}$$

[Figure 2 graph]

Fig. 2. Dependence of η and j_r on j_p (or μR_eff) for γ-radiation energies $E =$ 0.7; 1.25; 2.5 MeV

Let us determine the quantity j_r. Since the propagation of γ-radiation in the case of a spherical object and a central position of an isotropic γ-source is completely symmetric with respect to the source, one may assume that the energy-current buildup factor B'_j, determined for an elementary surface of the sphere in the elementary solid angle $d\Omega$, is equal to B_j. Consequently, by analogy with (9), one may assume that in the elementary solid angle of the macrosystem under consideration

$$dj_\text{r} = \frac{\mu_s}{\mu} (\mu\rho_\Omega) \, dj_\text{p}$$

and over the whole macrosystem

$$j_\text{r} = \frac{\mu_s}{\mu} \frac{1}{\Omega_k} \int_{\Omega_k} e^{-\mu\rho_\Omega} (\mu\rho_\Omega) \, d\Omega. \tag{13}$$

Calculation of the quantity j_r by formula (13) for a number of special cases shows that j_r, determined in this way, is equal to j_r for a sphere with R_eff determined by formula (12). Consequently, having computed for each specific case $j_\text{p} = \bar{\xi}$ by (14) and μR_eff by (12), we find j_r in the form

Fig. 3

Figure 2: Fig. 3

$$j_{\mathrm{r}} = \frac{\mu_s}{\mu}(\mu R_{\mathrm{eff}}) j_{\mathrm{p}}, \tag{14}$$

and equation (10) becomes

$$\eta = 1 - \left[1 + \frac{\mu_s}{\mu}\,(\mu R_{\mathrm{eff}})\right] j_p. \tag{15}$$

The expressions obtained, (12), (14), (15), make it possible to determine the efficiency and j_p as functions of the energy of the γ-radiation, of the absorbing-scattering properties of the medium, and of j_p. The dependence of η and j_p on the quantity j_p (or μR_{eff}) for various energies of the primary γ-radiation ($E = 0.7;\ 1.25;\ 2.5$ MeV), calculated from formulas (12), (14), (15), is shown in Fig. 2. Consequently, in order to determine the efficiency of a macrosystem by the proposed method, it is necessary to find the value of j_p.

Fig. 3. Comparison of the results of calculating the efficiency by the method proposed in the present work (curve) and by the Monte Carlo method (points). $E = 1.25$ MeV

The results of calculating the efficiency for various cases of macrosystems with a point source of γ-radiation were compared with the results of calculating the same quantities by the method of statistical trials (Monte Carlo method)*, which is essentially a numerical experiment ([7,8]). Processing the results of the efficiency calculation by the Monte Carlo method as a function of j_p showed that these results agree well with the values of η obtained from equation (18) (see Fig. 3). The agreement confirms the correctness of the method proposed in the present work for calculating the i.a.p. and the efficiency for γ-radiation in macrosystems.

The author takes this opportunity to express gratitude to A. Kh. Breger, N. P. Syrkus, V. A. El' tekov, and B. M. Terent' ev for fruitful discussion of the present work, and to S. I. Berestetskaya for assistance in carrying out the numerical calculations and in preparing the paper.

Physico-Chemical Institute
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Received
9 IX 1963

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* The Monte Carlo calculations were carried out by B. M. Terent' ev and V. A. El' tekov.

Note: Figure translations are in progress. See original paper for figures.