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Abstract

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GEOPHYSICS

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A Nonstationary One-Dimensional Model of a Convective Cloud

(Presented by Academician E. K. Fedorov, 29 I 1964)

The principal quantities characterizing a convective cloud include water content (volumetric or specific), the vertical velocity, and the temperature of the cloudy air. These quantities are mutually related and are connected with the state of the air surrounding the cloud. In solving the stationary problem ([1,2]) of a convective cloud, these relations are taken into account rather fully; however, not all of the principal quantities enter into the solution of the nonstationary problem ([3]). The nonstationary problem can be solved by taking into account not only the factors included in the solution of the stationary problem, but also certain others. One may additionally take into account the gravitational fall of drops ([4]), turbulent mixing in the vertical direction, consider, besides the processes in the cloud, the processes in the subcloud and above-cloud regions, and trace the development of convection up to the formation of a cloud and the evolution of the cloud from its inception to the fall of rain.

Let us find, as functions of height and time, the principal quantities that characterize the convective flow and the cloud contained in it. We shall regard as known the distributions with height of pressure and air density, and of the humidity of the air outside the cloud column, as well as the temperature at the moment convection begins; we shall prescribe the change with time of the air temperature near the ground. The vertical convective flow creating the cloud will be considered cylindrical. We shall consider this cylindrical vertical cloud column, containing the convective flow with a cloud or as yet without a cloud. Let the lower base of the cylinder under consideration be located at ground level, and the upper base at a level where the air no longer participates in convection and throughout the entire period considered has constant temperature and humidity. Let the horizontal dimensions of the cylinder correspond to the horizontal dimensions of the flow and be allowed to vary with time, while the height does not vary. During vertical motions of air in the cloud column, at some sections air from the surrounding space will be entrained into it through

Fig. 1

Figure 1: Fig. 1

Fig. 2

Figure 2: Fig. 2

the lateral surface, while at other sections air will be expelled. The entrained air influences the processes in the cloud column; the expelled air does not. We shall characterize the air in the column by quantities averaged over the area of the horizontal cross section of the column.

Introduce the notation: z is the vertical coordinate, directed upward, with origin at ground level; t is time; P is atmospheric pressure; ρ is air density; T and Q are, respectively, the temperature (absolute) and the specific moisture content of the air in the cloud column; T_c and Q_c are the same quantities outside the cloud column; u and W are the specific water content and the vertical velocity of the air in the cloud column; E is the saturated-vapor pressure at temperature T; η is the mean-weighted fall velocity of droplets; g is the acceleration of gravity; k is the coefficient of turbulence in the vertical direction; c_p is the heat capacity of air at constant pressure; L is the specific heat of condensation; H is the value of the coordinate z at the level of the upper base of the cylinder under consideration. The quantities P,

P, Q_c are specified as functions of z and will be regarded as independent of time: $P = P(z)$, $\rho = \rho(z)$, $Q_c = Q_c(z)$.

The specific water content in the cloud, assuming the vapor there to be saturated, can be determined from the known relation

$$u = Q - 0.622\,\frac{E}{P}. \tag{1}$$

The quantity u is positive in the cloud and negative outside it. Processes in the cloud differ from processes occurring before the formation of the cloud, beneath

Fig. 1

Fig. 2

the cloud and above the cloud. To take this difference into account, we introduce the function

$$\sigma = \begin{cases} 1, & \text{for } u > 0, \\ 0, & \text{for } u \le 0. \end{cases} \tag{2}$$

In order to take account of the influence of entrained air, we introduce the function

$$\mu = \begin{cases} \dfrac{1}{\rho}\dfrac{\partial(\rho W)}{\partial z}, & \text{for } \dfrac{\partial(\rho W)}{\partial z} > 0, \\ 0, & \text{for } \dfrac{\partial(\rho W)}{\partial z} < 0. \end{cases} \tag{3}$$

The quantity $\dfrac{1}{\rho}\dfrac{\partial(\rho W)}{\partial z}$ is the ratio of the mass of air entrained per unit time into a thin horizontal layer of the cloud column to the mass of air contained in this layer. We take the mean-weighted fall velocity of drops in the form [4]

$$\eta = \sigma B\left(1 - e^{-au\rho}\right), \tag{4}$$

where B and a are constants. The dependence of saturated-vapor pressure on temperature, as is known, can be represented in the form

$$E = E_0 \exp \frac{\alpha_1 T - \alpha_2}{T - \alpha_3}, \tag{5}$$

where E_0, α_1, α_2, α_3 are constants. Calculation of the moisture balance in an elementary horizontal layer of the cloud column leads to the equation

$$\frac{\partial Q}{\partial t} = -W\frac{\partial Q}{\partial z} - \mu(Q - Q_c) + \frac{1}{\rho}\frac{\partial(\eta u\rho)}{\partial z} + k\frac{\partial^2 Q}{\partial z^2}. \tag{6}$$

Here the transport of moisture by the vertical air flow is taken into account by the first term on the right-hand side, mixing of cloud air with entrained air by the second term, gravitational transport of drops by the third term (η depends on z through u_0), and turbulence by the last term ($k = \text{const}$).

We shall regard the air surrounding the cloud column as at rest, and write the equation of motion of the air in the form

$$\frac{\partial W}{\partial t} = -W\frac{\partial W}{\partial z} + g\frac{T - T_c}{T_c} - \sigma g u - \mu W + k\frac{\partial^2 W}{\partial z^2}. \tag{7}$$

The first term on the right-hand side of this equation represents the transport of velocity by the vertical air flow; the second, the Archimedean force; the third, the force

Fig. 3

Fig. 4

Fig. 3

Figure 3: Fig. 3

Fig. 4

Figure 4: Fig. 4

of gravity of the cloud water; the fourth, the braking of the flow by the entrained air; and the last, turbulent transport of velocity.

Changes in temperature are described by the equation:

$$\frac{\partial T}{\partial t} = -W\frac{\partial T}{\partial z} - \frac{1}{c_p + 0.622\,\sigma \frac{L}{P}\frac{\partial E}{\partial T}}\left[gW\left(1 + 0.622\frac{\sigma L_p E}{P^2}\right) + \right.$$

$$\left. + c_p\mu(T - T_c) + \sigma L\mu\left(0.622\frac{E}{P} - Q_c\right) - c_p k\frac{\partial^2 T}{\partial z^2} - 0.622\sigma Lk\frac{\partial^2}{\partial z^2}\left(\frac{E}{P}\right)\right]. \quad (8)$$

The first term on the right-hand side of this equation gives the transport of temperature by the vertical air flow; in the square brackets: the first term represents the dry-adiabatic and moist-adiabatic processes, the second the heating of the entrained air to the temperature of the air in the cloud column, the third the heat of evaporation of cloud water in the entrained air, the fourth the turbulent inflow of heat, and the last the heat of condensation of vapor redistributed in the cloud owing to turbulent exchange. Introduction of the function σ (2) makes it possible to apply the equations given both to the cloud and to the subcloud and above-cloud regions, and also in calculating the processes before the cloud forms. We shall assume that changes in the temperature of the air outside the cloud column occur only owing to turbulent mixing,

$$\frac{\partial T_c}{\partial t} = k\frac{\partial^2 T_c}{\partial z^2}. \quad (9)$$

The system of 9 equations (1)–(9), 4 of which are differential, contains 9 unknown functions: $u, \sigma, \mu, \eta, E, Q, W, T, T_c$. It was solved under the following boundary conditions: $T(0,t) = T_c(0,t) = T_0 + A\sin\omega t$, where T_0, A, ω are prescribed constants; $T(H,t) = T_c(H,t) = T$, where $T = \text{const}$; $Q(0,t) = Q_c(0)$, $Q(H,t) = Q_c(H)$, $W(0,t) = W(H,t) = 0$. The initial instant was taken to be the moment of the onset of convection, when the air in the cloud column did not differ from the surrounding air; the vertical velocities were zero everywhere, except for a small lower part of the cloud column, where a small velocity impulse was prescribed. Accordingly, the initial conditions were taken in the form: $T(z,0) = T_c(z,0) = f(z)$, where $f(z)$ is a prescribed function; $Q(z,0) = Q_c(z)$; $W(z,0) = 0$ for $z > 2h$, $W(z,0) = Dz$ for $0 < z < h$, $W(z,0) = D(2h - z)$ for

$h < z < 2h$; here D and h are prescribed constants, $h \ll H$. The temperature $f(z)$ was prescribed so that in the lower layers the vertical temperature gradient was close to adiabatic and so that subsequently, as the temperature near the ground increased, convection could develop up to sufficiently great heights, but not reach the height H. Corresponding to $f(z)$, P and ρ were prescribed in accordance with the barometric formula and the equation of state of a gas.

Integration of the system of equations was carried out by a numerical method on the BESM-2 electronic computer*. For the calculation of the example, the following values of the constants were taken: $g = 9.81$ m/sec^2, $c_p = 10^3$ J/kg $\cdot$ deg, $L = 2.5 \cdot 10^6$ J/kg, $k = 20$ m^2/sec, $B = 9$ m/sec, $a = 150$ m^3/kg, $E_0 = 6.108$ mb, $\alpha_1 = 17.57$, $\alpha_2 = 4798$°K, $\alpha_3 = 31.1$°K, $T_0 = 300$°K, $A = 2$°K, $\omega = \pi/21600$ sec^{-1}, $T = 236$°K, $H = 10^4$ m, $h = 10^2$ m, $D = 10^{-3}$ sec^{-1}, $P(0) = 10^3$ mb.

The results of the calculation of the vertical velocity W, the volumetric water content $\mu\rho$, the temperature T, and the excess of the temperature in the cloud column over the temperature of the surrounding air, $T - T_c$, are given respectively in Figs. 1, 2, 3, 4. Here the height above ground level is plotted vertically, and time horizontally; the curves are isolines of the calculated quantities; the numbers on the curves give $\mu\rho$ in g/m^3, W in m/sec, and T and $T - T_c$ in °C. In Fig. 2, corresponding to the heights, the dew-point values outside the cloud column are marked. Using the graphs presented, it is easy to find both the distribution of the corresponding quantity with height at any chosen instant and the change of this quantity with time at a given height. The isoline $\mu = 0$ in Fig. 2 encloses the region of existence of the cloud (positive values of water content), which arose half an hour after the onset of convection. In this region, in all figures the isolines are drawn as solid lines; in the region where there is no cloud, as dashed lines. The regions where $W = 0$ (Fig. 1) and $T - T_c = 0$ (Fig. 4) are hatched. The calculation was carried out up to the beginning of rainfall (4 hours 30 minutes), i.e., up to the moment when the water content became positive below the condensation level. The set of calculated quantities presented in the graphs makes it possible to find any other of the 9 sought functions by means of the simple relations entering into the system of equations.

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