ON A GENERAL STABILITY CONDITION FOR HIGHER CORRELATION FUNCTIONS IN A PLASMA

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Abstract

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Let us consider the equations describing the behavior of higher correlation functions in a plasma. These equations may be obtained (1) from the equation

$$\frac{\partial \delta N_{a_i}}{\partial t} + v_i \frac{\partial \delta N_{a_i}}{\partial r_i} + \frac{e_{a_i}}{m_{a_i}}\left(E + \frac{v_i}{c} \times H\right)\frac{\partial \delta N_{a_i}}{\partial v_i} +$$

$$+\frac{e_{a_i}}{m_{a_i}}\left(\delta E + \frac{v_i}{c} \times \delta H\right)\frac{\partial \overline{N}_{a_i}}{\partial v_i} + \frac{e_{a_i}}{m_{a_i}}\left[\left(\delta E + \frac{v_i}{c} \times \delta H\right)\frac{\partial \delta N_{a_i}}{\partial v_i} - \overline{\left(\delta E + \frac{v_i}{c} \times \delta H\right)\frac{\partial \delta N_{a_i}}{\partial v_i}}\right] = 0, \tag{1}$$

where E and H are the mean electric and magnetic fields, satisfying Maxwell' s equations, while by δE and δH in what follows we shall mean the expressions obtained from δN_{a_i} by the Coulomb and Biot-Savart formulas:

$$\delta E(r) = -\sum_{a_i} \int \frac{\partial}{\partial r'} \frac{e_{a_i}}{|r - r'|}\, \delta N_{a_i}\, d^6 x_i,$$

$$\delta H(r) = \sum_{a_i} \int \frac{\partial}{\partial r} \frac{e_{a_i}}{|r - r'|} \frac{v_i}{c}\, \delta N_{a_i}\, d^6 x_i, \tag{2}$$

where x_i denotes the aggregate $v_i r_i$. In the expressions (2) for $\delta E, \delta H$, retardation is not taken into account. By multiplying and averaging (1) from equalities (1), using (2), we obtain

$$\frac{\partial}{\partial t}\overline{\delta N_{a_1} \ldots \delta N_{a_s}} + \sum_{i=1}^{s} v_i \frac{\partial}{\partial r_i}\overline{\delta N_{a_1} \ldots \delta N_{a_s}} + \sum_{i=1}^{s} \frac{e_{a_i}}{m_{a_i}}\left(E + \frac{v_i}{c} \times H\right) \times$$

$$\times \overline{\delta N_{a_1} \ldots \delta N_{a_s}} - \sum_{i} \frac{e_{a_i}}{m_{a_i}} \sum_{a_{s+1}} \int \left[\frac{\partial}{\partial r_i} \frac{e_{a_{s+1}}}{|r_i - r_{s+1}|}\left(1 - \frac{v_i v_{s+1}}{c^2}\right) + \right. \tag{3}$$

$$+\frac{v_{s+1}}{c}\left(\frac{v_i}{c}\frac{\partial}{\partial r_i}\frac{e_{a_{s+1}}}{|r_i - r_{s+1}|}\right)\Bigg]\frac{\partial}{\partial v_i}\Big[\overline{N_a}\,\overline{\delta N_{a_i}\dots\delta N_{a_{i-1}}\delta N_{a_{i+1}}\dots\delta N_{a_{s+1}}}+$$

$$+\overline{\delta N_{a_1}\dots\delta N_{a_{s+1}}}-\overline{\delta N_{a_i}\delta N_{a_{s+1}}}\,\overline{\delta N_{a_1}\dots\delta N_{a_{i-1}}\delta N_{a_{i+1}}\dots\delta N_{a_s}}\Big]\,d^6x_{s+1}=0.$$

Obviously,

$$\overline{\delta N_{a_1}\dots\delta N_{a_m}}=\overline{N_{a_1}\dots N_{a_m}}-\sum_i\overline{N_{a_i}}\,\overline{N_{a_1}\dots N_{a_{i-1}}N_{a_{i+1}}\dots N_{a_s}}+\dots, \qquad (4)$$

where the ellipsis replaces terms containing more than two factors. We shall regard all arguments x_i $(i = 1, \dots, s)$ as distinct. In the integral terms of equality (3) one must take into account the terms containing δ-functions that arise when x_{s+1} coincides with the other arguments. In this case

$$\overline{N_{a_1}\dots N_{a_s}}=F_{a_1\dots a_s}+\sum_{i=1}^{s}\delta_{a_i a_{s+1}}\delta(x_i-x_{s+1})F_{a_1\dots a_s}. \qquad (5)$$

Let us express the distribution functions $F_{a_1\dots a_m}$ in terms of the irreducible correlation functions by formulas (2):

$$F_{a_1\dots a_m}=g_{a_1\dots a_m}+\sum_{(a_1\dots a_m)}{}' g_{a_1\dots}g_{\dots a_m}+\dots, \qquad (6)$$

where the second term represents the sum of products of pairs of correlation functions corresponding to all possible partitions of the indices $a_1\dots a_m$ into two groups. The ellipsis replaces terms containing more than two factors.

We shall now obtain equations for $g_{a_1\dots a_s}$, using the fact that the correlation functions tend to zero when any pair of spatial arguments is separated: $|r_i - r_j| \to \infty$.

In equation (3) let us first separate all arguments so that, for any pair i, j, $|r_i - r_j| \to \infty$. Then the sum of terms that do not tend to zero under such separation will be equal to zero by virtue of the equation. These terms may be crossed out in equation (3). Next, partitioning the arguments $r_1, \dots, r_s$ in an arbitrary way into $s-1$, $s-2$, and so on groups, we shall separate these groups of arguments without changing the relative arrangement of the arguments within the groups. The terms thereby obtained may be crossed out in equation (3). Finally, using equalities (4), (5), (6), we obtain for $g_{a_1\dots a_s}$ an equation each term of which tends to zero if $|r_i - r_j| \to \infty$ for at least one pair i and j:

$$
\begin{aligned}
&\frac{\partial}{\partial t} g_{a_1 \ldots a_s} + \sum_i v_i \frac{\partial}{\partial r_i} g_{a_1 \ldots a_s} + \sum_i \frac{e_{a_i}}{m_{a_i}} \left(E + \frac{v_i}{c} \times H \right) \frac{\partial}{\partial v_i} g_{a_1 \ldots a_s} \\
&\quad - \sum_{i,j;\, i \neq j} \frac{e_{a_i}}{m_{a_i}} \left[\frac{\partial}{\partial r_i} \frac{e_{a_j}}{|r_i - r_j|} \left(1 - \frac{v_i v_j}{c^2} \right) + \frac{v_i}{c} \left(\frac{v_i}{c} \frac{\partial}{\partial r_i} \frac{e_{a_j}}{|r_i - r_j|} \right) \right] \\
&\quad \times \frac{\partial}{\partial v_i} \left(g_{a_1 \ldots a_s} + \sideset{}{'}\sum_{(a_1 \ldots a_s)} g_{a_i \ldots} g_{a_j \ldots} \right) \\
&\quad - \sum_i \frac{e_{a_i}}{m_{a_i}} \sum_{a_{s+1}} \int \left[\frac{\partial}{\partial r_i} \frac{e_{a_{s+1}}}{|r_i - r_{s+1}|} \left(1 - \frac{v_i v_{s+1}}{c^2} \right) + \frac{v_{s+1}}{c} \left(\frac{\partial}{\partial r_i} \frac{e_{a_{s+1}}}{|r_i - r_{s+1}|} \right) \right] \\
&\quad \times \frac{\partial}{\partial v_i} \left(g_{a_1 \ldots a_{s+1}} + \sideset{}{'}\sum_{(a_1 \ldots a_{s+1})} g_{a_i \ldots} g_{a_{s+1} \ldots} - F_{a_{s+1}} g_{a_1 \ldots a_s} \right) d^6 x_{s+1} = 0.
\end{aligned}
\tag{7}
$$

By

$$
\sideset{}{'}\sum_{(a_1 \ldots a_s)} g_{a_i \ldots} g_{a_j \ldots}
$$

is denoted the sum of products of pairs of correlations corresponding to an arbitrary partition of the indices $a_1 \ldots a_s$ into

two groups, so that a_i and a_j do not fall into the same group. The expression

$$
\sideset{}{'}\sum_{(a_1 \ldots a_{s+1})} g_{a_i} \ldots g_{a_{s+1}} \ldots
$$

has an analogous meaning. If the mean field and δH are not taken into account, then system (7) coincides with system (3) of work [2]. In the fourth term of equation (7) the expression with $g_{a_1 \ldots a_s}$ is of the order of the plasma parameter μ relative to the last term. Taking $\mu \ll 1$, we shall neglect this quantity. Then the homogeneous part of equation (7) can be represented in the form

$$
\frac{\partial}{\partial t} g_{a_1 \ldots a_s} + \sum_i \sum_b L_{a_i b} g_{a_1 \ldots a_{i-1} b a_{i+1} \ldots a_s} = 0, \tag{8}
$$

where $\sum_b L_{a_i b}$ is an operator appearing in the linearized collisionless equation for the first distribution function

$$\frac{\partial f_{a_i}}{\partial t} + v_i \frac{\partial f_{a_i}}{\partial r_i} - \frac{e_{a_i}}{m_{a_i}}\left(E + \frac{v_i}{c}\times H\right)\frac{\partial f_{a_i}}{\partial v_i} - \frac{e_{a_i}}{m_{a_i}}\frac{\partial F_{a_i}}{\partial v_i}\sum_b \int \left[\frac{\partial}{\partial r_i}\frac{e_b}{|r_i - r'|}\left(1-\frac{v_i v'}{c^2}\right) + \frac{v'}{c}\left(\frac{v_i}{c}\frac{\partial}{\partial r_i}\frac{e_b}{|r_i-r'|}\right)\right] f_b(v',r')\,d^6x' = \frac{\partial f_{a_i}}{\partial t} + \sum_b L_{a_i b} f_b = 0, \tag{9}$$

in which the additions to the electromagnetic field are calculated by the Coulomb and Biot-Savart formulas (as in equalities (2)). Hence it follows that the product of solutions of equations (9) is a solution of equations (8), and that the Green' s function of system (8) is the product of the Green' s functions of equations (9). Thus, the behavior in time (growth or damping) of solutions of system (8) is determined by the properties of the solutions of system (8).

Consider some solution $G_{a_1 \ldots a_s}$ of system (7). We shall assume that, as in the stationary case, $G_{a_1 \ldots a_s} \sim \mu^{s-1}$. We shall study the behavior in time of the deviation from this solution $f_{a_1 \ldots a_m} = g_{a_1 \ldots a_m} - G_{a_1 \ldots a_m}$, regarding this deviation as small. If the initial data for the system (7) linearized (in f) are chosen so that at the initial moment only the s-th correlation is different from zero, then, analogously to how this was done in work [2], it can be shown that, accurate to terms of order μ, the function $f_{a_1 \ldots a_s}$ will satisfy system (8). Since arbitrary initial data can be represented as a superposition of the above initial data taken with different s, the question of the stability of solutions of equations (7) reduces to the question of the stability of the trivial solution of system (9).

Let us note that the result remains valid if δE and δH are calculated by exact formulas with allowance for retardation. In this case the fourth term will be absent in equation (7), while in the fifth term, in addition to integration over dx_{s+1}, integration will be carried out over $g_{a_1 \ldots a_{s+1}}(x_1, t; \ldots x_s, t;\ x_{s+1}, t')$ with respect to $t' < t$.

In equation (9), in the last term, which can be written in the form

$$\frac{e_{a_i}}{m_{a_i}}\frac{\partial F_{a_i}}{\partial v_i}\left(\delta E + \frac{v_i}{c}\times \delta H\right),$$

the electromagnetic field δE, δH will be calculated by exact formulas through f_{a_i}.

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CITED LITERATURE

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