S. P. LOMNEV

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Abstract

Full Text

S. P. LOMNEV

A VARIANT OF A MAGNETIC TRAP

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The confinement and accumulation of hot plasma is one of the fundamental problems of the thermonuclear problem ([1]). It is known from experiments that the traps now being used, of the "magnetic bottle" type, have not made it possible to accumulate particles to the required concentration: attempts to confine them in the transverse direction lead to losses in the longitudinal direction, and conversely. In some cases a sharp increase in concentration may lead to both types of losses.

As calculations have shown, these difficulties can be overcome to a considerable extent by using a time-increasing magnetic field of the type ([2]):

$$\mathbf{H} = \operatorname{grad} R_0 \sum_{n=1}^{\infty} \left(\frac{R_0}{r}\right)^{n+1} \sum_{m=0}^{\infty} \left(g_n^m \cos m\varphi + h_n^m \sin m\varphi\right) P_n^m(\cos\theta), \qquad (1)$$

where $P_n^m(\cos\theta)$ are the associated Legendre polynomials, and g_n^m and h_n^m are expressions determining the value of the magnetic field at $r = R_0$.

Since practical reproduction of (1) is very difficult, taking $n = 1,\ m = 0$, we obtain

$$\mathbf{H} = \frac{3R_0^3(\mathbf{H}_0\mathbf{r})\,\mathbf{r}}{r^5} - \left(\frac{R_0}{r}\right)^3 \mathbf{H}_0, \qquad (2)$$

i.e., the field of a magnetic dipole.

Some examples of the motion of nonrelativistic electrons in the magnetic field (2) have been considered earlier ([3]). For heavy particles with trajectories lying in a volume of order 1 m^3, calculations were not carried out. Here the motion of hydrogen ions is considered (see Table 1) with temperatures close to those used for producing thermonuclear reactions.

Table 1

x_0, cm	$\dot{z}_0$	r_{max}, cm	r_{min}, cm	z_{max}, cm	${}^1/_2\ T_z$	$\Delta\varphi$	T_φ	$\dot{\varphi}$
$H_0R_0^3 = -{}^5/_3$ 1840 erst·m^3	$H_0R_0^3 = {}^5/_3$ 1840 erst·m^3	$H_0R_0^3 = {}^5/_3$ 1840 erst·m^3	$H_0R_0^3 = {}^5/_3$ 1840 erst·m^3	$H_0R_0^3 = {}^5/_3$ 1840 erst·m^3	$H_0R_0^3 = {}^5/_3$ 1840 erst·m^3	$H_0R_0^3 = {}^5/_3$ 1840 erst·m^3	$H_0R_0^3 = {}^5/_3$ 1840 erst·m^3	$H_0R_0^3 = {}^5/_3$ 1840 erst·m^3
75	0,02	77,2	45,3	30	2700	0,32	8000	$0,4 \cdot 10^{-4}$
75	0,015	76,5	38,9	29,7	3700	0,30	12 000	$0,25 \cdot 10^{-4}$
75	0,01	76,2	28,9	29,3	5200	0,14	18 800	$0,7 \cdot 10^{-5}$
50	0,02	50,6	17,6	19,6	1800	0,14	6400	$0,22 \cdot 10^{-4}$
$H_0R_0^3 = -{}^1/_3$ 1840 erst·m^3	$H_0R_0^3 = {}^1/_3$ 1840 erst·m^3	$H_0R_0^3 = {}^1/_3$ 1840 erst·m^3	$H_0R_0^3 = {}^1/_3$ 1840 erst·m^3	$H_0R_0^3 = {}^1/_3$ 1840 erst·m^3	$H_0R_0^3 = {}^1/_3$ 1840 erst·m^3	$H_0R_0^3 = {}^1/_3$ 1840 erst·m^3	$H_0R_0^3 = {}^1/_3$ 1840 erst·m^3	$H_0R_0^3 = {}^1/_3$ 1840 erst·m^3
75	0,005	85,4	51	31,6	1200	0,35	33 000	$0,11 \cdot 10^{-5}$
50	0,02	65	38,7	22,8	1900	0,54	4400	$0,12 \cdot 10^{-3}$
50	0,015	52,6	37,6	22,4	3400	0,45	5800	$0,78 \cdot 10^{-4}$
50	0,01	50,95	33,7	21,9	5200	0,34	10 200	$0,33 \cdot 10^{-4}$
50	0,005	50,6	20,7	20,7	7200	0,18	23 600	$0,76 \cdot 10^{-5}$
25	0,02	25,5	10,3	9,74	900	0,20	3000	$0,67 \cdot 10^{-4}$
25	0,015	25,16	8,27	9,53	1200	0,12	4600	$0,28 \cdot 10^{-4}$
$H_0R_0^3 = -{}^5/_3$ 184 erst·m^3	$H_0R_0^3 = {}^5/_3$ 184 erst·m^3	$H_0R_0^3 = {}^5/_3$ 184 erst·m^3	$H_0R_0^3 = {}^5/_3$ 184 erst·m^3	$H_0R_0^3 = {}^5/_3$ 184 erst·m^3	$H_0R_0^3 = {}^5/_3$ 184 erst·m^3	$H_0R_0^3 = {}^5/_3$ 184 erst·m^3	$H_0R_0^3 = {}^5/_3$ 184 erst·m^3	$H_0R_0^3 = {}^5/_3$ 184 erst·m^3
40	0,015	45,5	30,6	17,9	1600	0,47	4200	$0,11 \cdot 10^{-3}$
40	0,01	41,6	27,9	17,3	3600	0,41	8500	$0,48 \cdot 10^{-4}$
40	0,005	41,4	19,8	16	5700	0,22	18 500	$0,12 \cdot 10^{-4}$
25	0,02	25,5	16	10	1200	0,27	2400	$0,11 \cdot 10^{-3}$
25	0,015	25,4	13,5	9,9	1300	0,24	3600	$0,65 \cdot 10^{-4}$
25	0,01	25,3	10,3	9,74	1800	0,16	6000	$0,27 \cdot 10^{-4}$

Figure 1

Figure 1: Figure 1

The results of the study of the motions of a single particle are as follows:

1. The field of a magnetic dipole is capable of retaining and accumulating heavy particles, and its magnitude is practically attainable.
2. The flight time of particles between the poles T_z and the magnitude of the oscillation in r can be regulated by means of the magnetic-field strength, the radius, and the injection energy.
3. For effective retention it is most advantageous to inject particles with the following initial values:

$$x_0 \neq 0, \qquad y_0 = 0; \qquad \dot{x}_0 \simeq \dot{y}_0 \simeq 0; \qquad \dot{z}_0 = V.$$

4. $\dot{z}_0$ is determined by the conditions for carrying out a thermonuclear reaction. As $\dot{z}_0$ increases, $z_{\max}$, $r_{\min}$, and $r_{\max}$ (the values of the oscillating functions) increase, while the period of oscillations $z(t)(T_z)$ and the mean angular velocity of rotation about the polar axis $\omega = \Delta\varphi/T_\varphi$ decrease.

Fig. 1. Example of dependences $r(t)$, $H_0 R_0^3 = -1840$ oersted $\cdot\,\mathrm{m}^3$. a—increasing field, $\omega = 10^{-5}$ sec^{-1}; —growing field, $\omega = 10^{-6}$ sec^{-1}; —decreasing field, $\omega = 10^{-5}$ sec^{-1}.

5. For the selected $\dot{z}_0$, there is a direct dependence between $H_0 R_0^3$ and x_0: as $H_0 R_0^3$ increases, the interval of values of x_0 suitable for retention and accumulation of particles increases. The upper value of x_0 is determined from the condition of particle retention, the lower one by the value of $r_{\min}$ ensuring insulation of the field-producing device.
6. Introducing an additional magnetic field, increasing in time, of the type

$$H_g = H_{g0} \sin \omega t, \tag{3}$$

we obtain the necessary conditions for particle capture, since $r(t) < r_0$ (see Fig. 1, a,) and the particles will not fall on the injector. At the same time the frequency and amplitude of the oscillations $r(t)$ decrease and, most importantly, $r_{\max} - r_{\min}$ decreases, i.e., the particle concentration can be increased. Simultaneously, heating by the field $E_\varphi = -\omega H_{g0} \cos \omega t$ will occur. The constant characterizing the change in H_g is determined by the accumulation time, and the value of H_{g0} by the dimensions of the installation.

If the strength of the main magnetic field decreases ($H_0 = H \cos \omega t$), the particles slow down and the amplitude of the oscillations $r(t)$ increases (Fig. 1,).

Fig. 2

Figure 2: Fig. 2

7. In practice, the dipole field can be produced by a current loop of finite radius a:

$$A_\varphi = I \int_0^\pi \frac{a \cos\varphi \, d\varphi}{\left(a^2 + \rho^2 + z^2 - 2a\rho\cos\varphi\right)^{1/2}}, \tag{4}$$

or

$$A_\varphi = C_0 \left\{ 1 - \frac{r}{a} P_1(\cos\theta) + \frac{1}{2}\left(\frac{r}{a}\right)^3 P_3(\cos\theta) - \dots \right.$$

$$\left. \dots + (-1)^{n+1} \frac{1 \cdot 3 \dots (2n-1)}{2 \cdot 4 \dots 2n} \left(\frac{r}{a}\right)^{2n+1} P_{2n+1}(\cos\theta) + \dots \right\} \quad \text{for } r < a;$$

(5)

$$A_\varphi = C_0 \left\{ \frac{1}{2}\frac{a^2}{r^2} P_1(\cos\theta) - \frac{3}{8}\frac{a^4}{r^4} P_3(\cos\theta) - \dots \right.$$

$$\left. \dots + (-1)^{n+1} \frac{1 \cdot 3 \dots (2n-1)}{2 \cdot 4 \dots 2n} \left(\frac{a}{r}\right)^{2n} P_{2n-1}(\cos\theta) + \dots \right\} \quad \text{for } r > a.$$

The field of such a loop will differ somewhat from a dipole field. As the calculations have shown, for $a/R_0 < 1/4$ this deviation may be neglected.

8. The introduction of an additional field of the type

$$H_r = \sum_{n=1}^{\infty} \frac{n(n+1)}{2n+1} C_n \left(\frac{a}{r}\right)^{n+2} P_n(\cos\theta), \tag{6}$$

$$H_\theta = -\sum_{n=1}^{\infty} \frac{n}{2n+1} C_n \left(\frac{a}{r}\right)^{n+2} P_n'(\cos\theta)$$

greatly narrows the region of $\dot{z}_0$ suitable for confinement, although for particular combinations of C_n we obtain sufficiently good trajectories (Fig. 2).

Fig. 2. Examples of projections of particle trajectories in the $r\theta$ plane, $C_1 = -1840$ erg $\cdot$ m^3.
a–$C_2 = -184$ erg $\cdot$ m^3; b–$C_3 = -184$ erg $\cdot$ m^3; c–$C_4 = -184$ erg $\cdot$ m^3; d–

$C_5 = -184$ erg $\cdot$ m^3; e—$C_6 = -184$ erg $\cdot$ m^3; f—$C_6 = C_2 = C_3 = C_4 = C_5 = -184$ erg $\cdot$ m^3. For comparison, the dashed line shows the dependence in the $r\theta$ plane for $C_2 = C_3 = C_4 = C_5 = C_6 = 0$.

9. An additional magnetic field of type (4) in the pole region at $z \simeq x_0$ reduces the possibility of particles penetrating through the plane of the main turn.

10. By using an additional magnetic field of type (4) in the injection region ($z_1 = 0$; $a_1 > x_0$), with a moment antiparallel to the moment of the main dipole and smaller by an order of magnitude, we improve the conditions for particle confinement at the initial instant of time.

11. Let us place, on the surface of a sphere of radius $a > x_0$, current-carrying rings ($\theta = \text{const}$). If the moments of these rings are parallel to the magnetic dipole, the condi-

initial confinement deteriorate, $r_{\min}$ increases; with antiparallel alignment, the opposite occurs. With moments alternating in sign, the picture is close to the original one, i.e., the case of one turn at the center.

22. Fields produced by a straight current along the polar axis (i.e., of the type $H_\varphi = C/(x^2 + y^2)^{1/2}$, where C is a constant) worsen the conditions for particle confinement.

Although the influence of the Coulomb interaction was not taken into account, the character of the motion when particles are concentrated near the poles, i.e., in the region of large magnetic-field strengths, while at the equator their densities are insignificant, gives grounds to hope that this difficulty will not be fundamental.

Fig. 3 Fig. 4

Fig. 3. Example of dependences $\bar{r}(t)$. a—$H_{g0} = 0$; b—$H_{g0} \neq 0$, $\omega = 10^{-5}$ sec^{-1}; v—without allowance for the interaction

Fig. 4. Example of the distribution of particles by velocity. $H_0 R_0^3 = -1840$ oerst $\cdot$ m^3. a—$t = 1000$; b—$t = 2000$; v—$t = 3500$

Indeed, an analysis of the motion of many enlarged charges (4) with initial conditions taken as type I and II from (4), for $N \simeq 1000$, made it possible to conclude:

1. Losses of particles of one sign (ions) begin at densities of the order of 10^{11} particles/cm^3 and occur both at the center, on the dipole wires, and on the chamber walls in the equatorial region.

2. The mean value of the radius

$$\bar{r} = \sum_{i=1}^{N} r_i / N$$

of the volume occupied by the particles increases with time (Fig. 3).

3. Use of a field of type (3) noticeably reduces the growth of $\bar{r}(t)$ (Fig. 3, b).
4. The energy spectrum of the particles broadens with time (Fig. 4). Particles from the tails of the distribution are lost: those with high velocities strike the chamber wall, and those with low velocities strike the dipole at the center.

Computing Center
Academy of Sciences of the USSR

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Note: Figure translations are in progress. See original paper for figures.