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Abstract

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ON THE COMPRESSION OF AN ANISOTROPICALLY HARDENING PLASTIC LAYER BY ROUGH PLATES

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The compression of an ideally plastic layer by rough plates was considered in ([1–3]). The compression of a strip made of a material obeying the linearized relation of the theory of anisotropic hardening was considered in ([4]). Below a solution of this problem is given for a hardening rigid-plastic material, with the relations of the theory of anisotropic hardening ([5–7]) being adopted.

The loading surface for an anisotropically hardening material, according to ([5–7]), in the case of plane deformation can be represented in the form

$$(\sigma_x - \sigma_y - c\varepsilon_x + c\varepsilon_y)^2 + 4(\tau_{xy} - c\varepsilon_{xy})^2 = 4k^2. \tag{1}$$

Equality (1) is identically satisfied by the substitution

$$\sigma_x = \sigma - k\sin 2\theta + c\varepsilon_x, \qquad \sigma_y = \sigma + k\sin 2\theta + c\varepsilon_y,$$
$$\tau_{xy} = k\cos 2\theta + c\varepsilon_{xy}. \tag{2}$$

Substituting these equalities into the equilibrium equations, we obtain

$$\frac{\partial\sigma}{\partial x} - 2k\left(\cos 2\theta\,\frac{\partial\theta}{\partial x} + \sin 2\theta\,\frac{\partial\theta}{\partial y}\right) + c\left(\frac{\partial\varepsilon_x}{\partial x} + \frac{\partial\varepsilon_{xy}}{\partial y}\right) = 0,$$
$$\frac{\partial\sigma}{\partial y} - 2k\left(\sin 2\theta\,\frac{\partial\theta}{\partial x} - \cos 2\theta\,\frac{\partial\theta}{\partial y}\right) + c\left(\frac{\partial\varepsilon_y}{\partial y} + \frac{\partial\varepsilon_{xy}}{\partial x}\right) = 0. \tag{3}$$

Applying to (1) the associated flow law, we have

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \qquad 2\frac{\partial u}{\partial x}\operatorname{ctg} 2\theta + \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = 0, \tag{4}$$

where u and v are the projections of the velocity vector on the axes x and y.

In what follows, when considering the deformation process, the Almansi strain tensor ([7]) is used, for which we have

$$\frac{\partial \varepsilon_x}{\partial t} + \frac{\partial \varepsilon_x}{\partial x} u + \frac{\partial \varepsilon_x}{\partial y} v = \frac{\partial u}{\partial x}(1 - 2\varepsilon_x) - 2\varepsilon_{xy}\frac{\partial v}{\partial x},$$
$$\frac{\partial \varepsilon_y}{\partial t} + \frac{\partial \varepsilon_y}{\partial x} u + \frac{\partial \varepsilon_y}{\partial y} v = \frac{\partial v}{\partial y}(1 - 2\varepsilon_y) - 2\varepsilon_{xy}\frac{\partial u}{\partial y},$$
$$\frac{\partial \varepsilon_{xy}}{\partial t} + \frac{\partial \varepsilon_{xy}}{\partial x} u + \frac{\partial \varepsilon_{xy}}{\partial y} v = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) - \varepsilon_x \frac{\partial u}{\partial y} - \varepsilon_y \frac{\partial v}{\partial x}. \tag{5}$$

Consider a plastic layer situated symmetrically along the x-axis between two rough plates approaching each other with constant unit velocity. Let the thickness of the layer at time t be denoted by $2h(t)$; then $h(t) = h(0) - t$. On the plates the magnitude of the tangential stress is assumed constant and equal to T, with $T \ll k$.

In what follows it is convenient to introduce the dimensionless coordinates

$$\alpha = \frac{x}{h(t)}, \qquad \beta = \frac{y}{h(t)}, \qquad t_1 = \frac{h(t)}{h(0)}.$$

When considering the flow of material sufficiently far from the edges of the plate and from the center, one may assume that v is a function only of β,

and θ does not depend on α. Then from (4) it follows that

$$\vartheta = -\beta, \qquad u = \alpha + \varphi(\beta, t_1), \qquad 2\operatorname{ctg} 2\theta + \frac{\partial \varphi}{\partial \beta} = 0. \tag{6}$$

From equations (5) we have

$$\varepsilon_x = \frac{1}{2}(1 - t_1^2), \qquad \frac{\partial \varepsilon_{xy}}{\partial t_1} = -\frac{1}{2}\frac{\partial \varphi}{\partial \beta} t_1,$$

$$t_1 \frac{\partial \varepsilon_y}{\partial t_1} = 1 - 2\varepsilon_y + 2\varepsilon_{xy}\frac{\partial \varphi}{\partial \beta}. \tag{7}$$

Passing in equations (3) to dimensionless coordinates, we obtain

$$\sigma - 2k \sin 2\theta \frac{\partial \theta}{\partial \beta}\alpha + c\frac{\partial \varepsilon_{xy}}{\partial \beta} = f_1(\beta, t_1),$$

$$\sigma + 2k \sin 2\theta + c\varepsilon_y = f_2(\alpha, t_1).$$

Comparing these equalities, we obtain

$$c\frac{\partial \varepsilon_{xy}}{\partial \beta} - 2k \sin 2\theta \, \frac{\partial \theta}{\partial \beta} = \mu(t_1). \tag{8}$$

For the hydrostatic pressure we have

$$\sigma = -\mu_1(t_1)\alpha - k \sin 2\theta - \nu_1(t_1). \tag{9}$$

Integrating equality (9), we have

$$\tau_{xy} = k \cos 2\theta + c\varepsilon_{xy} = \mu(t_1)\beta + \chi(t_1). \tag{10}$$

Taking into account that for $\beta = \pm 1$ the quantity $\tau_{xy} = \pm T$, we obtain

$$\mu(t_1) = T, \qquad \chi(t_1) = 0, \qquad c\varepsilon_{xy} + k \cos 2\theta = T\beta. \tag{11}$$

Eliminating the quantities ε_{xy} and $\varphi(\beta, t_1)$ from equalities (6), (7), and (11), we obtain

$$t_1 \operatorname{ctg} 2\theta = \frac{2k}{c} \sin 2\theta \, \frac{\partial \theta}{\partial t_1}.$$

Integrating this equation, we obtain

$$\frac{c}{2k} t_1^2 = \frac{1}{2} \ln \frac{1 + \sin 2\theta}{1 - \sin 2\theta} - \sin 2\theta + f(\beta).$$

Taking into account that for $t_1 = 1$, $\cos 2\theta = T\beta/k$, we obtain

$$\begin{gathered} \frac{c}{2k}(t_1^2 - 1) = \frac{1}{2} \ln \frac{1 + \sin 2\theta}{1 - \sin 2\theta} - \sin 2\theta \\ -\frac{1}{2} \ln \frac{1 - \sqrt{1 - \beta_1^2}}{1 + \sqrt{1 - \beta_1^2}} - \sqrt{1 - \beta_1^2} \qquad \left(\beta_1 = \frac{T\beta}{k}\right). \end{gathered} \tag{12}$$

Equation (12) determines the dependence of θ on β and t. After θ has been determined, the quantity ε_{xy} is determined from equation (11).

From equations (7) we have

$$t_1 \frac{\partial \varepsilon_y}{\partial t_1} + 2\varepsilon_y = 1 - \frac{4\varepsilon_{xy}}{t_1} \frac{\partial \varepsilon_{xy}}{\partial t_1}.$$

Fig. 1

Figure 1: Fig. 1

Fig. 2

Figure 2: Fig. 2

Integrating this equation, we obtain

$$\varepsilon_y = \frac{t_1^2 - 1}{2t_1^2} - 2\frac{\varepsilon_{xy}^2}{t_1^2}. \tag{13}$$

where in equality (13) it has been taken into account that for $t_1 = 1$ the material is in an undeformed state, i.e. $\varepsilon_y = \varepsilon_{xy} = 0$.

We obtain the pressure distribution over the plate from (2); we have

$$\sigma_y = -Ta - v(t_1) - c\frac{1 - t_1^2}{2t_1^2} - 2c\frac{\varepsilon_{xy}^2}{t_1^2}. \tag{14}$$

To determine the quantity $v(t_1)$, note that if the edge of the plate $x = 0$ is free, then the stresses σ_x in the vertical section must be balanced by the tangential stresses at the bases of the layer, i.e.

Fig. 1

Fig. 2

$$\int_0^1 \sigma_x \, d\beta + \int_0^a \tau_{xy}\big|_{\beta=1} \, d\alpha = 0. \tag{15}$$

For σ_x, from relations (2) and (9) we have

$$\sigma_x = -Ta - 2k \sin 2\theta - v_1(t_1) + \frac{c}{2}(1 - t_1^2).$$

Taking into account that $\tau_{xy}\big|_{\beta=1} = T$, from equation (15) we obtain

$$v(t_1) = -2k \int_0^1 \sin 2\theta \, d\beta + \frac{c}{2}(1 - t_1^2). \tag{16}$$

The dependence of $v(t_1)/k$ on t_1 for various T/k is shown in Fig. 1. Figure 2 presents the dependence of ε_{xy} on time for $\beta = 1$.

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REFERENCES

1. L. Prandtl, in: *Collected Translations. Theory of Plasticity*, IL, 1950.
2. V. V. Sokolovskii, *Theory of Plasticity*, 1950.
3. R. Hill, *Mathematical Theory of Plasticity*, 1956.
4. V. V. Dudukalenko, D. D. Ivlev, DAN, 152, No. 5 (1963).
5. W. Prager, ZAMM, 15, H. 1/2 (1935); in: *Collected Translations. Theory of Plasticity*, IL, 1948.
6. Yu. I. Kadashevich, V. V. Novozhilov, *Applied Mathematics and Mechanics*, 22, issue 1 (1958).
7. A. Yu. Ishlinskii, *Ukrainian Mathematical Journal*, 6, No. 3 (1954).
8. W. Prager, *Introduction to Mechanics of Continua*, IL, 1963.

Note: Figure translations are in progress. See original paper for figures.