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Fig. 1

Figure 1: Fig. 1

Abstract

Full Text

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A THREE-LAYER RECTANGULAR PLATE IN A SUPERSONIC GAS FLOW

The equations of work ([1]) are used to investigate, in the linear formulation, the stability of a thin elastic supported rectangular three-layer plate with a rigid core, loaded in the initial plane by normal and tangential forces and streamlined on one side by a supersonic gas flow. The plate has a structure asymmetric through the thickness; each of its layers takes up the action of tensile (or compressive) and bending loads, while the core, in addition, experiences transverse shear. It is assumed that the tangential displacements in the plate vary according to a linear law through the thickness and are expressed as gradients of a certain potential function. Application of the Bubnov method leads to a homogeneous system of algebraic equations. The eigenvalues of the matrix are analyzed by the Danilevsky and Routh methods. Values are given for the critical flow velocity in the case of a rectangular plate as a function of its geometric and physicomechanical parameters, as well as of the magnitude of the compressive and shearing forces in the initial surface; in particular, the influence of the stiffness of the core on transverse shear is assessed.

Fig. 1

1. Formulation of the problem. Small transverse vibrations of thin elastic three-layer plates of a structure asymmetric through the thickness, with a rigid core taking up transverse shear, are determined by the deflection function χ, which satisfies the differential equation ([1])

$$D\left(1-\frac{\vartheta h^2}{\beta}\nabla^2\right)\nabla^4\chi-\left(1-\frac{h^2}{\beta}\nabla^2\right)\left[N_{11}\frac{\partial^2\chi}{\partial x^2}-2N_{12}\frac{\partial^2\chi}{\partial x\,\partial y}+N_{22}\frac{\partial^2\chi}{\partial y^2}\right]+q=0. \tag{1,1}$$

Here the notation of ([1]) has been retained.

The transverse pressure q depends on the deflection w in the following way:

$$q = (q_1 + q_3)\frac{\partial w}{\partial t} + q_2\frac{\partial w}{\partial x} + \Omega\frac{\partial^2 w}{\partial t^2}, \qquad w = \left(1 - \frac{h^2}{\beta}\nabla^2\right)\chi, \tag{1,2}$$

where

$$q_1 = \frac{\chi p}{c}, \qquad q_2 = \chi p M, \qquad q_3 = 2\sum_{s=1}^{3}\varepsilon_s\rho_s h_s, \qquad \Omega = \sum_{s=1}^{3}\rho_s h_s; \tag{1,3}$$

p, c are the pressure and speed of sound in the undisturbed hypersonic flow moving with velocity V; $M = V/c$ is the Mach number; χ is the ratio of the specific heats of the gas; ε_s, ρ_s are, respectively, the coefficient of internal damping and the specific density of the material of the s-th layer; t is time. The last term of formula (1.3) determines the transverse inertia of the plate; the second term is the structural damping, proportional to the deflection velocity.

If a, b are the dimensions of the plate in plan, then, for simply supported edges, the following boundary conditions may be written:

for $x = 0, a$, for $y = 0, b$,

$$\chi = \nabla^2\chi = \nabla^2\nabla^2\chi = 0. \tag{1.4}$$

2. Solution of the initial equation

We seek the solution of the initial equation (1.1), taking (1.2) into account, under the boundary conditions (1.4), in the form (ψ is a complex quantity)

$$\chi = e^{\psi t}\sum_{k=1}^{n}\sum_{l=1}^{m} A_{kl}\sin\frac{k\pi x}{a}\sin\frac{l\pi y}{b}. \tag{2.1}$$

Introduce the dimensionless quantities (ρ is the density of the undisturbed gas flow)

$$\xi = \frac{x}{a}, \qquad \eta = \frac{y}{b},$$

$$\alpha = \frac{a}{b}, \qquad H = \frac{h}{a},$$

$$\omega = -\frac{a}{c}\psi,$$

Fig. 2

Figure 2: Fig. 2

$$n_{11} = \frac{N_{11}}{N_{11}^*},$$

$$n_{22} = \frac{N_{22}}{N_{22}^*}, \qquad n_{12} = \frac{N_{12}}{N_{12}^*},$$

$$\Gamma = \frac{1}{\rho}\sum_{s=1}^{3} \rho_s t_s, \qquad W = \sum_{s=1}^{3} \varepsilon_s \rho_s t_s,$$

$$k_{11} = \frac{a^2 N_{11}^*}{\pi^2 D}, \qquad k_{22} = \frac{b^2 N_{22}^*}{\pi^2 D}, \qquad k_{12} = \frac{a^2 N_{12}^*}{\pi^2 D}, \qquad r = \frac{\pi^2 H}{\beta}, \qquad f = \frac{E\theta}{12(1-\nu^2)\chi p}. \tag{2.2}$$

Fig. 2

Here $N_{11}^*, N_{22}^*, N_{12}^*$ are the critical forces corresponding to pure deformation; k_{11}, k_{22}, k_{12} are coefficients.

Using Bubnov' s method (3), from (1.1) with the aid of (2.1) we obtain a homogeneous linear algebraic system of equations for determining A_{kl}:

$$\sum_{k=1}^{n}\sum_{l=1}^{m} a_{lk}^{ij} A_{kl} = 0 \qquad (j = 1, \dots, m;\ i = 1, \dots, n). \tag{2.3}$$

The matrix A of this system has the elements

$$a_{lk}^{ji} = \begin{cases} b_{lk}^{ji} - \lambda, & \text{for } k = i \text{ and } l = j, \\ c_{lk}^{ji}, & \text{for } k + i \text{ and } l + j \text{ odd}, \\ d_{lk}^{ji}, & \text{for } k + i \text{ odd and } l = j, \\ 0, & \text{in the remaining cases}, \end{cases} \tag{2,4}$$

where

$$b_{lk}^{ji} = \pi\left[(i^2+\alpha^2 j^2)^2 - \frac{r(1-\vartheta)(i^2+\alpha^2 j^2)^3}{1+r(i^2+\alpha^2 j^2)} + k_{11} i^2 n_{11} + k_{22}\alpha^4 j^2 n_{22}\right], \tag{2,5}$$

$$c_{lk}^{ji} = 32\pi^2 \alpha n_{12} \frac{ijkl}{(i^2-k^2)(j^2-l^2)}, \qquad d_{lk}^{ji} = \frac{4Mf}{H^3}\frac{ik}{i^2-k^2}, \tag{2,6}$$

$$\lambda = -\frac{\Gamma f}{H^2}\omega^2 - \left(\frac{f}{H^3} + 2HW\right)\omega.$$

3. Stability parabola

From (2,6), for $\lambda = \tau + i\delta$ ($i = \sqrt{-1}$), there follows the "stability parabola"

$$\tau = z\delta^2, \qquad z = \frac{\Gamma f}{H^2\left(f/H^3 + 2HW\right)}, \tag{3,1}$$

and the values of λ lying inside the parabola correspond to values of ω in the left complex half-plane. In this case the plate will be stable in the flow.

When $M = n_{22} = n_{12} = 0$ and the plate is subjected to compression along the Ox axis ($n_{11} > -1$), all characteristic numbers of the matrix A lie inside the parabola on an interval of the real axis. The case $n_{11} = -1$ corresponds to the critical state for $M = n_{22} = n_{12} = 0$. Similarly, for $M = n_{11} = n_{12} = 0$, $n_{22} \neq 0$, or for $M = n_{11} = n_{22} = 0$, $n_{12} \neq 0$.

It is obvious that all λ_s will be inside the parabola also for sufficiently small M. Consider the matrix $M^{-1}A = B$. Represent it in the form of symmetric C and skew-symmetric D components: $B = C + D$. From (2,4), (2,5) it follows that the norm of C is proportional to M^{-1}, whereas the norm of D does not depend on M. Therefore, as $M \to \infty$, the characteristic (real) numbers of the matrix C will contract to zero, while all characteristic (imaginary) numbers of the matrix D will remain unchanged. Consequently, on the basis of Bendixson' s theorem (4), for some smallest value $M = M^*$ (critical), a characteristic number of the matrix B, and hence also of A, will intersect the parabola.

4. Calculation of the critical velocity

As the basic parameters we use M and r. Fixing the parameters, by Danilevsky' s method we find the coefficients of the characteristic polynomial of the matrix A

$$\sum_{s=0}^{N} R_s \lambda^{N-s} = \sum_{s=0}^{2N} Q_s \omega^{2N-s} \qquad (N = nm). \tag{4,1}$$

The roots of the polynomial (4,1) are functions of the form

$$\lambda_s = \lambda_s(M, r, \vartheta, H, n_{11}, n_{22}, n_{12}, \alpha, \Gamma, f, W) \quad (s = 1, \dots, N), \tag{4,2}$$

$$\omega_s = \omega_s(M, r, \vartheta, H, n_{11}, n_{22}, n_{12}, \alpha, \Gamma, f, W) \quad (s = 1, \dots, 2N).$$

Fig. 3

Figure 3: Fig. 3

Fig. 4

Figure 4: Fig. 4

Calculating λ_s, fixing them with respect to the parabola, or estimating $\operatorname{sign}\operatorname{Re}\omega_s$ by Routh' s method, we find the critical value M^* for fixed values of the parameters:

$$M^*(r, \vartheta, H, n_{11}, n_{22}, n_{12}, \alpha, \Gamma, f, W).$$

In the work, the influence of the first 7 parameters on M^* was evaluated. For in the calculations the parameters were taken in the following intervals:

$$0 \leqslant r \leqslant 1; \qquad 0 \leqslant \vartheta \leqslant 0.1; \qquad 0.004 \leqslant H \leqslant 0.02;$$

$$|n_{11}| < 1; \quad |n_{22}| < 1; \quad |n_{12}| < 1; \qquad 0.5 \leqslant \alpha \leqslant 5.$$

The computations were performed for an air flow at sea level ($\rho = 0.125\ \mathrm{kg} \cdot \mathrm{s}^2/\mathrm{m}^4,\ p = 103 \cdot 10^2\ \mathrm{kg/m}^2$) for a plate with duralumin outer layers $\Gamma = 506;\ W = 0;\ f = 0.460 \cdot 10^{-4};\ m = 2;\ n = 4.$

Estimates were made of the rate of convergence of Bubnov' s method: a) for $m = -2,\ n = 2$; b) for $m = 2,\ n = 4$; c) for $m = 2,\ n = 6$. If the critical velocity is conditionally estimated with respect to case c), then the error is 35% in case a) and 4% in case b).

Fig. 3

Fig. 4

The calculations show that $M^*(r)$ decreases sharply as r increases, i.e., as the stiffness of the core in transverse shear decreases. The plate aspect ratio α plays a substantial role only for a sufficiently stiff core. The critical numbers M^* are practically independent of compression of the plate in the direction perpendicular to the flow, i.e., of n_{22} when $n_{12} = 0$. Figure 1 presents the dependence $M^*(r)$ for $H = 0.01$ and $H = 0.008$ for a square plate ($\alpha = a/b = 1$) at $\vartheta = 0.1;\ 0.05;\ 0.025;\ 0.008;\ 0$. Figure 2 gives the dependences: a–$M^*(n_{11})$ for $n_{22} = n_{12} = 0$; b–$M^*(n_{22})$ for $n_{11} = 0.5$ and $n_{12} = 0$; c–$M^*(n_{22})$ for $n_{12} = 0.25$; d–$M^*(n_{22})$ for $n_{12} = 0.75$, for various r and $H = 0.01,\ \alpha = 1,\ \vartheta = 0.004$. The plot $M^*(n_{12})$ for $n_{11} = n_{22} = 0,\ \alpha = 1,\ H = 0.01,\ \vartheta = 0.004$ for $r = 0;\ 0.1;\ 0.2;\ 0.4;\ 0.6;\ 0.8;\ 1.0$ is given in Fig. 3. The dependence $M^*(r)$

for a number of values of α at $H = 0.01$ and $H = 0.008$ is given in Fig. 4 ($\vartheta = 0.004$; $n_{11} = n_{12} = n_{22} = 0$). In discussing the results of the work, it should be borne in mind that they are based on the linearized theory of the piston.

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Note: Figure translations are in progress. See original paper for figures.