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Abstract

Full Text

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INVESTIGATION OF SUPERSONIC FLOW PAST A SPHERE BY AIR AND CARBON DIOXIDE UNDER THERMOCHEMICAL EQUILIBRIUM

(Presented by Academician G. I. Petrov on 11 V 1964)

During 1962–1963 at Moscow State University, a series of computations was carried out on an electronic computer, making it possible to investigate the supersonic flow past bodies of various shapes, with a detached shock wave, by various gases and under various assumptions concerning the nature of the physicochemical processes taking place ([1]). Below we present some results of a numerical investigation of the supersonic flow past a sphere by air and carbon dioxide under the assumption of thermochemical equilibrium. In the computations, analytical approximations of the thermodynamic functions of the gases were used: for air, the approximation of A. N. Kraiko with a maximum error of 2.9% ([2]), and for carbon dioxide, the approximation of V. V. Mikhailov with a relative error less than 1% ([3]). The computations were carried out over a wide range of parameters of the oncoming flow: M_∞ from 3 to 50, pressure from 10^{-5} to 1 atm, and temperature from 200 to 1500°K, altogether about 100 cases.

If the origin of the polar coordinate system r, θ coincides with the center of the sphere of unit radius, the axis of the system is parallel to the vector of the oncoming flow, and $r = r_c(\theta)$ is the equation of the shock wave in it, then in the transformed coordinate system $\xi = \dfrac{r-1}{r_c-1}, \theta$ the equations of adiabatic motion of a gas under thermochemical equilibrium can be written in the form

$$(u - bv)\frac{\partial u}{\partial \xi} + \frac{1}{\rho}\frac{\partial p}{\partial \xi} = av\left(v - \frac{\partial u}{\partial \theta}\right),$$

$$(u - bv)\frac{\partial v}{\partial \xi} - \frac{b}{\rho}\frac{\partial p}{\partial \xi} = -av\left(\frac{\partial v}{\partial \theta} + u + \frac{1}{v\rho}\frac{\partial p}{\partial \theta}\right),$$

$$\frac{\partial u}{\partial \xi} - b\frac{\partial v}{\partial \xi} + \frac{u - bv}{\rho}\left(\frac{\partial \rho}{\partial p}\frac{\partial p}{\partial \xi} + \frac{\partial \rho}{\partial T}\frac{\partial T}{\partial \xi}\right) =$$

$$= -a\left(\frac{\partial v}{\partial \theta} + \frac{v}{\rho}\frac{\partial \rho}{\partial p}\frac{\partial p}{\partial \theta} + \frac{v}{\rho}\frac{\partial \rho}{\partial T}\frac{\partial T}{\partial \theta} + 2u + v\operatorname{ctg}\theta\right), \qquad (1)$$

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$$(u - bv)\left(\frac{\partial T}{\partial \xi} - \frac{dT}{dp}\frac{\partial p}{\partial \xi}\right) = av\left(\frac{\partial p}{\partial \theta}\frac{dT}{dp} - \frac{\partial T}{\partial \theta}\right),$$

where u and v are the projections of the velocity vector onto the unit vectors e_r and e_θ of the spherical system,

$$b = \frac{\xi\varepsilon'}{1+\xi\varepsilon}, \qquad a = \frac{\varepsilon}{1+\xi\varepsilon}, \qquad \varepsilon = r_c(\theta) - 1, \qquad \frac{dT}{dp} = \frac{1}{\partial h(p,T)/\partial T}\left(\frac{1}{\rho} - \frac{\partial h(p,T)}{\partial p}\right),$$

h is the specific enthalpy.

Fig. 1. $0 - \theta = 0$; $1 - \theta = 0.3125$

	M_∞	T_∞	p_∞
Air			
a	10	250	0.01
b	40	250	0.01
CO_2			
c	9	280	1
d	20	280	0.01

In the figure: left ordinate, $\frac{R_{\text{sph}}}{V_{\max}}\left(\frac{du}{d\xi}\right)_{\xi=0}$ (II); right ordinate, ξ_0/R_{sph} (I); abscissa, $\rho_\infty/\rho_{\text{osc}}$, $(\gamma-1)/(\gamma+1)$.

Fig. 2. a $-\gamma = \text{const}$, $M = 6 \div \infty$, $\gamma = 1.01 \div 1.4$

	M_∞	T_∞	p_∞		M_∞	T_∞	p_∞
Air	**Air**	**Air**	**Air**	**CO_2**	**CO_2**	**CO_2**	**CO_2**
	$8 \div 50$	250	0.01		$9 \div 30$	280	0.01
	$8 \div 15$	280	1		$7 \div 25$	280	1
	$10 \div 30$	250	10^{-5}		20	$250 \div 500$	0.01
	10	250	$10^{-4} \div 1$		10	$250 \div 500$	1
					20	280	$10^{-4} \div 1$
					10	280	$10^{-3} \div 0.1$

The boundary conditions have the usual form: on the surface of the sphere ($\xi = 0$), equality to zero of the normal component of velocity,

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Fig. 3. a—$\gamma = 1.4$

Figure 1: Fig. 3. a—$\gamma = 1.4$

$$u = 0, \tag{2}$$

and at the shock ($\xi = 1$), satisfaction of the conservation laws, which leads to two

transcendental equations

$$p_\infty - p + \rho_\infty V_{\infty n}^2 \left(1 - \frac{\rho_\infty}{\rho}\right) = 0, \quad 2(h_\infty - h) + V_{\infty n}^2 \left(1 - \frac{\rho_\infty^2}{\rho^2}\right) = 0, \tag{3}$$

$V_{\infty n}$ is the component of the velocity vector of the oncoming flow normal to the shock. In (1), (3), for the density and enthalpy behind the shock wave their approximate expressions are used as functions of pressure and temperature.

To solve system (1) with the corresponding boundary conditions, a finite-difference method was applied, based on repeated solution of the Cauchy problem in the direction from the shock to the body ([4]). The results presented were obtained with a scheme in which approximation of the derivatives with respect to θ by means of Lagrange polynomials was carried out using 5 points, and in the course of the solution 3 parameters in the shock equation were selected.

Fig. 3. a—$\gamma = 1.4$

T_∞	p_∞	T_∞	p_∞
Air	Air	CO_2	CO_2
250	0.01	280	0.01
280	1	280	1
250	10^{-5}		

An analysis of the flow fields in the flow around blunt bodies by perfect gases with different adiabatic exponents and by gases with allowance for their real properties shows that throughout the subsonic region, and especially near the axis of symmetry, the density changes comparatively weakly and differs little from its value behind the normal shock; this is illustrated in Fig. 1, where, for air and carbon dioxide and for different conditions in the oncoming flow (throughout the graphs the temperature of the oncoming flow T_∞ is given in °K, and the pressure p_∞ in physical atmospheres), density values are presented on two rays: the stagnation line $\theta_0 = 0$ and a ray intersecting the surface of the

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sphere at approximately half the distance from the critical point to the sonic point, $\theta_1 = 0.3125$. This property remains valid for other gases as well.

In Fig. 2, from the results of the calculations, the dependences are plotted of the shock stand-off distance along the axis of symmetry $\varepsilon_0/R_{\text{sph}}$, where R_{sph} is the radius of the sphere, and of the dimensionless derivative of the velocity along the arc length at the critical point

$$\frac{R_{\text{sph}}}{V_{\max}}\frac{dv}{ds}$$

($V_{\max}$ is the maximum value of the velocity) on the density ratio. These dependences with

with great accuracy and over a large interval can be represented by universal curves. In order not to complicate the drawing excessively, the results are plotted only for part of the computed variants. The data corresponding to the entire investigated range of temperatures and pressures of the oncoming flow at $M > 6$ fall on the same curves. In Fig. 2 the solid lines show the dependences

$$\frac{\varepsilon_0}{R_{\text{sph}}} = 0.8\,\frac{\rho_\infty}{\rho_{0\ \text{sk}}},$$

$$\frac{R_{\text{sph}}}{V_{\max}}\left(\frac{dv}{ds}\right) = \left(1.6 - 1.5\,\frac{\rho_\infty}{\rho_{0\ \text{sk}}}\right)\sqrt{\frac{\rho_\infty}{\rho_{0\ \text{sk}}}}.$$

Points obtained in calculations of the flow past a sphere by a perfect gas with various adiabatic exponents also fall on the universal curves. Only for the dimensionless velocity gradient in this case is the determining parameter not the density ratio, but the quantity $(\gamma-1)/(\gamma+1)$. As the investigation shows, in the case of any real gas the universal parameter determining, for a body of the given shape, the flow field in the vicinity of the axis of symmetry is the density ratio on the two sides of the normal shock, provided only that the properties of the gas are such that, at small M, large degrees of compression in the normal shock cannot be achieved for it. All gases of practical interest satisfy this condition.

Fig. 4

	M_∞	T_∞	p_∞
Air (a)			
1	10	250	
2	13	250	
3	15	250	
4	10		1
5	15		1

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	M_∞	T_∞	p_∞
6	20		0.01
CO_2 (b)			
1	7	280	
2	9	280	
3	10	280	
4	20	280	
5	5		1
6	10		1
7	20		0.01

In Fig. 3 the dependence of the standoff distance on the Mach number M of the oncoming flow is given. Its more complicated, nonmonotonic character in comparison with the case of a perfect gas is explained by the sequential course of the occurring physicochemical processes. The completion of the process is associated with a decrease in the compression ratio $\rho_{0\ \mathrm{sk}}/\rho_\infty$ and, consequently, with an increase in the standoff distance.

Fig. 4 illustrates the dependence of the standoff distance on pressure and temperature. For both gases, dissociation, and consequently also the dependence on pressure, begins to appear at $M_\infty = 6$–8.

In all the cases investigated, at $M > 4$ the ratio of the pressure on the surface of the sphere to the pressure at the critical point depends only weakly on the parameters of the oncoming flow. The pressure at the critical point itself is, with great accuracy, proportional to M_∞^2.

The accuracy analysis carried out showed that the difference between the computed data and their exact values, when the same approximation formulas are used, does not exceed 1%.

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1. G. S. Roslyakov, G. F. Telenin, in: *Numerical Methods in Gas Dynamics*, 2, Moscow State University Press, 1963, p. 5.

2. A. N. Kraiko, *Engineering Journal*, 4, 2 (1964).

3. V. V. Mikhailov, *Engineering Journal*, 2, No. 2, 239 (1962).

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4. S. M. Tilinskii, G. F. Telenin, G. P. Tinyakov, *Izv. AN SSSR, Mechanics and Machine Building*, No. 4 (1964).

Note: Figure translations are in progress. See original paper for figures.

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