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G. A. Bross

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Abstract

Full Text

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G. A. Bross

LOW-TEMPERATURE PROPERTIES OF AN ISOTROPIC FERROMAGNET IN THE CASE OF ARBITRARY SPIN

(Presented by Academician N. N. Bogolyubov on 16 V 1964)

In the last two years a number of attempts have been made ([1–4]) to calculate the magnetization of a ferromagnetic dielectric in the case of arbitrary spin. All the calculations are based on the method of two-time temperature Green's functions and differ from one another in the way the chain of equations for the Green' s functions is decoupled. Another difference appears when Bose operators are used instead of Pauli operators or spin operators. The first results of Tahir-Kheli and ter Haar ([1]) and of Tyablikov give only the zeroth approximation for the given problem, since in them the correlations of spin operators at different lattice sites are neglected. A similar decoupling procedure was used by Callen ([4]). Further calculations by Tahir-Kheli and ter Haar ([2]) were based on substituting, in place of Pauli operators, operators with Bose commutation relations, which may lead, as shown by Tyablikov ([5–7]), in the case of spin $s = 1/2$, to an incorrect result for the magnetization. In the present communication we briefly set out the results of certain calculations, which differ from Tyablikov' s calculations ([3]) by a better decoupling (for details see ([8,9])).

The Hamiltonian of our problem has the form

$$\mathcal{H} = E_0 - (\mu H - J(0)\langle S^z\rangle)\sum_f S_f^z - \frac{1}{2}\sum_{f,f'} I(f-f')S_f^- S_{f'}^+ - \frac{\varepsilon}{2}\sum_{f,f'} I(f-f')\left(S_f^z - \langle S^z\rangle\right)\left(S_{f'}^z - \langle S^z\rangle\right), \tag{1}$$

where H is the external field, which is assumed to be directed along the z-axis; μ is the Bohr magneton; f and f' are numbers of lattice sites; $I(f - f')$ is the exchange integral corresponding to the interaction of spins at the lattice sites f and f'; N is the number of lattice sites; ε is a parameter, which we regard as small; it is introduced for the decoupling of the chain of equations for the Green' s functions. Below we use the notation:

$$E_0 = \frac{\varepsilon}{2} N J(0)\langle S^z\rangle^2, \tag{2}$$

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$$J(p) = \sum_f I(f) \exp\left(-i(pf)\right); \tag{3}$$

$S_f^{\pm}, S_f^z$ are spin operators satisfying the commutation relations

$$[S_f^z, S_{f'}^{\pm}] = \pm S_f^{\pm} \Delta(f - f'); \tag{4}$$

$$[S_f^+, S_{f'}^-] = 2\{\langle S^z \rangle + \varepsilon(S_f^z - \langle S^z \rangle)\}\Delta(f - f'), \tag{5}$$

where $\Delta(x) = 1$ if $x = 0$, and $\Delta(x) = 0$ otherwise. Further, we use the relation

$$S_f^- S_f^+ = -\left[2\langle S^z \rangle - \varepsilon(2\langle S^z \rangle - 1)\right](S_f^z - S) - \varepsilon[(S_f^z)^2 - S^2], \tag{6}$$

which, together with conditions (4)–(5), becomes the generally known relation at $\varepsilon = 1$. To calculate the energy of the elementary excitations

(spin waves) up to terms of order ε^2, it is necessary to introduce the Green' s functions

$$G_1^n(g, f) = \left\langle\left\langle S_g^+ \mid S_f^-(S_f^z - \langle S^z \rangle)^n \right\rangle\right\rangle; \tag{7}$$

$$G_2^n(k, g, f) = \left\langle\left\langle (S_k^z - \langle S^z \rangle) S_g^+ \mid S_f^-(S_f^z - \langle S^z \rangle)^n \right\rangle\right\rangle; \tag{8}$$

$$G_3^n(l, k, g, f) = \left\langle\left\langle S_l^- S_k^+ S_g^+ \mid S_f^-(S_f^z - \langle S^z \rangle)^n \right\rangle\right\rangle; \tag{9}$$

$$G_4^n(l, k, g, f) = \left\langle\left\langle (S_l^z - \langle S^z \rangle)(S_k^z - \langle S^z \rangle) S_g^+ \mid S_f^-(S_f^z - \langle S^z \rangle)^n \right\rangle\right\rangle; \tag{10}$$

$$G_5^n(m, l, k, g, f) = \left\langle\left\langle S_m^-(S_l^z - \langle S^z \rangle) S_k^+ S_g^+ \mid S_f^-(S_f^z - \langle S^z \rangle)^n \right\rangle\right\rangle; \tag{11}$$

$$G_6^n(r, m, l, k, g, f) = \left\langle\left\langle S_r^- S_m^- S_l^+ S_k^+ S_g^+ \mid S_f^-(S_f^z - \langle S^z \rangle)^n \right\rangle\right\rangle \tag{12}$$

and to calculate the functions G_2^n and G_3^n from the equations of motion up to terms of order ε inclusive, and the functions G_4^n—G_6^n up to terms of zeroth order in ε. Therefore it is possible to terminate the chain of equations for the Green' s functions. To solve the equations of motion for G_2^n, let us pass to Fourier transforms in time and space. For example:

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$$G_1^n(g,f) = \frac{1}{N}\sum_p \int_{-\infty}^{\infty} dE\, G_1^n(p;E) \exp\left[i(p,q-f) - iE(x_0 - x_0')\right]. \tag{13}$$

If we introduce the mass operator $M_1^n(p;E)$, then the equation for the first Green' s function takes the form

$$\{E - E(p) - M_1^n(p;E)\}G_1^n(p;E) = I_1^n(p), \tag{14}$$

where

$$I_1^n(p) = -\frac{1}{2\pi}\left\langle [S_f^+, S_f^-(S_f^z - \langle S^z\rangle)^n]\right\rangle; \tag{15}$$

$$E(p) = \mu H + [J(0) - J(p)]\langle S^z\rangle. \tag{16}$$

The mass operator $M_1^0(p;E(p))$ can be determined in exactly the same way as in the case $S = \frac{1}{2}$ ([5,7]). Neglecting terms of order $T^{7/2}$ and higher, we obtain for it the expression

$$M_1^0(p;E(p)) = \varepsilon[J(0) - J(p)]p_S^0 + \varepsilon B_1 + \varepsilon^2 B_2 + i\varepsilon^2 B_2', \tag{17}$$

where

$$p_S^0 = \frac{1}{N}\sum_q N_S^0(p) = \frac{1}{N}\sum_p \left\{e^{\beta\widetilde{E}_1^0} - 1\right\}^{-1}; \tag{18}$$

$$\varepsilon B_1 = \frac{\varepsilon}{N}\sum_q \{J(p) + J(q) - J(p-q) - J(0)\}N_S^0(q); \tag{19}$$

$$\varepsilon^2 B_2 = \frac{\varepsilon^2}{2N^2}\sum_{q,q'} P\,\frac{[J(p+q-q') + J(q') - J(p-q') - J(q'-q)]^2}{E(p) + E(q) - E(q'-q) - E(p-q')}\,N_S^0(q); \tag{20}$$

$$\varepsilon^2 B_2' = \frac{\varepsilon^2\pi}{2N^2}\sum_{q,q'}[J(p+q-q') + J(q') - J(p-q')$$

$$- J(q'-q)]^2\,\delta[E(p) + E(q) - E(p-q') - E(q'-q)]\,N_S^0(q); \tag{21}$$

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P is the principal value. The expressions for B_1, B_2, B_2' are close to the expressions obtained by Tyablikov ([5,7]) for the case of spin $S = \frac{1}{2}$. For this reason we do not write out explicit expressions for them in the low-temperature case. Corrections to the spin-wave energy due to the interaction between them are deter-

are the real part of the mass operator

$$\widetilde{E}_1^0 = \widetilde{E}_1^0(p) = E(p) + M_1'^{\,0}(p; \widetilde{E}_1^0) =$$
$$= \mu H + [J(0) - J(p)]S + \varepsilon B_1 + \varepsilon^2 B_2 + O(T^{7/2}), \tag{22}$$

whereas the imaginary part determines their mean lifetime. Calculation of the magnetization by the method proposed by Tyablikov [(3)] for large values of the spin $s \gg 1/2$ requires considerable computations. Therefore we shall construct an approximate solution, using equation (6). With its help one can show that, in the case $s > 1/2$, the magnetization is determined by the expression:

$$\langle S^z \rangle = S - P_S^0 + O(T^{7/2}), \tag{23}$$

whereas in the case $s = 1/2$

$$\langle S^z \rangle = \frac{1}{2} - P_{1/2}^0 + 2(P_{1/2}^0)^2 + O(T^{7/2}). \tag{24}$$

It is easy to show that at low temperatures the first nontrivial correction to the spin-wave approximation (in the expression for $\langle S^z \rangle$) is proportional to T^3 in the case $s = 1/2$ and proportional to T^4 in the case $s > 1/2$. The same result was obtained by other methods by Tahir-Kheli and ter Haar [(2)] and by Callen [(4)].

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