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Abstract

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ON THE THEORY OF LASER RADIATION

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We shall formulate approximate equations that describe the processes of light generation in lasers. Following works ($^{1-3}$), let us consider a rectilinear sample of length l, of arbitrary cross section (in the case of a circular cross section the diameter is denoted by d), which contains active atoms producing luminescent or laser radiation. Reflecting coatings are applied to the end faces of the sample, with reflection coefficients r_1 and r_2 for the left and right ends, respectively. Generally speaking, r_1 and r_2 may be functions of time. We shall assume that the end faces of the sample are mirror-polished and strictly perpendicular to the axis of the generating element (the angle of nonparallelism of the end faces is less than $\alpha_1 \sim \frac{d}{l}(1-r)^2$), so that quanta moving at a small angle (less than $\alpha_2 \sim \frac{d}{l}(1-r)$) to the laser axis will not leave the amplification process upon reflection from the end faces of the sample. In calculating the intensity of laser radiation we shall disregard all questions connected with the width of the emission line, with consideration of phase relations, and with the excitation of various natural oscillations, since to solve these questions it is necessary to start from equations for the field strengths, whose averaging in a certain approximation leads to equations (1), (2), (5).

Let us proceed to the equations for the intensity of laser radiation. In view of the sharp directivity, the problem may be regarded as one-dimensional. We introduce the notation: $J_1(x,t)$ is the number of quanta moving along the x-axis and passing in 1 sec, at time t, through an area of 1 cm^2 located at the point x; $J_2(x,t)$ is the same quantity, but for quanta moving to the left, i.e., opposite to the direction of the x-axis. We choose the origin of the coordinate system at the middle of the element and take the x-axis to coincide with the direction of the axis of the sample.

Let us now consider the change in the flux of quanta moving parallel to the x-axis. Choose in the generating element a volume of thickness dx. The quanta that at time t have passed through the plane at a distance x from the origin and that at time $t+\Delta t$ leave through the area at a distance $x+dx$ from the origin are amplified by the amount $(\chi_1-\chi_0)J_1$. We have divided the absorption coefficient into two parts. χ_1 is connected exclusively with the processes of stimulated emission and absorption in the resonance region, whereas χ_0 takes into account all the remaining absorption processes of nonresonant character. All absorption coefficients are averaged over a small frequency region near the absorption frequency of interest to us. Thus, the equations obeyed by the intensities J_1 and J_2 have the form of the equations of nonstationary transport ([4])

$$\frac{\partial J_1}{\partial x} + \frac{1}{v}\frac{\partial J_1}{\partial t} = (\chi_1 - \chi_0)J_1; \tag{1}$$

$$-\frac{\partial J_2}{\partial x} + \frac{1}{v}\frac{\partial J_2}{\partial t} = (\chi_1 - \chi_0)J_2. \tag{2}$$

Naturally, in order to simplify the problem we neglect all processes of rescattering of J_1 into J_2 and conversely (the generating element is assumed⋯

is assumed to be optically thin). It is obvious that χ_1 is proportional to the population inversion Δ

$$\Delta = n_2 - n_1\frac{g_2}{g_1}, \tag{3}$$

where n_2 and n_1 are the numbers of atoms in 1 cm^3 on the upper and lower levels, and g_2 and g_1 are the corresponding statistical weights of the states, which we shall omit for convenience in writing formulas below. Transitions between levels $2 \rightleftarrows 1$ are responsible for the appearance and disappearance of quanta of laser radiation. Introducing the corresponding cross section σ, we may write

$$\chi_1 = \sigma\Delta. \tag{4}$$

The values of the inversion $\Delta(x,t)$ at a given point and at a given instant of time depend both on the external "pumping" and on the value of $J_1 + J_2$. For illustration we give the expression for the population inversion of an atom with three levels, which is established in the equilibrium regime, for a given external pumping intensity and for a given laser-radiation intensity. The "pumping" excites atoms from level 1 to level 3 with probability $W_{13} = J_{13}\sigma_{13}$; then the atoms pass spontaneously, in time τ_{32}, to level 2, from which either spontaneously, with lifetime τ_{21}, or by stimulated transition with probability $W_{12} = (J_1 + J_2)\sigma$, they pass to the ground level, forming luminescent or laser radiation. Assuming that $\tau_{21} \gg \tau_{32}$, and denoting by N_0 the total number of active atoms in 1 cm^3, we arrive at the well-known expression

$$\Delta = \frac{N_0(W_{13}\tau_{21} - 1)}{(3W_{13}\tau_{21}\tau_{32}\sigma + 2\sigma\tau_{21})(J_1 + J_2) + W_{13}\tau_{21} + 1}. \tag{5}$$

A case is possible in which at $t = 0$ the inversion is specified and is equal to Δ_0. If for $t > 0$ the external action during the laser flash may be neglected, then we arrive at the pulsed regime. If the external action is very intense and compensates all losses, then the regime becomes continuous. One may compose an equation taking account of both of these limiting cases. If $\tau_{32} W_{13} \ll 1$, then the change of inversion with time is given by the law

$$\frac{d\Delta}{dt} = -\Delta \left(2W_{12} + W_{13} + \frac{1}{\tau_{21}} \right) + N_0 W_{13} - \frac{N_0}{\tau_{21}}. \tag{6}$$

Putting $W_{13} = 0$, we arrive at the equation for the decay of the system under the action of laser radiation and luminescence, when at $t = 0$ the inversion $\Delta_0 > 0$ is specified. For the stationary regime we arrive at formula (5) with $\tau_{32} = 0$. Equations (1), (2), (6) or (1), (2), and (5) (we do not write here the time equation corresponding to the stationary condition (5)) together with the boundary conditions

$$J_2 \left(\frac{l}{2}, t \right) = r_2 J_1 \left(\frac{l}{2}, t \right), \qquad J_2 \left(-\frac{l}{2}, t \right) = r_1 J_2 \left(-\frac{l}{2}, t \right) \tag{7}$$

and the initial conditions

$$\text{at } t = 0 \qquad \Delta(t, x) = \Delta_0, \qquad J_1 = J_{10}, \qquad J_2 = J_{20}, \qquad \Sigma = \Sigma_0 \tag{8}$$

are sufficient for finding the characteristics of laser radiation. With the aid of the equations derived, let us consider two examples.

A. Stationary case (cf. [5,6]). In this example the time dependence of all quantities may be neglected. Using equations (1), (2), and (5), it is easy to carry through all the calculations to the end and, taking account of the boundary conditions (7), obtain a formula for the output of the intensity through the right-hand end. For simplicity we put $r_1 = 1$, $\chi_0 = 0$;

$$I = (1 - r_2) J_1 \left(\frac{l}{2} \right) = \frac{N_0 (J_{13} \sigma_{13} \tau_{21} - 1)}{3 J_{13} \sigma_{13} \tau_{21} \tau_{32} + 2\tau_{21}} \cdot l - \frac{(J_{13} \sigma_{13} \tau_{21} + 1)}{3 J_{13} \sigma_{13} \tau_{21} \tau_{32} + 2\sigma \tau_{21}} \ln \frac{1}{\sqrt{r_2}}. \tag{9}$$

From the positivity of the expression given above, two well-known conditions follow which are necessary conditions for generation. Using formula (9), one can obtain the dependence of the output on the pumping J_{13}, on τ_{21}, on the length l_{eff}, and on the reflection coefficient r_2. Let us note that taking true absorption into account leads, roughly speaking, to the need to substitute in formula (9), instead of $1/l$,

$$\frac{1}{l_{\text{eff}}} = \frac{1}{l} + \frac{2\chi_0}{1 - r_2}.$$

For r_2 close to unity (i.e., precisely where the allowance for absorption is important), the formula gives good accuracy.

B. Nonstationary case, averaged over the length of the generating element. We shall consider the pulsed mode of operation of the laser; however, we shall assume that all quantities vary little over the length of the sample l. Then we can average equations (1), (2) over the length x, replacing all quantities by their mean values, and replacing the mean of products of the type $J\Delta$ approximately by the product of the mean values. The values of the functions J_1 or J_2 at $x = \pm l/2$ (these terms are obtained in averaging the derivatives with respect to x) are replaced by the mean value of the functions. Then, instead of equations (1) and (2), we have the equation for $\Sigma(t) = \bar{J}_1 + \bar{J}_2$

$$\frac{\partial \Sigma}{\partial t} = v\Sigma \left(\sigma\bar{\Delta} - \chi_0 - \frac{2}{l}\frac{1 - r}{1 + r} \right). \tag{10}$$

The equations are valid for $1 - r \ll 1$.

The equations (10) and (6) derived in this way are in a certain sense equivalent to equations for the total number of photons in the resonator, used in the literature ([7,8]). In deriving (10), for simplicity we considered the symmetric case, when

$$r_1 = r_2 = r, \qquad J_1(x,t) = J_2(-x,t), \qquad \Delta(x,t) = \Delta(-x,t). \tag{11}$$

The second equation (see formula (6)) does not change its form (only an averaging sign is added, which in the following formulas we shall omit). Neglecting the dependence of the resonator properties on time, and also assuming that the processes of interest to us proceed considerably faster than spontaneous decays, and neglecting the latter as well as the pumping, from (10) and (6) we obtain the equation for $\Sigma(t)$

$$\frac{\partial^2 \ln\Sigma}{\partial t^2} + \frac{\partial \ln\Sigma}{\partial t} 2\sigma\Sigma + 2\sigma\Sigma v\chi = 0, \tag{12}$$

where

$$\chi = \chi_0 + \frac{1 - r_1}{2l} + \frac{1 - r_2}{2l} \qquad \text{for } r_1 \neq r_2.$$

If we introduce the new function $p = \partial \ln\Sigma/\partial t$, then equation (12) can be integrated once:

$$\frac{p}{2\sigma} - \frac{v\chi}{2\sigma}\ln(p + v\chi) = -\Sigma + C. \tag{13}$$

We shall determine the constant C from the initial conditions and equation (10). Using equation (10), we find the relation between Δ and Σ:

$$\Sigma_0 - \Sigma = -\frac{v}{2}(\Delta_0 - \Delta) - \frac{v\chi}{2\sigma}\ln\frac{\Delta}{\Delta_0}. \tag{14}$$

Putting $p = 0$ in (13), one can obtain the value of Σ at the maximum of the radiation. The solution of equation (13) can be represented in parametric form ([9]). It is simpler, however, to proceed from the equation for Δ, after finding

from which, by formula (14), Σ is recovered. The equation for Δ (for $\tau_{21} \gg 1/W_{12}$ and $W_{13} \ll W_{12}$) takes the simple form

$$\frac{\partial}{\partial t}\ln\Delta = v\sigma(\Delta - \Delta_0) - \chi v\ln\frac{\Delta}{\Delta_0} - 2\Sigma_0\sigma. \tag{15}$$

The solution reduces to the calculation of the integral

$$\int_{\Delta_0}^{\Delta(t)} \frac{d\ln\Delta}{v\sigma(\Delta - \Delta_0) - \chi v\ln\dfrac{\Delta}{\Delta_0} - 2\Sigma_0\sigma} = t.$$

It follows from the solution of equations (15) and (11) that the times characterizing pulsed radiation are of the order of the larger of the two times $t_1 \sim \dfrac{1}{v\sigma\Delta_0}\ln\dfrac{v\Delta_0}{2\Sigma_0}$ and $t_2 \sim \dfrac{l}{(1-r)v}$. These time intervals must be considerably greater than the time of flight of a quantum over the length l. Then all functions in fact depend only weakly on x, and our approximation is justified. On the other hand, they must be less than the luminescence time, so that it may be neglected. In addition, the "pumping" energy during the flash must be considerably less than the radiated energy. Otherwise we shall have quasi-continuous radiation, and this case is readily investigated by perturbation-theory methods.

For $r \ll 1$, reflections may be neglected and, starting from (1) and (6), a closed expression may be obtained for the problem of the passage of quanta through an overpopulated medium.

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Note: Figure translations are in progress. See original paper for figures.