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Abstract

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FREQUENCIES OF FREE OSCILLATIONS OF THE EARTH' S ATMOSPHERE

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The frequencies of free oscillations of the atmosphere on a rotating spherical Earth have been calculated for a real temperature stratification, taken from the so-called CIRA 1961 standard atmosphere ([1]). In the resulting graph (Fig. 3), oscillations of all types are brought together—from waves of synoptic scales to fast acoustic waves.

The basis is the usual system of hydrodynamic equations, linearized with respect to the state of rest; the time dependence is assumed to be periodic ($e^{i\sigma t}$):

$$i\sigma u = -\frac{1}{a\rho}\frac{\partial p'}{\partial \theta} + 2\omega\cos\theta\, v, \qquad i\sigma\rho' = -\bar{\rho}\chi - \omega\rho',$$

$$i\sigma v = -\frac{1}{a\rho\sin\theta}\frac{\partial p'}{\partial \varphi} - 2\omega\cos\theta\, u, \qquad i\sigma p' = -c^2\bar{\rho}\chi + \bar{g}\rho\omega, \tag{1}$$

$$i\sigma w = -\frac{1}{\rho}\frac{\partial p'}{\partial z} - g\frac{\rho'}{\rho}.$$

A spherical coordinate system is used: φ is longitude, θ is colatitude; ω is the angular velocity of the Earth' s rotation; $\bar{\rho}(z)$ is the density in the basic state of rest. Here the traditional neglect of part of the Coriolis acceleration is made (see ([2]), Chap. VII); $c^2 = \varkappa R\bar{T}(z) = \varkappa p/\rho = \varkappa gH$ is the adiabatic speed of sound;

$$\chi = \frac{1}{a\sin\theta}\frac{\partial}{\partial\theta}(u\sin\theta) + \frac{1}{a\sin\theta}\frac{\partial v}{\partial\varphi} + \frac{\partial w}{\partial z}$$

is the three-dimensional divergence.

The system can be reduced to a single equation for χ:

$$c^2\chi_{zz} + \left[\frac{dc^2}{dz} - \varkappa g\right]\chi_z + \frac{g}{\sigma^2 a^2}F\left\{\frac{\sigma^2 c^2}{g}\chi - \left[(\varkappa - 1)g + \frac{dc^2}{dz}\right]\chi\right\} = 0. \tag{2}$$

Fig. 1

Figure 1: Fig. 1

Here F is the differential operator on the surface of the sphere

$$F = \frac{f^2}{\sin\theta}\frac{\partial}{\partial\theta}\left[\frac{\sin\theta}{f^2-\cos^2\theta}\left(\frac{\partial}{\partial\theta} - \frac{i\operatorname{ctg}\theta}{f}\frac{\partial}{\partial\varphi}\right)\right] +$$

$$+\frac{f^2}{f^2-\cos^2\theta}\left[\frac{i\operatorname{ctg}\theta}{f}\frac{\partial^2}{\partial\theta\,\partial\varphi} + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}\right], \qquad f = \frac{\sigma}{2\omega}. \tag{3}$$

The substitution

$$x = \int_0^z \frac{dz}{H(z)}\left(= -\ln\frac{\bar{p}}{p_0}\right), \qquad \chi = \exp\left(\frac{1}{2}x\right)y$$

and separation of variables $y = \Psi(\varphi,\theta)y(x)$ reduce the equation to the system

$$F\Psi + \frac{a^2\sigma^2}{gh}\Psi = 0; \tag{4}$$

$$y'' + \left\{-\frac{1}{4} + \frac{\sigma^2 H}{\varkappa g}\left(1 - \frac{\varkappa H}{h}\right) + \frac{H\left[(\varkappa-1)g + dc^2/dz\right]}{\varkappa g h}\right\}y = 0. \tag{5}$$

The separation constant h is usually called the dynamically equivalent height of the atmosphere. The second equation requires boundary

conditions at $x = 0$ and at infinity. The condition that the vertical velocity vanish leads to $y' + (H/h - 1/2)y = 0$ (for $x = 0$). At infinity we impose the condition that $|y|$ be bounded. Each of equations (4) and (5) is solved independently. They contain two eigenparameters, σ and h. Therefore, for each of the equations one obtains a family of eigen-curves $\sigma(h)$. At the intersections of the curves of these two families we obtain the eigenvalues σ and h. Let us note that if H increases without bound as $x \to \infty$ (the thermosphere), then, independently of the form of $H(x)$, the curve $\sigma^2 = g/h$ proves to be an eigen-curve of the "vertical" equation (5). However, as can be shown, the behavior of the solutions for such values of σ and h differs sharply from all the others: the velocity w increases extremely rapidly at infinity. This casts doubt on the practical significance of such oscillations.

Fig. 1

Fig. 2

Fig. 2

Figure 2: Fig. 2

In equation (4) one can separate off one more variable, the longitude, by putting $\Psi = \exp(is\varphi)\Psi(\cos\theta)$. For Ψ one obtains the so-called Laplace tidal equation (see ([2])). We have computed several eigen-curves of this equation for $s = 2$ by the Galerkin method; they are presented in Fig. 1. For the eigen-curves, asymptotic formulas of two kinds are known:

a) $\frac{1}{\sigma} \sim \frac{a}{\sqrt{gh}\sqrt{n(n+1)}}$ as $h \to \infty$,

b) $\frac{1}{\sigma} \sim \frac{n(n+1)}{s \cdot 2\omega}$ as $h \to \infty$,

where n are arbitrary natural numbers. Several such asymptotic curves are drawn in Fig. 1 with dashed lines: the curves of type a)—for $n = 2, 3, 4$, and the straight line b)—for $n = 3$. If one passes to the model of a nonrotating Earth, i.e., lets ω tend to zero, then the horizontal asymptotes of type b) rise to infinity. These asymptotes are also absent for $\sigma/s < 0$, i.e., for waves propagating from west to east.

We now turn to the "vertical" equation (5). In Fig. 2, for comparison, the eigen-curves are shown for the simplest isothermal case $H = \text{const}$, studied in ([3]). Because H is constant, a continuous spectrum appears here, occupying the hatched region in the graph. The corresponding eigenfunctions y have the simple form $\sin mx$. One may regard the region of the continuous spectrum as consisting of a continuum of eigen-curves, each of which is characterized by one value of the wave number m or of the wave type. Several such curves are shown in the figure. In addition, there are two curves of the discrete spectrum: $\sigma^2 = g/h$ and $h = \varkappa H$. The eigenfunctions of the discrete spectrum do not oscillate with height and are called two-dimensional waves. In the same figure, dashed lines···

one of the eigen-curves of the "horizontal" equation has been plotted, and the points of intersection of the "horizontal" and "vertical" eigen-curves are shown. Points of intersection of type 1 correspond, according to (3), to acoustic waves; type 2—to gravitational waves (the terminology is established by their limiting behavior as $\varkappa \to \infty$, i.e., in the transition to incompressibility, and as $\varkappa \to 1$, i.e., in the transition to indifferent equilibrium). Type 3—two-dimensional fast waves, or Lamb waves. Type 4 was first indicated by Pekéris (4). Type 5—two-dimensional long-period waves associated with ω, i.e., with the gyroscopic effect—these are Rossby waves. Finally, type 6—gravitational-gyroscopic internal waves (see (5)).

Fig. 3

Figure 3 shows curves computed on a computer for a real temperature strat-

Fig. 3

Figure 3: Fig. 3

ification. In the calculations we imposed, for simplicity, the upper boundary condition $y' = 0$ at an altitude of 200 km. The spectrum turns out to be discrete. Beside each eigen-curve a numeral indicates the wave type, i.e., the number of nodes of the eigenfunction with height. We see that the basic character of the curves is preserved; consequently, the possibility remains of separating the curves into classes—acoustic and gravitational. The differences between an isothermal and a real atmosphere are especially noticeable for intermediate wave types, corresponding to periods of the order of several minutes and dynamically equivalent heights of the order of 10 km.

Let us also note that the often adopted condition of quasistatics, i.e., neglect of vertical acceleration, leads to all curves being replaced by their vertical asymptotes, if such exist, and disappearing altogether otherwise. Thus the acoustic curves disappear completely. The gravitational curves are strongly distorted for periods less than 20 min and are practically unchanged for periods greater than half an hour.

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Note: Figure translations are in progress. See original paper for figures.