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1964

SovietRxiv

Abstract

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THE INFLUENCE OF THE SCATTERING INDICATRIX ON TRANSPARENCY

(Presented by Academician A. A. Lebedev on 6 X 1963)

1. Measurements of transparency in turbid media, especially those containing large particles, are often interpreted inaccurately and lead to erroneous values for the attenuation coefficient. The error in question is associated with neglecting the part of the light scattered by the medium directly into the receiver. The general theory of the problem was studied in [^1] (pp. 149–154).

The task of the present work is to indicate a method for calculating corrections that take the indicatrix into account in transparency measurements and to estimate the magnitude of the effect for specific, typical cases. We assume that we are dealing with a sufficiently transparent medium, so that we shall confine ourselves only to taking single scattering into account. The effect we study has different significance for different forms of the illuminated volume. We shall consider three typical cases.

2. Parallel beam. The beam length is L, and its radius is $R_{\max}$. The intensity at the entrance ($x = 0$) is E_0, and the intensity recorded by the receiver ($x = L$) is E. The attenuation coefficient is usually determined by the formula

$$\alpha = \frac{1}{L} \ln \frac{E_0}{E}. \tag{1}$$

Calculations by formula (1) are inaccurate. In reality, E can be represented as the sum $E = E_1 + E_2$, where E_1 is the theoretically calculated intensity, and E_2 is the scattered "parasitic" intensity. The exact formula for the attenuation coefficient will be

$$\alpha_1 = \frac{1}{L} \ln \frac{E_0}{E_1}. \tag{2}$$

Let us consider the correction coefficient:

$$\Delta\alpha = \alpha_1 - \alpha = \frac{1}{L} \ln(1 + c), \qquad c = \frac{E_2}{E_1}. \tag{3}$$

For E_1 we have:

$$E_1 = E_0 e^{-n\alpha_0 L}; \tag{4}$$

here n is the number of particles in 1 cm^3, and α_0 is the polydisperse scattering coefficient. It is determined through $f(a)$, the function of the droplet-size distribution. For a γ-distribution with mean radius $\bar{a}$ and parameter ν, we have

$$\alpha_1 = n\alpha_0, \qquad \alpha_0 = 2\pi \left(\frac{\nu+2}{\nu+1}\right) \bar{a}^2.] \tag{5}$$

Let us find an expression for E_2. Denote the luminous intensity sent by the element dV in the direction β by $I(\beta) n\, dV$. Let ψ be the angle between the normal to the receiver and the scattered ray of light, and ρ the distance from dV to the receiver. It is not difficult to show that the "parasitic" intensity E_2 will be

$$E_2 = \frac{F_2}{S} = \int_V n \frac{1}{\rho^2} \cos\psi\, I(\beta)\, e^{-n\alpha_0 \rho}\, dV. \tag{6}$$

This is the general formula.

In the case of a parallel beam, introducing cylindrical coordinates (x, R), and representing the scattering indicatrix $I(\beta)$ in the form

$$I(\beta) = I_0 \Phi(\beta), \tag{7}$$

$$I_0 = J \frac{\pi \Gamma(\nu+5) \bar{a}^4}{(\nu+1)^4 \lambda^2 \Gamma(\nu+1)} \tag{8}$$

(I_0 is the intensity of the light scattered directly forward; $\Phi(\beta)$ is the angular distribution of the intensity of the scattered beam), we obtain:

$$E_2 = B' e_2', \qquad B' = \frac{2\pi^3}{\lambda^2} n \bar{a}^4 Y_\nu E_1, \qquad Y_\nu = \frac{\Gamma(\nu+5)}{(\nu+1)^4 \Gamma(\nu+1)}, \tag{9}$$

$$e_2' = e^{n\alpha_0 L} \int_0^L (L-x) e^{-n\alpha_0 x}\, dx \int_0^{R_{\max}} \frac{R \Phi(\beta) e^{-n\alpha_0 \rho}\, dR}{[R^2 + (L-x)^2]^{3/2}}. \tag{10}$$

For $(L - x) \gg R$ we have

$$e_2' \simeq \int_0^L \frac{dx}{(L-x)^2} \int_0^R \Phi(\beta)\, R\, dR. \tag{11}$$

For the quantity c in formula (3), denoted here by us as c', we obtain

$$c' = Ne_2', \qquad N = \frac{2\pi^3}{\lambda^2} n\bar{a}^4 Y_\nu. \tag{12}$$

For analogous quantities in the case of a **point source** (e_2'' and c''), producing a beam uniformly bright over the cross section with divergence angle φ_1, and in the case of a **projector** (e_2''' and c'''), for which the cone of rays converges beyond it at a distance l, we obtain

$$e_2'' = e^{n\alpha_0 L} \int_0^L \frac{(L-x)e^{-n\alpha_0 x} L^2 dx}{x^2} \int_0^{R_{\max}} \frac{R\Phi(\beta) e^{-n\alpha_0 \rho}\, dR}{[R^2 + (L-x)^2]^{3/2}}; \tag{13}$$

$$e_2' \simeq \int_0^L \frac{L^2 dx}{x^2 (L-x)^2} \int_0^{x\varphi_{\max}} \Phi(\beta)\, R\, dR, \qquad c'' = \frac{E_2}{E_1} = Ne_2''; \tag{14}$$

$$e_2''' = e^{n\alpha_0 (L-l)} L^2 \int_0^L \frac{(L-x)}{x^2} e^{-n\alpha_0 (x-l)}\, dx \int_0^{R_{\max}} \frac{R\Phi(\beta) e^{-n\alpha_0 \rho}\, dR}{[R^2 + (L-x)^2]^{3/2}}; \tag{15}$$

$$e_2''' \simeq L^2 \int_l^L \frac{dx}{x^2 (L-x)^2} \int_0^{x\varphi_{\max}} \Phi(\beta)\, R\, dR, \qquad c''' = \frac{E_2}{E_1} = Ne_2'''. \tag{16}$$

3. To calculate the quantities $\Phi(\beta)$ we used the theory developed in ([2–4]). Three cases were considered, $\nu = 2;\ 5$ and 10, three wavelengths $\lambda = 0.5;\ 3$ and 9μ, four values $\bar{a} = 2.5;\ 5;\ 10$ and 20μ, for a total of 108 experimental models.

The length of the illuminated region was taken equal to 100 m, the divergence angle φ equal to 1.5°. The radius of the parallel beam and the radius of the projector were taken equal to 1 m. For the quantity l in observations with a projector this gave 38.2 m; accordingly, in this case $L = 138.2$ m.

The quantities e_2 were determined numerically by double integration over x and over R. The trapezoidal formula was used. The region in x was divided into 20 intervals, 5 m each. For 19 intervals (counting from the source) the integral over R was found for each cross section, assuming the condition $(L - x) \gg R$. For the last 5-meter interval, the exact formulas were used.

4. The results of the calculations for $\nu = 5$ are presented in Table 1 in the form of values of η—the ratio of the theoretical attenuation coefficient to the optically measured one,

$$\eta = \frac{\alpha_1}{\alpha} = \frac{\alpha_1}{\alpha_1 - \Delta\alpha}.$$

Table 1

Fig. 1

Figure 1: Fig. 1

λ, μ	$\bar{a}$, μ	Parallel beam	Point source	Projector	λ, μ	$\bar{a}$, μ	Parallel beam	Point source	Projector
0.5	2.5	1.28	1.42	1.59	3	10	1.16	1.24	1.31
0.5	5	1.51	1.54	1.62	3	20	1.16	1.19	1.21
0.5	10	1.52	1.40	1.44	9	2.5	1.00	1.02	1.01
0.5	20	1.26	1.20	1.20	9	5	1.01	1.04	1.06
3	2.5	1.01	1.04	1.05	9	10	1.02	1.04	1.06
3	5	1.08	1.17	1.24	9	20	1.01	1.01	1.01

Let us note the variation of η with λ. For $\lambda = 0.5\,\mu$, the value of η, depending on a, has values 1.20–1.62. On passing into the infrared region, η practically becomes equal to unity. For the characteristic size $a = 5\mu$, $\eta \simeq 1.6$. The structure of the fog, i.e., the value v, does not greatly affect η. Nor is the change of η with a change in the geometry of the experiment very significant. With a change in $\bar{a}$, the value of η for all three experimental schemes passes through a maximum. This can also be understood qualitatively. For the other dependences, a qualitative explanation is hardly possible: the value η is the result of the influence of a large number of different, often oppositely acting, factors.

Fig. 1. α_1' and α' –parallel beam for $L = 100$ m, $R = 1$ m, $v = 5$, $\bar{a} = 5\mu$, $n = 120$; (α_1'' and α'' –parallel beam for $L = 100$ m, $R = 1$ m, $v = 5$, $\bar{a} = 5\mu$, $n = 60$

5. The value η also depends on the concentration of droplets, i.e., on the water content of the cloud, and, importantly, η varies sharply with wavelength in the visible region of the spectrum. The values of η were calculated for a parallel beam with $L = 100$ m and $R = 1$ m, $v = 5$, $\bar{a} = 5\mu$; for five wavelength values $\lambda = 0.3$; 0.5; 0.7; 1.0 and 1.5μ; and four concentration values $n = 30$; 60; 90 and 120 droplets per 1 cm^3. The results of the calculations are given in Table 2. For $n = 30$, the value of η decreases from 1.74 at $\lambda = 0.3\,\mu$ to $\eta = 1.17$ at $\lambda = 1.5\,\mu$; the variation of η as a function of droplet concentration is likewise significant.

Table 2

n, cm^{-3}	λ, μ: 0.3	λ, μ: 0.5	λ, μ: 0.7	λ, μ: 1.0	λ, μ: 1.5	n, cm^{-3}	λ, μ^3: 0.3	λ, μ^3: 0.5	λ, μ^3: 0.7	λ, μ^3: 1.0	λ, μ^3: 1.5
30	1.74	1.51	1.38	1.27	1.17	90	1.55	1.41	1.32	1.23	1.16
60	1.63	1.45	1.34	1.25	1.16	120	1.49	1.37	1.29	1.22	1.15

Taking account of the scattering indicatrix leads to a sharp change in the spectral attenuation coefficient in the direction of increasing the anomaly of the curve $\alpha(\lambda)$ (see Fig. 1), i.e., to better transparency for blue rays.

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Received
5 X 1963

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Note: Figure translations are in progress. See original paper for figures.