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## Abstract

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# RADIATION OF GRAVITATIONAL WAVES BY BODIES MOVING IN THE FIELD OF A COLLAPSING STAR

Recently, the conclusion has been gaining ever wider acceptance that the final stage in the evolution of stars with mass considerably exceeding the solar mass is unlimited contraction–collapse ([1,2]). In this case, for an external observer the radius of the star asymptotically (but very rapidly) tends to the gravitational radius $r_g = 2GM/c^2$. For the analysis of processes that may occur near the surface of a contracting star, it is of interest to consider the dynamics of the motion of bodies in the field of this star with allowance for the effects of the general theory of relativity. In the present note the radiation of gravitational waves by a body of small mass $m$, moving in the spherically symmetric field of a large mass $M$, is considered, as is the influence of this radiation on the motion of the mass $m$ and the possible observable effects caused by such gravitational radiation.

Radiative gravitational friction causes the appearance of a force acting on the body. This force is caused by the interaction of the mass $m$ with its own gravitational field and therefore is proportional to $m^2$, in contrast to the force of interaction with an external gravitational field, proportional to $m$. Thus, the change in the motion of the body as a consequence of the radiation of gravitational waves may be regarded as a small correction to the motion under the action of the force of the external field, when $m/M \ll 1$.

The standard theory of gravitational radiation is applicable only to processes in a weak gravitational field. However, the estimates in order of magnitude given below should also be valid for the motion of $m$ near the gravitational radius of $M$*. The constant of gravitation $G$, characterizing the smallness of the gravitational interaction, enters in an obvious way into the formula for the power of gravitational radiation. When bodies approach one another under the action of mutual gravitation to distances of the order of the sum of their gravitational radii, the total amount of radiated energy must be a function only of their masses, $G$, and $c$. From dimensional considerations it follows at once that $G$ cannot enter the formula. Thus, the total radiation of gravitational energy is not small! Additional considerations of symmetry and correspondence with the formulas for $M \gg m$ make it possible immediately to write the sought formulas to within a dimensionless constant not greatly different from unity.

The question of the role of the radiation of gravitational waves in the collapse of supermassive stars was posed by I. S. Shklovskii and N. S. Kardashev ([3]) (see also ([10])).

The problem of the motion of a test body in a Schwarzschild gravitational field has long been solved and analyzed in detail (see, for example, ([4–6])). The general picture of the motion of a test particle is easily represented by considering the differential equation of the trajectory ([7]):

$$d\varphi = \frac{a\,dr}{r^2\sqrt{E^2 - (1 + a^2/r^2)(1 - 1/r)}},$$

* Analogously to the way in which a charge moving uniformly in a circle with velocity $v \simeq c$ radiates mainly in the visible harmonics, the radiation of gravitational waves at $v \simeq c$ should have the same features. However, in the case under consideration the problem is posed near the gravitational radius itself, where the radiation is cut off by effects of the general theory of relativity. For $r$ somewhat substantially exceeding $r_g$, the indicated effects do not change the order-of-magnitude estimates.

where all quantities are dimensionless: $r$ is the distance measured in units of $r_g$; $a$ is the angular momentum measured in units of $mcr_g$; $E$ is the energy measured in units of $mc^2$.

The vanishing of the expression in the denominator corresponds to the replacement of approach toward the center by recession. Equating this expression to zero, we find $\dot{E} = E(r, a)$. This dependence is shown in Fig. 1. The motion of a test body having, at a given $r_0$, energy $E_0$ and angular momentum $a_0$ is represented by the horizontal line drawn through $r_0, E_0$ until it meets the corresponding curve $\dot{E} = E(r, a_0)$, or until $r = r_g$ and $r = \infty$ (see Fig. 1). Stable circular orbits correspond to minima of the curves, unstable ones to maxima. From the figure it is evident that a test body which at infinity has a velocity $v_\infty$ negligible in comparison with $c$, i.e. $E = 1$, will fall onto $r_g$ for $a < 2$, will asymptotically approach $r = 2$ for $a = 2$, and for $a > 2$ will again go off to infinity. In the Newtonian two-body problem gravitational capture is impossible; in general relativity the capture cross section of a test particle $\sigma_{\text{capt}}$, in our units, is $4\pi(c/v_\infty)^2$. Let us note that the fact of gravitational capture of a test particle possessing finite angular momentum, and its eventual fall onto the gravitational radius, apparently indicates that the rotation of a massive star cannot prevent its gravitational collapse. This is an additional argument in favor of Hoyle' s idea ([1]) that in general relativity, in contrast to Newtonian theory, the rotation of large masses cannot prevent their gravitational collapse. Stable circular orbits are possible only for $r > 3$, correspondingly with $a > \sqrt{3}$, $v_{\text{circ}} < c/2$, and $E = \sqrt{8/9} = 0.943$.

The classical picture considered is altered to a considerable degree by radiative gravitational friction. To take this effect into account, let us consider gravitational radiation in the motion of $m$ in the field of $M$. Let $m$ approach $M$ from a large distance, where its velocity is $v_\infty$. Denote by $r_{\min}$ the minimum distance from $M$ of the points of the flyby trajectory of $m$. The number of revolutions which the particle, arriving from infinity, makes near $r_{\min}$ for $a$ close to 2 is given by the asymptotic formula $N = -\ln(a-2)/2^{1/2}\pi$. Thus, already for $a$ only substantially greater than 2 (and hence $r_{\min} > 2$) the particle does not make many revolutions. We shall consider motion along trajectories satisfying this requirement.

According to the character of the gravitational radiation, the motion is divided into three stages: 1) the velocity of $m$ has not yet changed greatly under the influence of the attraction of $M$; in this case $r > 2GM/v_\infty^2 \equiv r_0$; 2) the velocity of $m$ changes substantially, $r_0 > r \gg r_{\min}$; 3) flight at the vertex of the trajectory, $r \sim r_{\min}$.

The radiation power, calculated by the usual formulas ([7]), and the total amount of radiated energy at the different stages are as follows:

$$1)\quad \frac{d\varepsilon}{dt} = 0.03\,\frac{v_\infty^5}{G}\left(\frac{m}{M}\right)^2\left(\frac{r_g}{r}\right)^4\left(\frac{v_\infty}{c}\right)^5, \qquad \Delta\varepsilon = 0.2\,\frac{v_\infty^2 m^2}{M}\left(\frac{v_\infty}{c}\right)^5;$$

$$2)\quad \frac{d\varepsilon}{dt} = 0.03\,\frac{c^5}{G}\left(\frac{m}{M}\right)^2\left(\frac{r_g}{r}\right)^5, \qquad \Delta\varepsilon = 0.02\,\frac{c^2 m^2}{M}\left(\frac{r_g}{r}\right)^{3.5};$$

$$3)\quad \frac{d\varepsilon}{dt} = 0.4\,\frac{c^5}{G}\left(\frac{m}{M}\right)^2\left(\frac{r_g}{r}\right)^5, \qquad \Delta\varepsilon = \frac{c^2 m^2}{M}\left(\frac{r_g}{r}\right)^{3.5}.$$

Thus, almost all the energy is radiated at the vertex of the trajectory, and the characteristic radiation time is $\Delta T = r_{\min}^{3/2}/\sqrt{2GM}$. The loss of energy through radiation leads to the body being captured by the mass $M$ for values of $a$ considerably exceeding $a = 2$, at which capture occurs in the purely mechanical problem. Taking radiation into account, the critical

the values of $a$ and $\sigma_{\text{capt}}$ are determined as follows. For $\chi = \dfrac{c^2}{v_\infty^2}\dfrac{m}{M} > 100$

$$a_{\text{capt}} = (2\chi)^{1/7}, \qquad \sigma_{\text{capt}} = \pi\left(c/v_\infty\right)^2(2\chi)^{2/7};$$

for $\chi < 1$

$$a_{\text{capt}} = 2 + e^{-20/\chi}, \qquad \sigma_{\text{capt}} = 4\pi\left(c/v_\infty\right)^2\left(1 + e^{-20/\chi}\right).$$

For $v \simeq 10^6$ cm/sec, $m/M \simeq 0.1$, $\chi = 10^8$, we obtain $a_{\text{capt}} \simeq 10$. As a result of capture the body is carried away from $M$ not to infinity, but to a distance

Figure 1 diagram

Figure 1: Figure 1 diagram

$$L \simeq \frac{r_g}{2\left[\frac{m}{M}\left(\frac{r_g}{r_{\min}}\right)^{3.5} - \frac{v_\infty^2}{2c^2}\right]}.$$

For $v_\infty = 0$ and $r_{\min} \simeq 3$, $L \simeq 600 r_g$.

When a body $m$ moves with $a = 0$, the radiation is determined by formulas (2); the main part of the energy is radiated at $r \sim r_g$: $\Delta\varepsilon_\parallel = \alpha c^2 m^2/M$, where $\alpha \lesssim 0.01$, and the radiation time is $\Delta T \sim r_g/c$. The expression for $\Delta\varepsilon_\parallel$ is valid only for $m/M \ll 1$, but it also gives an order-of-magnitude estimate for $m/M \sim 1$. In view of the fact that in this case the formula must be symmetric with respect to $m$ and $M$, its structure must be as follows:

$$\Delta\varepsilon_\parallel = \beta \frac{c^2 m^2 M^2}{(m+M)^3}, \tag{1}$$

where $\beta$ is of the same order as $\alpha$.

Circular motion* is represented by the minima of the curves in Fig. 1. As a result of gravitational radiation, the point representing the motion moves on the diagram along the minima of the curves, reaching the critical circle with $r = 3$ and $a = \sqrt{3}$. The energy for motion along this circle is $\sqrt{8/9}$ of the energy when the body is formed at a large distance. Consequently, the total amount of radiated energy $\Delta\varepsilon_\perp = 0.06 mc^2$ and does not depend on the mass of the central body. In one revolution on the critical circle, an energy $\Delta\varepsilon_\perp = 0.1 c^2 m^2/M$ is radiated. The body, continuing to lose energy, approaches the attracting center and passes to an orbit which spirals toward the gravitational radius, no longer because of loss of energy due to radiation, but as the orbit of a test particle with $r < 3$ in the purely mechanical problem (see Fig. 1). Before passing to this "mechanical" spiral orbit, after reaching $r = 3$, $m$ makes near this critical circle $0.1(M/m)^{1/3}$ revolutions, approaching the center still due to gravitational radiation. Then, already along the spiral "mechanical" orbit, it approaches the gravitational radius, making another $\sim (M/m)^{1/3}$ revolutions. $\Delta\varepsilon_\perp$ per revolution remains all the time of the same order as at $r = 3$. Thus, after reaching the critical orbit the body falls to the sphere of the gravitational radius, adding practically nothing to the energy already radiated before this, if $m/M \ll 1$. If, however, $m/M \sim 1$, then the number of revolutions after reaching $r = 3$ is of order unity, and the radiated energy is of the same order as before reaching the critical orbit. Proceeding from the same considerations that were used for the deri-

**Fig. 1.** 1 $-a = 0$; 2 $-a = \sqrt{3}$; 3 $-a = 2$, 4 $-a = \sqrt{6}$, $r \to \infty$, $E \to 1$; $a \to \infty$, $r_{\max} \to 3/2$, $E_{\max} \to \infty$

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* Radiation during motion along an ellipse is considered in detail in the work of Peters and Mathews (8). We note that, owing to the much more intense radiation at pericenter, the eccentricity of the orbit will decrease with time.

from formula (1), one can obtain a formula for the total amount of radiated energy $\Delta\varepsilon_\perp$ in the finite motion of two masses comparable in magnitude:

$$\Delta\varepsilon_\perp = \gamma \frac{c^2 mM}{m+M},$$

where $\gamma$ is of order 0.06, i.e., of the same order as $\alpha$ and $\beta$.

Thus, as a result of gravitational radiation the system loses no more than a few percent of its energy. Returning to the question posed by I. S. Shklovskii and N. S. Kardashev on the role of gravitational radiation in processes associated with the collapse of superstars, we note that this radiation is clearly insufficient for the Michel mechanism ([9]) to operate—the ejection of the outer envelope associated with a sharp weakening of the gravitational field of the superstar' s core. Apparently, the electromagnetic radiation of superstars and the birth of cosmic rays are connected with the processes described in the work of one of the authors ([2]). In addition, strong variable gravitational fields near moving collapsing masses (in the absence of spherical symmetry in the distribution and motion of matter) may by themselves, irrespective of the system' s emission of gravitational waves, lead to the acceleration of individual clumps of matter and to processes of cosmic-ray production.

Let us briefly dwell on the problem of detecting the gravitational radiation of collapsing stars.

When a mass $m$, moving radially, falls onto $M$, the gravitational radiation in the wave zone, i.e., at distances from the source $R \gg c\Delta T \simeq r_g$, has the form of a pulse of width $r_g$; and in finite motion along an orbit with radius comparable to $r_g$, it has the form of a train of such pulses. Each pulse contains energy $\sim \alpha c^2 m^2/M$. It is easy to estimate the field strength $F$ in the wave:

$$F = \frac{\alpha c^2}{R} \frac{m}{M}.$$

We note that the field depends only on the mass ratio. Detection of gravitational radiation is associated with the differential action of the wave field on test masses separated in space. Consequently, in order to detect the radiation described above, the test masses should be separated by distances $\sim r_g$.

If it is assumed that instruments can register a difference of accelerations with an accuracy up to $10^{-5}$ cm/sec$^2$, then gravitational radiation can be recorded from the collision of two collapsed stars of equal mass occurring at a distance of 300 kiloparsecs ($10^{24}$ cm).

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*Note: Figure translations are in progress. See original paper for figures.*