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Abstract

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ONE-DIMENSIONAL UNSTEADY MOTION OF A LIQUID WITH THE ONSET AND DEVELOPMENT OF CAVITATION

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Observation of cavitation arising near the surface of a vibrator immersed in water is characterized by the fact that almost all cavitation bubbles arise and disappear during each cycle of oscillation of the vibrator ([1]). Records of the pressure in the region of passing waves indicate a distortion of the wave form when cavitation appears; in this case the moment of collapse of the cavitation bubbles corresponds to a sharp pressure peak. Similar phenomena are also observed when sound waves are focused in a narrow region of a liquid.

Below we consider the problem of the development of a cavitation region near a piston moving nonuniformly in a liquid, under the assumption that the motion of the cavitating liquid is described by the system of equations proposed in ([2])

$$\frac{d\rho}{dt} + \rho \operatorname{div} \vec{v} = 0, \qquad \rho = \frac{\rho_0}{1 + b(R^3 - R_0^3)}, \qquad b = \frac{4}{3}\pi n \rho_0,$$

$$\frac{d\vec{v}}{dt} = -\frac{1}{\rho} \operatorname{grad} p, \qquad p = p \;\; - \rho_0 \left[R\frac{d^2R}{dt^2} + \frac{3}{2}\left(\frac{dR}{dt}\right)^2 \right], \tag{1}$$

where n is the number of cavitation "nuclei" per unit volume of the droplet liquid; R_0 is the initial radius of the cavitation "nuclei" ; R is the radius of the cavitation bubbles; $p, \rho, \vec{v}$ are, respectively, the pressure, density, and velocity of the cavitating liquid.

The solution found makes it possible, for certain particular motions of the piston, to describe the development of the cavitation region near the oscillating surface during one cycle of oscillation.

Let the piston bound on the left a semi-infinite tube filled with liquid (Fig. 1). At time $t = 0$ the piston has coordinate $x = 0$ and begins to move out of the tube according to some law $x = F(t) \leq 0$.

Fig. 1

Figure 1: Fig. 1

The liquid is assumed to be compressible and to remain continuous until the pressure in it falls to some value p . After that we neglect the change in the density of the droplet liquid in comparison with the change in the density of the mixture of liquid with bubbles due to the change in the volume of the bubbles. The pressure in the bubbles throughout the entire process is assumed to be constant and equal to p . From the piston to the right, a sound wave will propagate through the liquid. If in this wave the pressure can fall to the value p , then the process of nucleation and development of cavitation bubbles will begin in it. The cavitation region will be enclosed between the surface of the piston and the phase surface on which the pressure is equal to p and which, by virtue of the assumptions made, will propagate to the right with the speed of sound c. In what follows we assume that, on crossing this surface, the pressure, density, and velocity of the liquid particles

continuous. Since we are interested chiefly in the development of the cavitation region, we shall assume that the initial pressure in the liquid is equal to p_{cr} and that cavitation begins immediately after the piston starts moving.

Equations (1) for one-dimensional motion with plane waves in the Lagrangian form reduce to

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho_0}\frac{\partial p}{\partial \xi}, \qquad \rho = \frac{\rho_0}{1 + b\left(R^3 - R_0^3\right)}, \tag{2}$$

$$\frac{\partial(1/\rho)}{\partial t} = \frac{1}{\rho_0}\frac{\partial u}{\partial \xi}, \qquad p = p_{\text{cr}} - \rho_0\left[R\frac{\partial^2 R}{\partial t^2} + \frac{3}{2}\left(\frac{\partial R}{\partial t}\right)^2\right];$$

ξ is the Lagrangian coordinate.

Boundary conditions:

$$\text{for } \xi = 0 \qquad u = u_n \quad \left(u_n = \frac{dF}{dt}\right), \tag{3}$$

$$\text{for } \xi = ct \qquad p = p_{\text{cr}}, \quad \rho = \rho_0, \quad R = R_0, \quad u = 0.$$

Let us indicate a particular solution of the system of nonlinear equations (2), satisfying all the conditions of the posed problem (3), which will correspond to a certain one-parameter family of laws of piston motion:

Fig. 1

Fig. 2

Figure 2: Fig. 2

Fig. 2

$$p = p_{\text{cr}} - b\rho_0 c^2 \left(R^3 - R_0^3\right),$$

$$u = -bc\left(R^3 - R_0^3\right),$$

$$\frac{\partial R}{\partial t} = \sqrt{\frac{1}{3}bc^2R^3\left[1 - \left(\frac{R_0}{R}\right)^3\right]^2 + \chi R^{-3}}, \tag{4}$$

$$\frac{\partial^2 R}{\partial t^2} = \frac{1}{2}bc^2R^2\left[1 - \left(\frac{R_0}{R}\right)^6\right] - \frac{3}{2}\frac{\chi}{R^4},$$

$$t - \frac{\xi}{c} = \int_{R_0}^{R} \frac{dR}{\sqrt{\frac{1}{3}bc^2R^3\left[1 - (R_0/R)^3\right]^2 + \chi R^{-3}}}.$$

In this family of solutions the parameter is the dimensionless quantity χ, which is expressed in terms of the piston acceleration at the initial moment $\dot{u}_0$ and the product $b^2R_0c^2$ as follows:

$$\chi = \frac{\dot{u}_0^2}{9b^2R_0c^2}. \tag{5}$$

The corresponding equation of motion of the piston can be determined with the aid of the relation

$$x = \int_0^t u(t, \xi = 0)\,dt = -bc\int_0^t \left(R_n^3 - R_0^3\right)\,dt, \tag{6}$$

where the function $R_n(t)$ is found from the last equation of system (4) for $\xi = 0$.

From the first equation of system (4) and the third equation of system (2) it follows that in the cavitation region there is a linear relation between p and $1/\rho$,

$$p_{\text{cr}} - p = \frac{c\rho_0^2}{\rho} - \rho_0 c^2. \tag{7}$$

Thus, we obtain Chaplygin gas. As is known, in this case a progressive wave propagates without changing its shape. Consequently, the profile of the cavitation wave is not distorted in the present case.

In practice, when ultrasonic cavitation is excited, the amplitude of oscillations of the end face of the vibrator is limited to very small values, while the frequency of the oscillations is large. With a sharp deceleration of the piston and its return motion to the right, compression of the cavitation bubbles will begin. Owing to the inertia of the liquid, one may expect that a narrow zone will be formed in which the process of collapse of the cavitation bubbles will occur. This zone may be regarded as a shock wave. Let us determine how the presence of bubbles affects the characteristics of the wave being formed. We shall assume that the pressure and density of the liquid in region (2) between the piston and the shock wave (Fig. 2) are everywhere the same, and that the velocity of the liquid coincides with the velocity of the piston. As the equation of state of the liquid we take the known relation

$$p = B\left[\left(\frac{\rho}{\rho_0^*}\right)^n - 1\right] \quad \left(B = 3045\ \frac{\text{kg}}{\text{cm}^2},\ n = 7.15\right), \tag{8}$$

where ρ_0^* is the density of the liquid at zero pressure. We assume that it is satisfied in the liquid both before cavitation begins in it and in region (2).

Retaining only first-order terms with respect to p/Bn, equation (8) may be written in the form

$$\frac{\rho}{\rho_0} = 1 + \frac{p - p_{\text{cr}}}{Bn}, \tag{9}$$

where ρ_0 is the density of the liquid at pressure p_{cr}.

It follows from (9) that the speed of sound in the drop liquid is determined as follows:

$$c = \sqrt{\frac{Bn}{\rho_0}}. \tag{10}$$

The conditions at the discontinuity are:

$$\rho_1(u_1 - D) = \rho_2(u_2 - D),$$

$$p_1 + \rho_1 u_1(u_1 - D) = p_2 + \rho_2 u_2(u_2 - D), \tag{11}$$

$$\frac{\rho_2}{\rho_0} = 1 + \frac{p_2 - p_{\text{cr}}}{\rho_0 c^2}.$$

Here D is the shock-wave velocity; the quantities denoted by subscript 2 are the corresponding quantities in the region between the piston and the shock-wave front; the quantities with subscript 1 are determined from (4) through the radius of the cavitation bubbles R_1 immediately ahead of the shock wave.

Let us introduce the dimensionless quantities

$$x = b\left(R_1^3 - R_0^3\right), \quad y = \frac{p_2 - p_{\text{cr}}}{\rho_0 c^2}, \quad a = \frac{u_2}{c}. \tag{12}$$

From (11) and (12) we find the relation

$$y = \frac{\frac{1}{2}a^2 + ax - x + \sqrt{(a+x)^2 + \frac{1}{4}a^2(a+2x)^2}}{1+x}. \tag{13}$$

Taking a and x to be quantities of the same order of smallness (much less than unity), we obtain from (13)

$$y = a + \frac{a^2}{2} + \frac{1}{8}\,\frac{a^4}{a+x} \tag{14}$$

or, in dimensional form,

$$p_2 = p_{\text{cr}} + \rho_0 c u_2 + \frac{1}{2}\rho_0 u_2 \,\frac{u_2}{c} + \frac{1}{8}\,\frac{(u_2/c)^4}{u_2/c + b\left(R_1^3 - R_0^3\right)}. \tag{14'}$$

It is seen that the magnitude of the pressure in the wave that is formed practically does not depend on the presence of cavitation bubbles and is determined by the usual acoustic formula.

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CITED LITERATURE

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Note: Figure translations are in progress. See original paper for figures.