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Abstract

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NEUTRINO LUMINOSITY OF A STAR DURING GRAVITATIONAL COLLAPSE IN GENERAL RELATIVITY

The theory of gravitational collapse, whose principles were developed as early as 1937-1938 ([1,2]), has recently been attracting attention: the question is being posed of the number of collapsed stars in the Universe ([3–5,10]), and of the dynamics of collapse and the energy losses, in particular by neutrino emission in the course of collapse ([4,6–8]).

Oppenheimer and Snyder ([2]) already established that the surface of a star crosses the gravitational radius r_0 in a finite proper time τ, but for an external observer the surface only asymptotically approaches r_0, $r \to r_0$ as $t \to \infty$ (for a distant observer the time coincides with the coordinate time t). The luminosity of the surface asymptotically tends to zero despite the finite temperature of the surface. Gravitational self-closing of the star occurs.

In the present note the law of gravitational self-closing is considered for neutrino radiation, which is produced mainly at the center of the star. In this case there is neither a Doppler effect depending on the motion of the surface, nor a change in the solid angle.

In works ([4,6]) the effect of gravitational self-closing was taken into account roughly, to order of magnitude: a characteristic density was found, $\rho_g = M/v_g$, where $v_g = {}^4/_3\pi r_0^3$, and the integration of the energy losses by a mass element was cut off at the moment when the density reached ρ_g.

Having a solution describing collapse, it is easy to find the trajectories of light or neutrino rays ($ds = 0$) propagating along the radius. The moment τ_1 at which the last ray leaves the center, crossing the surface of the star exactly at $r = r_0$ and reaching the external observer at $t = \infty$, does not coincide with the moment at which ρ_g is reached.

The intensity of the radiation is transformed according to the law

$$I(t) = Q(\tau)\left(\frac{d\tau}{dt}\right)^2, \tag{1}$$

where τ is the proper time at the beginning of the ray, at the center of the star; t is the coordinate time of a stationary distant observer at the moment when

Fig. 1

Figure 1: Fig. 1

the same ray passes him; $Q(\tau)$ is the energy power of the radiation by matter at the center of the star, $dE = Q\,d\tau$; $I(t)$ is the power calculated from the energy passing through a distant sphere on which the observer is located;

$$dE = 4\pi r^2 i(t)\,dt = I(t)\,dt.$$

Let us note that formula (1) contains the square of the quantity $d\tau/dt$, which is less than 1 and tends to zero as $t \to \infty$, therefore *

$$\int_{-\infty}^{\tau_1} Q(\tau)\,d\tau < \int_{-\infty}^{+\infty} I(t)\,dt.$$

* An error was made here in work ([4]).

When calculating the number of particles—neutrinos—conserved along the ray, the following relation holds

$$n(t) = \nu(\tau)\frac{d\tau}{dt}, \qquad N = \int_{-\infty}^{\tau_1} \nu(\tau)\,d\tau = \int_{-\infty}^{\infty} n(t)\,dt,$$

where ν is the number of neutrinos emitted per unit proper time τ; n is the number of neutrinos flying through a distant sphere per unit coordinate time; the total number of particles N is an invariant.

However, when calculating the energy, it is also necessary to take into account the redshift, i.e., the change in the energy of each individual neutrino as it moves outward from the gravitational potential well to a distant observer. The energy changes in the same ratio as the frequency (in accordance with $\varepsilon = \hbar\omega$); this gives an extra factor $d\tau/dt$.

Fig. 1

Thus, in order to calculate the luminosity of the star by formula (1), one must know the course of the radial light rays both inside the star and outside it; inside the star it is more convenient to use not the Schwarzschild coordinate system (t, r), but the comoving one (τ, R). With this in mind, we multiply and divide the right-hand side of (1) by $(dt_1/d\tau_1)^2$, where dt_1 is the change in time between two rays along the boundary of the star, and $d\tau_1$ is the same interval of proper time. Then

$$I(t) = Q(\tau)\left(\frac{d\tau}{d\tau_1}\right)^2 \left(\frac{d\tau_1}{dt_1}\right)^2 \left(\frac{dt_1}{dt}\right)^2. \tag{2}$$

The factor $d\tau_1/dt_1$ is determined from the law of transformation of time along the boundary and is equal to

$$\frac{d\tau_1}{dt_1} = \frac{r_1 - r_0}{r_1} \frac{1}{\sqrt{\dot{r}_1^2 + 1 - r_0/r_1}}; \tag{3}$$

here r_1 is the radius of the boundary, and the dot denotes differentiation with respect to proper time τ_1.

The factor dt_1/dt is easily found, knowing the equations of the trajectories of light rays in vacuum,

$$r + r_0 \ln(r/r_0 - 1) - t = \text{const}$$

(here and below $c = 1$):

$$\frac{dt_1}{dt} = \frac{\sqrt{\dot{r}_1^2 + 1 - r_0/r_1}}{\sqrt{\dot{r}_1^2 + 1 - r_0/r} - \dot{r}_1}. \tag{4}$$

Combining (2), (3), and (4), we obtain

$$I(t) = Q(\tau) \left(\frac{d\tau}{d\tau_1}\right)^2 \left(1 - \frac{r_0}{r}\right)^2 \frac{1}{\left[\sqrt{\dot{r}_1^2 + 1 - r_0/r} - \dot{r}_1\right]^2}. \tag{5}$$

All the quantities entering here are functions of τ_1, and therefore t must also be expressed through τ_1. This relation has the form

$$t = r + r_0 \ln\left(\frac{r}{r_0} - 1\right) - r_1 - r_0 \ln\left(\frac{r}{r_0} - 1\right) + t_1. \tag{6}$$

It is easy to see that in the limit, as $r_1 \to r_0$,

$$I(t) \sim \left(\frac{r_1}{r_0} - 1\right)^2, \qquad t \sim 2r_0 \ln \frac{1}{r/r_0 - 1} \tag{7}$$

(the latter is easily obtained from (3)). Therefore the asymptotic form of the luminosity of a collapsing star is

$$I(t) \sim e^{-t/r_0}. \tag{8}$$

We see that, although the collapse process for an external observer lasts indefinitely, the characteristic time of luminosity decline is very small; it is equal to the gravitational radius of the star divided by the speed of light. For the Sun

$r_0 \simeq 3\cdot 10^5$ cm, and the time for the luminosity to decrease by a factor e is 10^{-5} sec.

Consider numerical examples. For the rate of energy loss we take $Q \sim T^9/\rho$, which is valid for $T_9 > 5$. Here T is the temperature, ρ is the number density of particles. If the adiabatic exponent of the matter is $\gamma = 4/3$, then $Q \sim \rho^2$.

Let us analyze two cases:

1) **Collapse of dust** ($p = 0$). The motion is described by the well-known Tolman solution[9], which may be taken in the particular, simplest form

$$r = \left(\frac{3}{2}\right)^{2/3} r_0^{1/3} R\tau^{2/3}, \qquad e^{\omega/2} = \left(\frac{3}{2}\right)^{2/3} r_0^{1/3}\tau^{2/3}, \qquad \rho = \frac{1}{6\pi\tau^2}. \tag{9}$$

Here $k = 1$, $R = 1$ at the boundary of the star. The interval is $ds^2 = d\tau^2 - e^{\omega} dR^2 - r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$. Collapse occurs at $\tau = 0$, and gravitational self-closure at $\tau = -\frac{2}{3}r_0$.

2. **Collapse of a star made of a cold Fermi gas.** The initial state of the star is rest. The initial density profile differs slightly from the profile in a star with the maximum possible mass $M_{\max} \simeq 0.73 M_\odot$, so that the total mass is $0.98\dot{M}_\odot$. The results of a numerical calculation of the collapse of such a star are given in ([6]). In the case of dust, owing to the simplicity of the solution, formulas (5) and (6) can be written in finite form

$$I(t) = q\frac{1}{\left[\tau_1^{1/3} - \frac{1}{3}\left(\frac{3}{2}\right)^{2/3} r_0^{1/3}\right]^8}\frac{1}{\tau_1^{4/3}}\left[1 + \left(\frac{2}{3}\right)^{1/3} r_0^{1/3}\tau_1^{-1/3}\right]^2,$$

$$t = r + r_0 \ln\left(\frac{r}{r_0} - 1\right) + \tau_1 - \left(\frac{3}{2}\right)^{2/3} r_0^{1/3}\tau_1^{2/3} + 3\left(\frac{4}{9}\right)^{1/3} r_0^{2/3}\tau_1^{1/3} - r_0 \ln\left[1 + \left(\frac{3}{2}\right)^{1/3} r_0^{-1/3}\tau_1^{1/3}\right]^2. \tag{10}$$

Here q is a proportionality factor, and the arbitrary constant in $t(\tau_1)$ is chosen so that as $\tau_1 \to -\infty$ one has $\tau_1 = t_1$.

In Fig. 1 the dependences of the source power $Q(\tau)$ and of the stellar luminosity $I(t)$ on proper and coordinate time, respectively, are presented. Along the abscissa are plotted $\tau^{1/3}$ and $t^{1/3}$, and along the ordinate $\log Q(\tau)$ and $\log I(t)$. The unit of length (and time) is taken to be $\frac{9}{4}r_0$, and the unit for Q and I is $\frac{q}{(9/4\, r_0)^4}$. On the curve $Q(\tau)$ the point a is marked, above which

Fig. 2

the second part of the curve $Q(\tau)$ is inaccessible to an external observer. Point a corresponds to $\tau_1 = -1$. Gravitational self-closure occurs at the instant $\tau_2 = -{}^8/_{27}$. In the adopted units

$$\int_{-\infty}^{-8/27} Q\,d\tau = 12.81, \qquad \int_{-\infty}^{-1} Q\,d\tau = {}^1/_3, \qquad \int_{-\infty}^{+\infty} I\,dt = 0.0882.$$

Figure 2 gives analogous curves for a collapsing star with $M = 0.98\,M_\odot$ made of a cold Fermi gas. Here it is assumed that for $\tau < 0$ the star was at rest. The power of the source in the star at rest is chosen as the unit of luminosity.

Appendix. We give a table characterizing the mass losses of stars due to neutrino emission in the process of collapse. Also given here are the values of T and ρ at the moment of gravitational self-closure. The table is taken from [6]. In the last row there have been added to it the values of the losses, reduced by the ratio

$$\int_{-\infty}^{\infty} I\,dt \Big/ \int_{-\infty}^{\tau_2} Q\,d\tau$$

$M/M_\odot$	10^2	10^4	10^5	10^6	10^8
T_9	360	36	11.3	3.6	0.5
ρ, g/cm^3	$1.8 \cdot 10^{12}$	$1.8 \cdot 10^8$	$1.8 \cdot 10^6$	$1.8 \cdot 10^4$	1.8
$\Delta M/M$	55	0.055	$1.7 \cdot 10^{-3}$	$4.5 \cdot 10^{-5}$	10^{-12}
$(\Delta M/M)_1$	0.38	$3.8 \cdot 10^{-4}$	$1.17 \cdot 10^{-5}$	$3.1 \cdot 10^{-7}$	$7 \cdot 10^{-15}$

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