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PHYSICS

G. A. SOKOLIK, N. P. KONOPLEVA

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Abstract

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PHYSICS

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CONSTRUCTIVE THEORY OF COMPENSATING FIELDS

(Presented by Academician N. N. Bogolyubov, 2 IX 1963)

1. In this paper an attempt is made to find the effective form of compensating fields as connection coefficients in the space of internal degrees of freedom ([1−3]).

Consider the gauge local group $\psi' = S(x)\psi$,

$$S = \exp[\varepsilon_a(x) I_a]; \qquad [I_a I_b] = C^c_{ab} I_c.$$

Generalizing Noether' s theorem to the case of a local group ([1−3]), we have:

$$\mathcal{L} = \mathcal{L}(\psi; \nabla_i \psi), \qquad \nabla_i = \partial_i - A^a_i I_a,$$

$$\delta\mathcal{L} = \frac{\partial \mathcal{L}}{\partial \psi}\delta\psi + \frac{\partial \mathcal{L}}{\partial \nabla_i \psi}\delta \nabla_i \psi = 0. \tag{1}$$

To compensate the terms arising owing to $\varepsilon_a = \varepsilon_a(x)$, it is necessary to introduce a nontensor field A^a_i:

$$\delta A^a_i = \varepsilon_b C^a_{bc} A^c_i + \partial_i \varepsilon_a; \tag{2}$$

the derivative ∇_i can be obtained from the condition of covariance of the wave equation $\mathcal{L}_i \partial_i \psi + m\psi = 0$ with respect to $\psi' = S\psi$. Then

$$S\mathcal{L}_i S^{-1} = \mathcal{L}_i, \qquad \partial_i \to \nabla_i = \partial_i - A^a_i I_a, \qquad \overline{A^a_i} = e^{\varepsilon_e C^b_{ea}} \left(A^b_i + N^b_c \partial_i \varepsilon_c\right),$$

$$N^b_a = \int_0^1 \exp[-t\varepsilon_e C^b_{ea}]\, dt$$

(passing to infinitesimal transformations, we obtain (2)).

Expanding $\delta\mathcal{L}$ and taking into account

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$$\nabla_i \frac{\partial \mathcal{L}}{\partial \nabla_i \psi} - \frac{\partial \mathcal{L}}{\partial \psi} = 0, \qquad [\delta \partial] = 0,$$

we obtain

$$\frac{\partial}{\partial x_i} \left(\frac{\partial \mathcal{L}}{\partial \nabla_i \psi} I_a \psi \right) = A_i^c C_{cb}^a J_i^b. \tag{3}$$

Thus the conservation current has the form

$$J_i^a = \frac{\partial \mathcal{L}}{\partial \nabla_i \psi} I_a \psi,$$

where

$$\nabla_i J_i^a = \partial_i J_i^a + A_i^c C_{cb}^a J_i^b. \tag{4}$$

From $\mathcal{L} = \bar{\psi}(\mathcal{L}_i \partial_i \psi + m)\psi$ it follows that

$$J_i^a = \bar{\psi} \mathcal{L}_i I_a \psi.$$

Taking into account the Lagrangian of the free field $\mathcal{L}_0[A_i^0]$, in which A_i^0 enters only in the form of the tensor

$$F_{ik}^a = (\partial_i A_k^a - \partial_k A_i^a) - \frac{1}{2} C_{bc}^a (A_i^b A_k^c - A_k^b A_i^c),$$

transforming according to the adjoint group of the gauge group $\psi' = S\psi$ (1),

$$\delta F_{ik}^a = \varepsilon_b(x) C_{bc}^a F_{ik}^c,$$

as a result of which $\mathcal{L}_0$ preserves gauge invariance, and the homogeneous conservation law $\partial_i j_i^a = 0$ is replaced by the inhomogeneous one $\partial_i J_i^a + f^a = A_i^c C_{cb}^a I_i^b$, where $f^a = \dfrac{\partial \mathcal{L}_0}{\partial F_{ik}^b} C_{ac}^b F_{ik}^c$.

It is interesting that in the case of the gravitational field, for $\mathcal{L}_0 = F_{ik}^a F_{ik}^a$, $f^a \equiv 0$, since $f^a = C_{ac}^b F_{ik}^a F_{ik}^c = 0$ (^{1}), because for compact groups C_b^{ac} are completely antisymmetric: $C_{bc}^a = -C_{cb}^a$; $C_{bc}^a = -C_{ba}^c$. Thus only a deviation from the Euclidean metric, which is connected with allowance for gravitation, leads to $f^a \neq 0$. In this case:

$$\mathcal{L}_0 = g_{bc}(x) F_{ik}^b F_{ik}^c.$$

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2. In order to express A_i^a effectively, we use the relation $\delta A_a = \varepsilon_e C_{eb}^a X_b$, where X_a define the basis of the adjoint group ([4]): $X_a = C_{ac}^b X_b \partial_c$. Then

$$A_i^a = C_{ac}^b \Omega_b \partial_i \Omega_c, \\ C_{ac}^{b'} C_{ec}^b \Omega_{b'} \Omega_{c'} = \delta_{ae}. \tag{5}$$

The quantities Ω_a may be regarded as a frame specified in the space of internal degrees of freedom. The adjoint group

$$\delta\Omega_a = \varepsilon_e(x) C_{eb}^a \Omega_b,$$

where S is regarded as its representation, induces, as is easy to see, the transformation A_i^a (2), if one takes into account the Jacobi identity.

The results obtained are close to the ideas of ([5]) on the connection between ordinary and isotopic space. Indeed, note that A_i^a can be represented as a mass operator by substituting (5) into

$$\mathcal{L}_i(\partial_i - A_i^a I_a)\psi + m\psi = 0$$

and introducing the coordinates of isospace ω ([6,7]) and the periodicity condition $\omega_a = \exp\left[\frac{2m_a}{\Lambda}\right]$. Then

$$\mathcal{L}_i \partial_i \psi(x, \omega) + M\psi(x, \omega) = 0. \tag{6}$$

The expression for the mass operator is:

$$M = m + \frac{m_i}{\Lambda} \mathcal{L}_i I_a C_{ab}^c \left(\omega_b \frac{\partial}{\partial \omega_c} - \omega_c \frac{\partial}{\partial \omega_b}\right),$$

where Λ is a constant with the dimension of length; it coincides with that given in ([6]) for particles of arbitrary spatial and isotopic spins, and in the present work the term corresponding to the isospin group arises because of its locality, i.e., through the connection with ordinary space.

3. In the case of an Abelian gauge group $S = \exp[ia(x)]$, the compensating field has the form

$$A_i = \overline{\Omega} \partial_i \Omega, \qquad \overline{\Omega}\Omega = 1.$$

For $\Omega' = S\Omega$, $A_i' = A_i + i\partial_i \alpha$ (gradient transformations). Passing to the variables $\Omega = e^{i\varphi}$ ($\varphi' = \varphi + \alpha$), we have $A_i = i\partial_i \varphi$. In other words, in the case of an Abelian group, A_i is given by a gradient ([8]).

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4. Comparing expression (5) for A_i^a as a connection coefficient in an isospace with the Ricci coefficient

$$\Delta_\sigma(i,k) = \Omega^\tau(j)\Omega^\lambda(k) M^{\gamma\alpha\beta}_{\sigma\tau\lambda} \Omega_\gamma(j)\partial_\alpha \Omega_\beta(j),$$

$$M^{\gamma\alpha\beta}_{\sigma\tau\lambda} = \frac{1}{2}\left(\delta^\gamma_\sigma\delta^\alpha_\tau\delta^\beta_\lambda + \delta^\gamma_\lambda\delta^\alpha_\tau\delta^\beta_\sigma + \delta^\gamma_\tau\delta^\alpha_\sigma\delta^\beta_\lambda - \delta^\gamma_\sigma\delta^\alpha_\lambda\delta^\beta_\tau - \delta^\gamma_\lambda\delta^\alpha_\sigma\delta^\beta_\tau - \delta^\gamma_\tau\delta^\alpha_\lambda\delta^\beta_\sigma\right),$$

which is the compensating field induced by the local Lorentz group:

$$\mathcal{L}_{ii'} = \exp\left[\varepsilon_{lk}(x) M^{lk}_{ii'}\right],$$

$$\delta\Delta_\sigma(i,k) = \varepsilon_{ln} C^{ln}_{ik;sp}\Delta_\sigma(s,p) + \partial_\sigma\varepsilon_{ik}$$

or

$$\Delta'_\sigma(i,k) = \mathcal{L}_{ii'}\mathcal{L}_{kk'}\Delta_\sigma(i',k') + M^{ik}_{sp}\partial_\sigma\varepsilon_{sp},$$

where

$$[I_{ik}I_{ls}] = C^{nm}_{ik;ls} I_{nm}, \qquad M^{ik}_{sp} = \frac{1}{2}\left(\delta^i_s\delta^k_p - \delta^i_p\delta^k_s\right).$$

Let us note that gravitation is the only compensating field induced by a group specified in the same space as the field itself. In other words, gravitation has a universal character. If this circumstance is connected with the idea of a hierarchy of interactions, according to which the weakening of an interaction is associated with an enlargement of the symmetry group, then it follows from the universality of gravitation that the gravitational interaction is minimal and does not correspond to any degree of homogeneity.

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Note: Figure translations are in progress. See original paper for figures.

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