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Abstract

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PHYSICS

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ACCOUNTING FOR DAMPING IN WEAK INTERACTIONS

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As is known, the theory of the 4-component neutrino, developed in works ([1]), makes it possible to explain the nonconservation of parity in processes involving a neutrino of definite orientation of the neutrino spin. In this theory the neutrino is described by the Dirac equation with zero rest mass. Two solutions with $\varepsilon = 1$ ($\varepsilon = -1$) describe the electron neutrino with left helicity (the electron right-handed antineutrino $\bar{\nu}_R$). The other two solutions with $\varepsilon = 1$ ($\varepsilon = -1$) correspond to the muon neutrino ν'_R (the muon antineutrino $\bar{\nu}'_L$). In the 4-component theory, positive leptonic charge is assigned to e^-, μ^+, ν_L, ν'_R, and negative charge to e^+, μ^-, $\bar{\nu}_R$, $\bar{\nu}'_L$. In the present work we consider νe-scattering (or $\nu'\mu$-scattering) in the 4-component theory according to the damping theory ([2]). The amplitude of this process, calculated in the lowest order of perturbation theory, leads to divergent results at high energies, since the total cross section grows proportionally to the neutrino energy in the laboratory system. This indicates that at sufficiently large energies ($E_\nu \geq 10^3$ BeV in the center-of-inertia system (c.i.s.)) the use of perturbation theory becomes illegitimate, since the weak interaction ceases to be weak ([3]). This difficulty does not arise in the damping theory. As shown in ([4]), the solution of the equations of the damping theory for scattering amplitudes is equivalent to the summation of a series of chain diagrams in which the propagation functions of intermediate particles are replaced by δ-functions. This series can be summed in the region of its convergence $GE_\nu^2 < 1$, and the amplitude obtained can be regarded as the analytic continuation of the series into the region $GE_\nu^2 > 1$.

We note that the summation of the series is simplified if one uses the expansion of the process amplitude in Wigner d-functions ([5]), thanks to which it is possible to decompose the matrix element into orthogonal combinations, as is necessary for solving the equations of the damping theory. The amplitude obtained differs from the amplitude in perturbation theory by the presence of a denominator, which leads to the fact that partial cross sections never exceed unity.

The relativistically invariant amplitude (Fig. 1) depends on the variables $w = E + p$ and $z = \dfrac{(p, p_{\nu'})}{p^2}$. The matrix element of νe-scattering, corresponding to the $V - A$ interaction, in the lowest order of perturbation theory can be written in the following form in the c.i.s.:

$$M^{(1)}(w, z) = -\sum_{j=V,A} G_j (b^+_{e'} O_j b_e)(b^+_{\nu'} O_j b_\nu)(2p)(2E), \tag{1}$$

where $\hbar = c = 1$, $p_{\nu,e}$, $s_{\nu,e}$ are the momenta and helicities of the incident and scattered (the latter will be denoted by primes) neutrino and electron, $E = \sqrt{p^2 + m^2}$ is the electron energy, and m is its mass. Coeffi-

coupling constants $G_V = -G_A = G\sqrt{2}$; $O_V = (i\alpha, I)$, $O_A = (\vec{\sigma}, i\rho_1)$ are Dirac matrices; b are spinor amplitudes (see (6)). Let us expand the matrix elements in d-functions:

$$M^{(1)\,s'_\nu s'_e}_{s_\nu s_e}(w, z) = \frac{1}{4\pi^2 \xi} \sum_J (2J + 1)\, C^{s'_\nu s'_e}_{J\, s_\nu s_e}(w)\, d^J_{\frac{s_\nu - s_e}{2},\, \frac{s'_\nu - s'_e}{2}}(z), \tag{2}$$

$$\xi = \frac{1}{4(2\pi)^3} \frac{p}{w};$$

$$C^{-1-1}_{J\,-1-1}(w) = C^{11}_{J\,11}(w) = -\frac{G\sqrt{2}}{\pi}\, p^2 \delta_{J0}, \tag{3}$$

while the remaining C_J are equal to zero, i.e., only the S-wave participates in the scattering.

Multiplying two terms $M^{(1)}$ and then summing over helicities, as well as integrating over the angles of the intermediate particles, one can obtain the expansion in d-functions of the diagram shown in Fig. 2 (the cross on a line means replacement of the propagation function of the corresponding particle by a δ-function):

Fig. 1 **Fig. 2**

$$M^{(2)\,s'_\nu s'_e}_{s_\nu s_e}(w, z) = \frac{1}{4\pi^2 \xi} \sum_J (2J + 1) \left[C^{s'_\nu s'_e}_{J\, s_\nu s_e}(w)\right]^2 d^J_{\frac{s_\nu - s_e}{2},\, \frac{s'_\nu - s'_e}{2}}(z). \tag{4}$$

Repeating this procedure n times, we obtain the expansion of the diagram with n loops, in which all propagation functions are replaced by δ-functions:

$$M^{(n)\,s'_\nu s'_e}_{s_\nu s_e}(w, z) = \frac{1}{4\pi^2 \xi} \sum_J (2J + 1) \left[C^{s'_\nu s'_e}_{J\, s_\nu s_e}(w)\right]^n d^J_{\frac{s_\nu - s_e}{2},\, \frac{s'_\nu - s'_e}{2}}(z).$$

The solution of the damping-theory equation is obtained by summing the series (4)

$$M^{s'_\nu s'_e}_{s_\nu s_e}(w,z) = \sum_{n=1}^{\infty} i^{\,n-1} M^{(n)\,s'_\nu s'_e}_{s_\nu s_e}(w,z) =$$

$$= \frac{1}{4\pi^2\xi}\sum_J (2J+1)\frac{C^{s'_\nu s'_e}_{J\,s_\nu s_e}(w)}{1-iC^{s'_\nu s'_e}_{J\,s_\nu s_e}(w)} d^J_{\frac{s_\nu - s_e}{2},\,\frac{s'_\nu - s'_e}{2}}(z). \tag{5}$$

The total cross sections for scattering of a right- and left-handed neutrino by an unpolarized electron in the c.m.s. are equal to

$$\sigma = \frac{4G^2p^2}{\pi}\,\frac{1}{1+2G^2p^4/\pi^2}. \tag{6}$$

It follows from (6) that perturbation theory is inapplicable at neutrino energies

$$E_\nu = \left(\frac{\pi^2}{2G^2}\right)^{1/4} \simeq 10^3 \text{ BeV}$$

and, as $E_\nu \to \infty$, the cross section decreases to zero.

Let us pass to antineutrino scattering on an electron. The matrix element of $\bar\nu e$-scattering in the lowest order of perturbation theory has the form

$$M^{(1)}(w,z) = -G\sqrt{2}\{b^+_{e'}Ovb_e b^+_{\bar\nu}Ovb_{\bar\nu'} - b^+_{e'}O_A b_e b^+_{\bar\nu}OAb_{\bar\nu'}\}(2p)(2E). \tag{7}$$

For antineutrino scattering ($\varepsilon = -1$) we shall have:

$$C^{1}_{J\,11}(w) = C^{J-1}_{J\,-1-1} = \frac{G}{\pi\sqrt{2}}p^2\left(\frac{E-p}{w}\right)\left(\delta_{J0} - \frac{1}{3}\delta_{J1}\right),$$

$$C^{1-1}_{J\,1-1}(w) = C^{-11}_{J\,-11} = -\frac{G}{\pi\sqrt{2}}\frac{2}{3}p^2\delta_{J1}, \tag{8}$$

$$C^{-11}_{J\,-1-1}(w) = C^{-1-1}_{J\,-11} = -C^{-1}_{J\,11} = -C^{11}_{J\,1-1} = -\frac{G}{\pi\sqrt{2}}p^2\left(\frac{m}{w}\right)\frac{\sqrt{2}}{3}\delta_{J1}.$$

The scattering amplitude in the damping theory is given by the expression

$$M^{s'_{\bar\nu},\,s_{e'}}_{s_{\bar\nu}\,s_e}(w,z) = \frac{1}{4\pi^2\xi}\sum_J (2J+1) M^{s'_{\bar\nu},\,s_{e'}}_{J\,s_{\bar\nu}\,s_e}(w) d^J_{\frac{s'_{\bar\nu}-s_e}{2},\,\frac{s_{\bar\nu}-s_{e'}}{2}}(z), \tag{9}$$

where M_J satisfies the equation

$$M^{s'_{\bar\nu}, s_{e'}}_{J\, s_{\bar\nu}\, s_e} - i \sum_{s', s''} M^{s'_{\bar\nu}, s_{e'}}_{J\, s'\, s''} C^{s'\, s''}_{J\, s_{\bar\nu} - s_e} = C^{s'_{\bar\nu}\, s_{e'}}_{J\, s_{\bar\nu} - s_e}, \tag{10}$$

which has the approximate solution

$$M^{s'_{\bar\nu}, s_{e'}}_{J\, s_{\bar\nu}\, s_e}(w) = \frac{C^{s'_{\bar\nu}\, s_{e'}}_{J\, s_{\bar\nu} - s_e}(w)}{1 - iC^{11}_{J\,11}(w) - iC^{-1-1}_{J\,1-1}(w)}. \tag{11}$$

The scattering cross sections of both the right (electron) and the left (muon) antineutrino coincide and are equal to:

$$\sigma = \frac{G^2}{\pi}\left(\frac{p}{w}\right)^2 \left\{ \frac{(E-p)^2}{1 + \frac{G^2}{2\pi^2} p^4 \left(\frac{E-p}{w}\right)^2} + \frac{1}{3}\, \frac{(E-p)^2 + 4w^2 + 4m^2}{1 + \frac{G^2}{2\pi^2} p^4 \left(\frac{3E+p}{3w}\right)^2} \right\}. \tag{12}$$

At sufficiently small energies the denominator becomes unity, and (12) goes over into the well-known expression for the cross section in perturbation theory, found in ([7,8]).

The difference between the total cross sections for neutrino and antineutrino scattering on an electron is connected with the circumstance that in antineutrino scattering the S- and P-waves must be taken into account.

We now investigate the polarization properties in neutrino scattering on polarized electrons, whose spin in the c.m.s. is directed either along ($s_e = 1$), or opposite to ($s_e = -1$), the initial electron momentum. We have been able to find not only the longitudinal s_3^0 (see (8)), but also the transverse (s_1^0, s_2^0) components of the recoil-electron spin, which in the laboratory system are equal to:

$$s_3^0 = \pm s\, \frac{\beta - \cos\Phi}{1 - \beta\cos\Phi}; \qquad s_1^0 = \pm s\sqrt{1-\beta^2}\, \frac{\sin\Phi}{1 - \beta\cos\Phi}, \tag{13}$$

while the component directed perpendicular to the plane of the scattering angle vanishes ($s_2^0 = 0$). Here s is the helicity, for which in the case of a neutrino ($s = s_\nu$) one should take the upper sign (+), and in the case of an antineutrino ($s = s_{\bar\nu}$) the lower sign (−). This helicity does not change in scattering. The velocity β of the recoil electron is related to the electron scattering angle θ and to the energy of the incident particle E (all quantities are taken in the labo-

in the laboratory system) relation:

$$\beta = -\frac{2E(E+m)\cos\theta}{(E+m)^2 + E^2\cos^2\theta}. \tag{14}$$

From (13) it is clear that only in the ultrarelativistic case will the recoil electrons be completely longitudinally polarized. In antineutrino scattering we must put $\Phi = \theta$, while in the case of a neutrino the angle Φ should be taken equal to the sum of the angles of the scattered electron and neutrino. It is interesting to note that neutrinos will not be scattered when the initial spins of the particles are parallel. If, however, they are antiparallel, then in the c.m.s. the electrons also remain longitudinally polarized, preserving their initial helicity (the rotation for the spin in passing from one system to another can be found with the aid of the formulas given in ([6])).

For an antineutrino, however, when the spins of the particles before scattering are parallel, the cross section reaches a maximum, while when they are antiparallel it becomes minimal, decreasing by a factor $m^2/8E_\nu^2$ (E_ν is the antineutrino energy in the c.m.s.). For any longitudinal orientation of the spin of the initial electron ($s_e = \pm 1$), the spin of the recoil electrons will have one and the same direction, determined in the c.m.s. by the relations

$$s_3^0 = -s_\nu \left[1 - \frac{m^2(1-z)}{2p^2(1+z) + \frac{m^2}{2}(1-z)}\right],$$

as well as a transverse component:

$$s_1^0 = s_\nu \frac{pm(1-z^2)^{1/2}}{p^2(1+z) + \frac{m^2}{4}(1-z)}, \qquad s_2^0 = 0.$$

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Note: Figure translations are in progress. See original paper for figures.