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Yu. A. ARKHANGELSKII

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Abstract

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MECHANICS

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NEW PARTICULAR SOLUTIONS OF THE PROBLEM OF THE MOTION OF A HEAVY RIGID BODY ABOUT A FIXED POINT

(Presented by Academician A. Yu. Ishlinskii, 11 V 1964)

In paper ([1]) new particular solutions were found of the Euler–Poisson equations

$$A\frac{dp}{dt} + (C - B)qr = Mg(y_0\gamma'' - z_0\gamma'), \qquad \frac{d\gamma}{dt} = r\gamma' - q\gamma''$$

$$\begin{pmatrix} ABC, \ pqr \\ \gamma\gamma'\gamma'', \ x_0y_0z_0 \end{pmatrix}, \tag{1}$$

under the conditions

$$r_0 \text{ large}, \qquad \gamma_0'' \neq 0, \ \pm 1; \qquad \lim_{r_0\to\infty}(p_0^2 + q_0^2) < \infty \qquad (u_0 = u(t)_{t=0}) \tag{2}$$

and two theorems were formulated concerning the motion about a fixed point of a heavy rigid body set into rapid rotation about the greater or the smaller axis of its ellipsoid of inertia, under the assumption

$$z_0 \neq 0, \qquad \omega \neq \frac{1}{2}\left(\omega^2 = \frac{(A-C)(B-C)}{AB}\right).$$

In paper ([2]), for $\omega = 1/2$, periodic solutions of system (1) were found under conditions (2), satisfying the additional relations

$$A \geq B > C, \qquad p(t,\mu)_{\mu=0} = q(t,\mu)_{\mu=0} = 0 \qquad \left(\mu^2 = \frac{Mg\sqrt{x_0^2 + y_0^2 + z_0^2}}{r_0^2 C}\right).$$

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1. It can be shown that, for $\omega = 1/2$, when the relations

$$s^2 = z_0(C-A)R_1^2 \pm \frac{\sqrt{1-\gamma_0''^2}}{\gamma_0''}\sqrt{x_0^2L_2^2 + y_0^2L_3^2} > 0; \tag{3}$$

$$(x_0^2+y_0^2)(3B-2C) \neq 0; \tag{4}$$

$$R_1^2 = \frac{3(A+B)C - 4AB - 2C^2}{AB(C-B)}, \qquad L_2 = \frac{3B-2C}{4B}, \qquad L_3 = \frac{3B-2C}{8(B-C)}$$

are fulfilled, system (1) under conditions (2) has periodic solutions of period T

$$T = \frac{4\pi}{r_0} - \frac{2\pi}{r_0^3}\left[Ap^2(0,0) + Bq^2(0,0) + 2Mg\left(z_0\gamma_0'' - x_0\sqrt{1-\gamma_0''^2}\right)\right] + \frac{1}{r_0^4}(\cdots),$$

for which

$$p^2(0,0) + q^2(0,0) \neq 0, \tag{5}$$

$$p(0,0) = \pm R_2 s\sqrt{1 \pm s_1}, \qquad q(0,0) = \pm\frac{2CR_2 s}{3B-4C}\sqrt{1 \mp s_1},$$

$$R_2^2 = \frac{MgB\gamma_0''}{(A-C)(A+B-2C)}, \qquad s_1^2 = \frac{x_0^2 l_2^2}{x_0^2L_2^2 + y_0^2L_0^2}.$$

In this case, as the z-axis of the moving coordinate system one chooses an axis for which the inequality $\gamma_0'' > 0$ is satisfied.

The expressions for the Euler angles θ, φ, ψ corresponding to these solutions will be

$$\begin{gathered}
r_0(\theta-\theta_0) = \frac{s_2}{6(C-B)}[\theta_1(t+h) - \theta_1(h)] + \frac{1}{r_0}(\dots),\\
r_0(\psi-\psi_0) = -\frac{Mgz_0}{C}t + \frac{s_2}{6(C-B)\sqrt{1-\gamma_0''^2}}[\psi_1(t+h) - \psi_1(h)] + \frac{1}{r_0}(\dots),\\
\varphi - \varphi_0 = r_0 t + \frac{1}{r_0}(\dots),
\end{gathered} \tag{6}$$

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$$\theta_1(t) = (A-2B+2C)\cos\left({}^3/_2\, r_0 t - \varepsilon\right) - 3(A+2B-2C)\cos\left({}^1/_2\, r_0 t + \varepsilon\right),$$
$$\psi_1(t) = (A-2B+2C)\sin\left({}^3/_2\, r_0 t - \varepsilon\right) - 3(A+2B-2C)\sin\left({}^1/_2\, r_0 t + \varepsilon\right),$$
$$\operatorname{tg}\varepsilon = \pm\sqrt{\frac{1\mp s_1}{1\pm s_1}}, \qquad s_2^2 = p^2(0,0) + \frac{(3B-4C)^2}{4C^2} q^2(0,0),$$
$$r_0 h = \varphi_0 + \frac{\pi}{2} + \frac{1}{r_0}(\ldots).$$

Moreover, when the relation

$$z_0(C-A)R_1^2 \pm \frac{\sqrt{1-\gamma_0''^2}}{\gamma_0''}\sqrt{x_0^2 L_2^2 + y_0^2 L_3^2} \neq 0 \tag{7}$$

is fulfilled, for the cases $A \geq B > C$ and $C > B \geq A$ ($\omega = {}^1/_2$) the system (1) under conditions (2) has periodic solutions of period T

$$T = \frac{4\pi}{r_0} - \frac{4\pi}{r_0^3} Mg\left(z_0\gamma_0'' - x_0\sqrt{1-\gamma_0''^2}\right) + \frac{1}{r_0^4}(\ldots),$$

for which

$$p(0,0) = q(0,0) = 0. \tag{8}$$

The expressions for the Euler angles corresponding to these solutions are determined from formulas (7.4) of paper ([2]).

Let us note that condition (7), for any fixed values of x_0, y_0, z_0 ($x_0^2 + y_0^2 + z_0^2 \neq 0$); A, B, C ($\omega = {}^1/_2,\ A \geq B > C,\ C > B \geq A$) may hold for arbitrary values of θ_0 ($0 < \theta_0 < \pi/2$), with the exception of one value $\theta_0 = \theta_0^*$,

$$\operatorname{tg}\theta_0^* = \frac{|z_0(C-A)|R_1^2}{\sqrt{x_0^2 L_2^2 + y_0^2 L_3^2}}, \qquad \text{if } z_0(x_0^2 + y_0^2)(3B-2C) \neq 0.$$

Condition (3) may hold for all values of θ_0, or on the whole interval $0 < \theta_0 < \pi/2$, or on one of its parts $0 < \theta_0 < \theta_0^*$, $\theta_0^* < \theta_0 < \pi/2$, depending on the sign of the quantity $z_0(C-A)$ and on the sign before the radical in the expression for s^2.

Formulas (6), (7.4) ([2]), which depend on four arbitrary constants θ_0 (in the indicated domains), φ_0, ψ_0, r_0 (r_0 large), make it possible to study the motion of a heavy rigid body in the case under consideration.

2. From the results obtained and the results of paper ([2]), corresponding to the case $\omega = {}^1/_2$, and of paper ([3]), the following theorems follow.

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Theorem 1. The equations (1), under the conditions (2) and the condition $\omega = \frac{1}{2}$, possess periodic solutions satisfying the relations (5) or (8) when the conditions (3), (4), or (7), respectively, are fulfilled.

To these periodic solutions there correspond expressions for the Euler angles (6), (7.4) [2], depending on four arbitrary constants θ_0 (in the indicated domains), φ_0, ψ_0, r_0 (r_0 large).

For any periodic solution of the equations (1) satisfying the conditions (2) and the condition $\omega = \frac{1}{2}$ and the corresponding relations (3), (4), or (7), the expressions for the Euler angles are given by formulas (6), (7.4).

Theorem 2. For the system (1), under the conditions (2) and the condition $\omega = \frac{1}{2}$, to have new periodic solutions to which: 1) there correspond expressions for the Euler angles depending on five arbitrary constants, or 2) the relation (5) is satisfied, it is necessary that

$$z_0(C - A) > 0, \qquad (x_0^2 + y_0^2)(3B - 2C) = 0.$$

In the Kovalevskaya case ($A = B = 2C,\ y_0 = z_0 = 0$), the relations between the moments of inertia satisfy the condition $\omega = \frac{1}{2}$. Therefore, on the basis of the results obtained, we have

Theorem 3. In the Kovalevskaya case, any periodic solutions of the equations (1) under the conditions (2) satisfy the relation $p(0,0)q(0,0) = 0$, and the corresponding expressions for the Euler angles can depend only on four arbitrary constants.

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CITED LITERATURE

[1] Yu. A. Arkhangel' skii, DAN, **158**, No. 2 (1964).
[2] Yu. A. Arkhangel' skii, Prikl. matem. i mekh., **27**, no. 5 (1963).
[3] Yu. A. Arkhangel' skii, Prikl. matem. i mekh., **28**, no. 5 (1964).

Note: Figure translations are in progress. See original paper for figures.

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