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Abstract

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ON THE QUESTION OF ELECTRON HEATING IN AN ANISOTROPIC PLASMA

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It is known that a low-pressure plasma $\beta = 8\pi p/H^2 \ll 1$ with an anisotropic ion velocity distribution may be unstable with respect to the buildup of potential oscillations ([1–4]). Thus, for example, in ([4]) the cyclotron instability of a strongly anisotropic ($T_{\perp i} \gg T_{\parallel i}$) collisionless plasma in the intermediate region of phase velocities ($v_{\parallel i} \ll \omega/k_\parallel \ll v_{\parallel e}$) was considered; here ω and k are the wave frequency and its wave vector. We shall show that this instability is built up on a branch of oscillations with negative energy, $W = \frac{1}{8\pi}\frac{d}{d\omega}(\varepsilon\omega)|E|^2 < 0$ ([5]);

$$\varepsilon = \varepsilon_{pq}\frac{k_p k_q}{k^2}$$

is the longitudinal dielectric permittivity of the plasma, and its development should be accompanied by intensive absorption of energy by resonant electrons (Landau damping). Therefore it is possible that the anomalous heating of electrons observed in adiabatic traps (see, for example, ([6,7])) is caused by the buildup of this instability. In the case of a dense and cold plasma, collisional dissipation may lead to an analogous buildup.

Let us consider a collisionless anisotropic plasma, homogeneous in space, in a constant magnetic field H. The dispersion equation for longitudinal oscillations for a Maxwellian distribution of ions and electrons over longitudinal and transverse velocities with $T_\perp \neq T_\parallel$, as is known, has the form

$$\varepsilon \equiv 1 + \sum_{j=e,i}\varepsilon_j = 1 + \sum_{j=e,i}\frac{1}{k^2 r_{dj\parallel}^2}\sum_n \xi_n(\rho_j)\left[1 + i\sqrt{\pi}\left(z_{nj} + \frac{nh_j}{\tau_j}\right)W'(z_{nj})\right] = 0. \tag{1}$$

Here

$$r_{dj}^2 = v_j^2/\omega_{pj}^2; \qquad \omega_{pj}^2 = 4\pi n e^2/m_j$$

is the plasma frequency;

$$\xi_n(\rho_j) = I_n(\rho_j)e^{-\rho_j};$$

$I_n(\rho)$ is a Bessel function of imaginary argument,

$$\rho_j = T_{\perp j}/m_j\omega_j^2;$$

$$\omega_j = eH/m_{jc}$$

is the cyclotron frequency;

$$z_{nj} = (\omega - n\omega_j)/k_{\parallel}v_{\parallel j}; \qquad h_j = \omega_j/k_{\parallel}v_{\parallel j};$$

$$\tau_j = T_{\perp j}/T_{\parallel j};$$

$$W(z) = e^{-z^2}\left(1 + \frac{2i}{\sqrt{\pi}}\int_0^z e^{\xi^2}\,d\xi\right)$$

is the probability integral of a complex argument.

In the simplest case of oscillations with

$$|x_n| \equiv \left|\frac{\omega - n\omega_i}{\omega_i}\right| \ll 1, \qquad z_{e0} \ll 1,$$

$$z_{ni} \gg 1, \qquad \rho_e \ll 1, \qquad \tau_i \gg \frac{z_{ni}}{x_n} \gg 1, \qquad \tau_e = 1,$$

we have

$$\varepsilon_e = \frac{1}{k^2 r_{de\parallel}^2}\left(1 + i\sqrt{\pi}\,z_{e0}\right),$$

$$\varepsilon_i = -\frac{\omega_{pi}^2}{(\omega - n\omega_i)^2}\,\frac{k_{\parallel}^2}{k^2}\,\xi_n(\rho_i) + i\sqrt{\pi}\,\frac{1}{k^2 r_{di\parallel}^2}\,z_{ni}e^{-z_{ni}{}^2}\xi_n(\rho_i).$$

For $z_{e0} \ll 1$, $z_{ni} \gg 1$, when $\operatorname{Im}\varepsilon_j \ll \operatorname{Re}\varepsilon_j$, the dispersion equation $\varepsilon = 0$ splits into two: $\operatorname{Re}\varepsilon = 0$, $\gamma\,\partial\operatorname{Re}\varepsilon/\partial\omega + \operatorname{Im}\varepsilon = 0$. Consequently, γ–the increment (decrement) of the oscillations–is equal to $\gamma = -\operatorname{Im}\varepsilon/(\partial\operatorname{Re}\varepsilon/\partial\omega)$.

Usually (see, for example, [2,3]) the stability of oscillations with $\partial \operatorname{Re} \varepsilon / \partial \omega > 0$ was investigated; these are amplified when the resonant particles emit waves ($\operatorname{Im} \varepsilon < 0$). In our case the equation $\operatorname{Re} \varepsilon = 0$ has two roots

$$\omega = n\omega_i \pm \omega_{pi} \frac{k_\parallel}{k} \zeta_n^{1/2} \left(1 + \frac{1}{k^2 r_{de\parallel}^2}\right)^{-1},$$

and for the root with the minus sign ($\omega < n\omega_i$), $d \operatorname{Re} \varepsilon / d\omega < 0$. Such oscillations are unstable when the resonant particles absorb energy ($\operatorname{Im} \varepsilon > 0$). Oscillations with negative dispersion are possible only in a thermodynamically nonequilibrium plasma [5]. In our case $\partial \operatorname{Re} \varepsilon_e / \partial \omega = 0$, and the derivative $\partial \operatorname{Re} \varepsilon / \partial \omega$ is negative owing to the contribution of the anisotropic ions.

Let us now consider the energy balance for these oscillations, writing the energy equation [8] in the form

$$\frac{\gamma}{8\pi} \left(1 + \sum_{j=e,i} \operatorname{Re} \varepsilon_j\right) |E_0|^2 + \frac{\omega}{8\pi} \left(\gamma \sum_{j=e,i} \frac{\partial \operatorname{Re} \varepsilon_j}{\partial \omega} + \sum_{j=e,i} \operatorname{Im} \varepsilon_j\right) |E_0|^2 = 0. \quad (2)$$

Here the relation

$$\sigma = \frac{\omega(\varepsilon - 1)}{4\pi i}$$

has been used; in this case

$$\operatorname{Re} \sigma = \operatorname{Im} \frac{\omega(\varepsilon - 1)}{4\pi} = \frac{\gamma}{4\pi} \left(\operatorname{Re} \varepsilon - 1 + \omega \frac{\partial \operatorname{Re} \varepsilon}{\partial \omega}\right) + \frac{\omega}{4\pi} \operatorname{Im} \varepsilon,$$

and it has been taken into account that the electric field is

$$\mathbf{E} = \mathbf{E}_0 e^{-i\omega t + \gamma t + i\mathbf{kr}}.$$

In equation (2), for the natural oscillations both brackets vanish separately by virtue of the relations $\operatorname{Re} \varepsilon = 0$, $\gamma \, \partial \operatorname{Re} \varepsilon / \partial \omega + \operatorname{Im} \varepsilon = 0$. The terms proportional to γ, which are usually combined together, take into account the change in the oscillatory energy. Their sum, for $\partial \operatorname{Re} \varepsilon / \partial \omega < 0$, is negative:

$$\frac{dW_k}{dt} = \frac{\gamma}{8\pi} \frac{\partial}{\partial \omega} (\omega \operatorname{Re} \varepsilon) |E|^2 < 0.$$

Thus, in our case the energy of the plasma in the presence of oscillations is less than the energy of the unperturbed plasma. To increase the amplitude of the oscillations, it is necessary to absorb energy. The energy is absorbed by resonant electrons ($\operatorname{Im}\varepsilon_e > 0$). Resonant ions at $\omega < n\omega_i$ ($z_{ni} < 0$) emit energy ($\operatorname{Im}\varepsilon_i < 0$), promoting damping of the oscillations; however, their contribution for $|z_{ni}| \gg 1$ is negligibly small. It should be noted that the fraction of the energy transferred by the ions into the oscillatory degrees of freedom constitutes only a small part, of order

$$\left|\frac{\omega - n\omega_i}{\omega_i}\right| \ll 1$$

of the energy dissipated on the resonant electrons

$$\left(\frac{\partial \operatorname{Re}\varepsilon_i}{\partial\omega} = \frac{\omega_i}{\omega - n\omega_i} \operatorname{Re}\varepsilon_i\right).$$

The instability under consideration can develop in adiabatic traps, where the velocity distribution of the injected ions is strongly anisotropic, approaching a δ-functional distribution near some v_0 in the transverse velocities and of the same character in the longitudinal ones.

It is interesting that the onset of cyclotron oscillations is usually accompanied by intense heating of the electrons (6,7), and this is precisely what follows from the consideration carried out by us. If the plasma density is sufficiently high ($\omega_{pe}^2/\omega_i^2 \gg 1$), then an instability of hydrodynamic type with $Z_e \gg 1$ can be amplified simultaneously (1). However, in the linear approximation this instability does not lead to electron heating, since in such oscillations their energy is determined simply by the oscillation amplitude in the wave field. In particular, in the stationary case this energy is constant. The dissipative instability considered by us "heats" the electrons and

in the stationary case, and this heating promotes the further development of the instability, since the interval of wave numbers of unstable oscillations is broadened ($k_\parallel > \omega_i/v_e$).

In this work the boundaries of the region of dissipative instability of a quasineutral plasma have been determined numerically in terms of the parameters $\tau_i^{-1} = T_{\parallel i}/T_{\perp i}$, $\mu = T_{\parallel i}/T_{\parallel e}$; see Fig. 1.

Ion-acoustic oscillations are unstable at approximately the same values of τ, μ (2). Although the region of ion-acoustic instability is wider than the dissipative region (see Fig. 1), its increment may prove to be substantially smaller. Thus, for example, at $T_{\parallel i} \to 0$, which occurs in adiabatic traps, the increment of the ion-acoustic instability is exponentially small, whereas for the dissipative instability it remains quite appreciable, $\gamma \sim \sqrt{m/M}\,\omega_i$.

In conclusion we note that in a dense plasma the dissipative instability can develop owing to collisional dissipation of energy. Thus, for example, in the case $T_i/T_e \gg M/m$, when only electron-electron collisions are important, the previous expression for ε_i remains valid, while for ε_e, at $\omega \gg \nu_{ee}$, $z_e \gg 1$, it is not difficult to obtain

$$\varepsilon_e = -\frac{\omega_{0e}^2}{\omega^2}\frac{k_\parallel^2}{k^2}\left[1 + \frac{1}{z_e^2}\left(3 - \frac{4}{3}\frac{i\nu_{ee}}{\omega}\right)\right]. \quad (3)$$

From the equation $\varepsilon(\omega) = 0$ it is easy to see that cyclotron oscillations are unstable in this case as well.

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Fig. 1. Boundary of the instability region of an anisotropic plasma. **1** – boundary of instability of ion-acoustic oscillations; **2** –boundary of dissipative instability.

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Note: Figure translations are in progress. See original paper for figures.