THREE-DIMENSIONAL FLOW PAST BLUNT BODIES WITH ALLOWANCE FOR EQUILIBRIUM PHYSICOCHEMICAL REACTIONS

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Fig. 1

Figure 1: Fig. 1

Abstract

Full Text

AERODYNAMICS

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THREE-DIMENSIONAL FLOW PAST BLUNT BODIES WITH ALLOWANCE FOR EQUILIBRIUM PHYSICOCHEMICAL REACTIONS

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In many problems of modern gas dynamics, high temperatures arise in the flow, at which physicochemical transformations of the gas play an essential role. Generally speaking, in parallel with the solution of the basic gas-dynamic problem one can calculate the thermodynamic functions or store a table of these functions. However, this leads to a considerable increase—approximately by an order of magnitude—in the computing time as compared with the case of a perfect gas. The use of various analytical approximations of the thermodynamic functions of air ([1,2]) also leads to a noticeable increase in computing time (approximately by a factor of $2 \div 4$), because of the need to calculate exponential functions many times. In addition, one more parameter—the temperature—has to be introduced into the equations.

Fig. 1

In the present work a new approximation of the equilibrium thermodynamic functions of gases is proposed, convenient for solving problems of gas dynamics on electronic computers. With the aid of this approximation, the mesh method has been used to solve the problem of three-dimensional flow past blunt bodies with allowance for equilibrium physicochemical reactions.

Let us introduce for consideration a new thermodynamic function χ, which we shall call the **effective isentropic exponent**:

$$\chi = \left(\frac{d\ln p}{d\ln \rho}\right)_S = \frac{\rho a^2}{p}. \quad (1)$$

Suppose that the dependence of the quantity χ on pressure and density is known to us. Then the system

Fig. 2

Figure 2: Fig. 2

$$(\mathbf{w}, \nabla)\mathbf{w} = -\frac{1}{\rho}\operatorname{grad} p, \qquad \operatorname{div}(\rho\mathbf{w}) = 0,$$

$$(\mathbf{w}, \operatorname{grad}\ln p) = (\mathbf{w}, \chi \operatorname{grad}\ln\rho), \qquad \chi = \chi(p, \rho) \tag{2}$$

can be regarded as a closed system of exact equations of gas dynamics in the case of steady flow of a nonviscous and non-heat-conducting gas in the presence of equilibrium physicochemical reactions. In this case, as in the case of a perfect gas, the unknown functions are the velocity, pressure, and density.

Figure 1 shows the dependence of χ on pressure and temperature, calculated from the results of ([3]). As is seen from the data presented, the dependence $\chi(p, T)$ has a complicated, nonmonotonic character both with respect to pressure and with respect to temperature.

An analysis of the relations for the speed of sound, enthalpy, molecular weight, and certain other thermodynamic functions of gases that are in

state of complete thermodynamic equilibrium shows that, as the determining parameters for these functions, one should take the quantities p and p/ρ, or certain simple combinations of them. In this case the principal parameter will be p/ρ, while the dependence on pressure will be weak. Investigations carried out with the aid of the tables [3] confirmed this conclusion.

Figure 2 presents the dependence of χ on pressure and on a certain parameter q, which is expressed in the following way through p, ρ:

$$q = (1 + \varphi_1 |\lg p|)^{-\operatorname{sign}\lg p}\beta_1, \tag{3}$$

where $\beta_1 = p/(10^6\rho) - \varphi_2$; $\varphi_1 = 0.1667$; $\varphi_2 = 0.45$ in the case of air; the dimension of $[p/\rho] = \mathrm{m}^2/\mathrm{sec}^2$, and the pressure p is referred to the value $p_0 = 1.01325$ bar. From the data of Fig. 2 it is seen that, in the variables p, q, the difference between the values of

Fig. 2

χ at fixed q and different p over the entire range of tabulated values $p = 10^{-3} \div 10^3$, $T = 200 \div 20000^\circ$K [3] does not exceed $8 \div 10\%$, and the dependence on pressure is monotone, in contrast to the data of Fig. 1. The dependence of the enthalpy, molecular weight, electron concentration, and a number of other thermodynamic functions of air in the variables p, q has an analogous character.

Because in these variables the thermodynamic functions depend only weakly on pressure, for approximating, for example, χ, the enthalpy, or the molecular weight of air over the entire range [3] with an accuracy of $1 \div 2\%$, only $100 \div 200$ memory cells of an electronic digital computer are required.

Other gases investigated—CO_2 and N_2—possess similar properties. This allows one to hope that the indicated properties are, to one degree or another, common to most gases and their mixtures over a wide range of temperatures and pressures.

We note that, for calculating vortex flows of an equilibrium gas in which no changes of entropy occur along streamlines, it is sufficient to approximate one quantity $\chi = \chi(p, \rho)$.

We shall solve system (2), as applied to the problem of the three-dimensional flow past blunt bodies, by means of the method proposed in [4]. In the solution we use the approximation of the equilibrium thermodynamic functions of gases proposed in the present work.

Let us write system (2) in the form

$$\frac{\partial Z}{\partial z} + A\frac{\partial Z}{\partial \xi} + B\frac{\partial Z}{\partial \psi} + Y = 0, \tag{4}$$

where Z is a column vector whose components are the projections of the velocity on the axes z, r, ψ of the body-fixed cylindrical coordinate system, the pressure, and the density; A, B are coefficient matrices; Y is the column vector of free terms. The variable $\xi = \dfrac{r - \Phi}{F - \Phi}$, where $r = F(z, \psi)$; $r = \Phi(z, \psi)$ are the equations of the surfaces of the shock wave and of the body.

If everywhere in the flow field the condition $u > a$ is satisfied, where $a = \sqrt{(\chi p)/\rho}$ is the speed of sound, then system (4) is a z-hyperbolic Cauchy-Kovalevskaya system.

Suppose that in the plane $z = 0$ the vector Z and the position of the shock wave are specified. Then in the region $z \geq 0$; $0 \leq \xi \leq 1$; $0 \leq \psi \leq 2\pi$ the vector Z is determined as the solution of a mixed problem for system (4), with initial data at $z = 0$ and boundary conditions on the body and on the shock wave.

Introduce in the indicated region a grid analogous to that given in (4), and pass from system (4) to a system of difference equations. In solving it we use the iteration method according to the following scheme: for the first iteration, when determining the coefficients of the difference equations, the values of all functions are taken equal to their values at the preceding step in z. The resulting value of Z is used for the next iteration, etc. The value of χ is computed from the corresponding values of p and ρ at the given point. To solve the equations on the layer $z = \text{const}$, the sweep method is used.

Fig. 3

Fig. 3

Figure 3: Fig. 3

Let us consider in somewhat more detail the boundary conditions on the shock wave. In the case of a real-gas flow, the basic conservation laws must be supplemented by the relation

$$h = h(p, \rho). \tag{5}$$

In the present work, instead of the energy equation and (5), the dependence of the density ratio in the direct compression jump was specified:

$$\frac{\rho}{\rho_\infty} = \frac{\rho}{\rho_\infty}(M_\infty, p_\infty, T_\infty). \tag{6}$$

However, in view of what has been said, it is clear that the enthalpy h can be approximated very simply and economically over the entire range (3). Then there is no need to prescribe (6) in advance, and with the aid of the method described one will also be able to compute flows in which equilibrium physico-chemical transformations of the gas already play an essential role in the incoming stream.

In accordance with the algorithm set forth, a universal program for an electronic computer was compiled, by means of which calculations were carried out for the flow about blunt bodies by a stream of a perfect gas ($\gamma = 1.4$) and of an equilibrium gas (air).

The dependence $\chi(p, q)$ (see Fig. 2) was approximated with respect to q by second-degree polynomials (piecewise), and with respect to p linearly in the value $\lg p$. The accuracy of the approximation was of the order of 1%. At the same time, for the approximation of χ over the entire range (3), about 150 computer memory cells were required. The duration of the computation time for equilibrium flow about a blunt body exceeds the computation time for ideal flow about the same body by only 3–5%.

Let us consider some results. First, on the basis of data of G. F. Telenin, G. P. Tinyakov, and S. M. Gilinskii, calculations were made of the supersonic flow region around a sphere. These results were used in determining the initial data in the plane $z = 0$, necessary for calculating the flow about bodies with spherical bluntness.

Figure 3 shows the position of the shock wave in flow past a cone with semiangle $\beta = 0.3491$ (20°), $\alpha = 0$, and the pressure distribution on the surface of this same cone at $\alpha = 0.08727$ (5°). The pressure p_s is referred to the quantity $\rho_\infty a_*^2$, where a_* is the critical speed of sound calculated for a perfect gas.

Fig. 4

Figure 4: Fig. 4

Figure 4 gives the pressure distribution on the cone at $\alpha = 0;\ 0.1745(10^\circ),\ \beta = 0.3491$. The value $\psi = 0$ corresponds to the windward side of the cone in the plane of the angle of attack. Measurement along the z-axis is taken from the plane of the initial data, in units of the sphere radius. The plane $z = 0$ passes through the junction line of the sphere and the cone. The data in Figs. 3 and 4 were calculated at $M_\infty = 20,\ p_\infty = 0.0101325$ bar, $T_\infty = 300^\circ$ K; the solid lines correspond to the equilibrium case, and the dashed lines to the ideal case of flow ($\gamma = 1.4$).

Fig. 4

In equilibrium flow, the inflection point on the shock wave shifts closer to the nose of the body (Fig. 3); the shock layer becomes thinner, and as a result the region influenced by blunting is noticeably reduced (Figs. 3 and 4). The difference in the values of p_s at certain z in the case of flow of a perfect and of equilibrium air reaches values of the order of $15 \div 20\%$. A number of features of flow past blunt bodies, investigated in work (5) for the example of a perfect gas, also occur in flows of equilibrium air. At large z, where the influence of blunting and of disturbances reflected from the shock wave ceases to be felt, the pressure p_s and the inclination angle of the shock wave become equal to the corresponding values for a sharp cone; moreover (see (6)), the values of these parameters in the case of real air are somewhat smaller than in the case of perfect-gas flow (Figs. 3 and 4).

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Note: Figure translations are in progress. See original paper for figures.