PHYSICS

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1964

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Abstract

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SPINOR NOTATION FOR THE GRAVITATIONAL FIELD

(Presented by Academician N. N. Bogolyubov on 2 IX 1963)

1. In the present paper the method of compensating fields is extended to spinor quantities.

In work ([1]) attempts were made to construct a spinor theory of gravitation, but the results of ([1]) are incorrect, since, according to ([2]), tensor and spinor quantities can be related by the formulas

$$g^{p\dot{\alpha}\beta} g^{q}_{\dot{\alpha}\gamma} + g^{q\dot{\alpha}\beta} g^{p}_{\dot{\alpha}\gamma} = 2g^{pq}\delta^{\beta}_{\gamma}, \qquad \Phi_{\dot{\alpha}\beta} = g_{p\dot{\alpha}\beta}x^p$$

only in an orthogonal basis. In what follows we shall use the method of compensating fields ([3]); moreover, the relation between the invariant derivative induced by the local Lorentz group $\mathcal{L}$ and spinor invariant derivatives will be indicated.

2. The expression for the invariant derivative that specifies the gravitational interaction ([3]) follows from the condition of covariance of the wave relativistic equation in an orthogonal basis:

$$(\Omega_\sigma(i)\mathcal{L}_i\partial_\sigma + m)\psi = 0, \tag{1}$$

where $\Omega_\sigma(i)$ are Lamé coefficients relating orthogonal and world coordinates, with respect to the local Lorentz group,

$$\mathcal{L}_{ii'} = \exp[\varepsilon_{lk}(x)M^{lk}_{ii'}],$$

$$M^{lk}_{ii'} = \frac{1}{2}(\delta^k_i g^l_{i'} - g^l_i \delta^k_{i'})$$

(u_σ are world coordinates; $x_i = \Omega_\sigma(i)u_\sigma$),

$$\partial_\sigma \to \nabla_\sigma = \partial_\sigma - \frac{1}{2}\Delta_\sigma(i,k)I_{ik},$$

where $\Delta_\sigma(i,k)$ are the Ricci coefficients:

$$\Delta_\sigma(i,k) = \Omega^\tau(i)\Omega^\lambda(k) M^{\gamma\alpha\beta}_{\sigma\tau\lambda}\Omega_\gamma(i)\partial_\alpha\Omega_\beta(j),$$

$$M^{\gamma\alpha\beta}_{\sigma\tau\lambda} = \frac{1}{2}(\delta^\gamma_\sigma\delta^\alpha_\tau\delta^\beta_\lambda + \delta^\gamma_\lambda\delta^\alpha_\tau\delta^\beta_\sigma + \delta^\gamma_\tau\delta^\alpha_\sigma\delta^\beta_\lambda - \delta^\gamma_\sigma\delta^\alpha_\lambda\delta^\beta_\tau - \delta^\gamma_\lambda\delta^\alpha_\sigma\delta^\beta_\tau - \delta^\gamma_\tau\delta^\alpha_\lambda\delta^\beta_\sigma),$$

which transform according to the formula

$$\delta\Delta_\sigma(i,k) = \varepsilon_{ln} C^{ik}_{ln;\,sp}\Delta_\sigma(s,p) + \partial_\sigma\varepsilon_{ik}$$

([3]); I_{ik} are the generators of the representation $\mathcal{L}$ according to which ψ transforms:

$$\varepsilon_{ik} = \varepsilon_{ik}(u), \qquad [I_{ik}I_{ls}] = C^{nm}_{ik;\,ls}I_{nm}.$$

3. We shall use the well-known decomposition that reduces the Lorentz group to the contracted direct product of second-order unimodular groups, according to which dotted and undotted semispinors (1/20) and (01/2) transform ([2]). Then the generators of the representation are reduced to

$$\tau_i = iI_{4i} + I_{jk}; \qquad \nu_i = iI_{4i} - I_{jk} \quad (ijk = \text{cycl}); \qquad [\tau_i\nu_j] = 0.$$

$$[\tau_i\tau_j] = \tau_k \quad \text{and} \quad [\nu_i\nu_j] = \nu_k$$

correspond to the sets of weights $(j_1 j_2)$ and $(j_2 j_1)$.

Equation (1) in this case takes the form of a second-order equation:

$$\mathcal{L}_{\alpha\dot\alpha}\Omega_\sigma(\alpha)\Omega_{\dot\sigma}(\dot\alpha)\,\partial_{\sigma\dot\sigma}\psi + m\psi = 0, \tag{2}$$

where $\Omega_\sigma(\alpha)$ and $\Omega_{\dot\sigma}(\dot\alpha)$ are the spinor components of $\Omega_\alpha(i)$; $u_\sigma = \Omega_\sigma(\alpha)\xi_\alpha$, $\partial_\sigma = \partial/\partial u_\sigma$, while $u_{\dot\sigma} = \Omega_{\dot\sigma}(\dot\alpha)\xi_{\dot\alpha}$ (a 4-component vector has the meaning of the bispinor $x_\alpha = \xi_\gamma\xi_{\dot\delta}$).

Assuming the locality of $\varepsilon_a(u_\sigma)$ and $\dot\varepsilon_a(u_{\dot\sigma})$, $S = 1 + \varepsilon_a\tau_a$, $\dot S = 1 + \dot\varepsilon_a\nu_a$, and requiring covariance of (2) with respect to $S, \dot S$, we obtain:

$$\partial_\sigma \to \partial_\sigma - A^a_\sigma\tau_a, \qquad \partial_{\dot\sigma} \to \partial_{\dot\sigma} - A^a_{\dot\sigma}\nu_a,$$

where A^a_σ and $A^a_{\dot\sigma}$ transform in such a way as to compensate the terms:

$$(\partial_\sigma\varepsilon_a)\tau_a\partial_{\dot\sigma}, \qquad (\partial_{\dot\sigma}\dot\varepsilon_a)\nu_a\partial_\sigma,$$

$$\tau_a \tau_c \varepsilon_b C^c_{pb} A^a_\sigma A^a_{\dot\sigma}, \qquad \tau_c \nu_p \varepsilon_b A^a_\sigma A^p_{\dot\sigma} C^c_{ab}, \tag{3}$$

where C^a_{bc} define the algebra of the 3-dimensional orthogonal group: $[A_{iA} j] = A_k$ $(ijk = \text{cycl})$. The covariant form of (2) is

$$\mathcal{L}_{\alpha\dot\alpha} \Omega_\sigma(\alpha) \Omega_{\dot\sigma}(\dot\alpha)(\partial_\sigma - \tau_a A^a_\sigma)(\partial_{\dot\sigma} - \nu_a A^a_{\dot\sigma})\psi + m\psi = 0. \tag{4}$$

The formula $\nabla_{\sigma\dot\sigma} = \nabla_\sigma \nabla_{\dot\sigma}$ expresses $\Delta_\sigma(i,k)$ through A^a_σ and $A^a_{\dot\sigma}$. The latter can be written as connection coefficients in spinor spaces:

$$A^a_\sigma = C^b_{ac} \Omega_\alpha(b)\, \partial_\sigma \Omega_\alpha(c),$$

$$A^b_{\dot\sigma} = C^b_{ac} \Omega_{\dot\alpha}(b)\, \partial_{\dot\sigma} \Omega_{\dot\alpha}(c). \tag{5}$$

Then A^a_σ and $A^a_{\dot\sigma}$ transform according to formula (3), if $\Omega_\alpha(b)$ and $\Omega_{\dot\alpha}(b)$ transform according to the adjoint groups of the unimodular groups S and $\dot S$,

$$\delta\Omega_\sigma(b) = \varepsilon_e C^b_{ec} \Omega_\sigma(b),$$

$$\delta\Omega_{\dot\sigma}(b) = \dot\varepsilon_e C^b_{ec} \Omega_{\dot\sigma}(c),$$

isomorphic to S and $\dot S$ themselves.

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Received
31 VIII 1963

CITED LITERATURE

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2. É. Cartan, *Theory of Spinors*, IL, 1947.
3. G. A. Sokolik, *DAN*, **148**, No. 3 (1963).

Note: Figure translations are in progress. See original paper for figures.