Soviet-era science, translated into English

PARAMETRIC INVARIANCE OF LINEAR DYNAMICAL SYSTEMS

1964

SovietRxiv

View the original and related papers at https://sovietrxiv.org/items/ru-196401.22889

Source: Math-Net.Ru and CyberLeninka. Machine translation. Verify with the original.

SovietRxiv

Abstract

Full Text

CYBERNETICS AND CONTROL THEORY

R. S. RUTMAN, M. S. EPELMAN

PARAMETRIC INVARIANCE OF LINEAR DYNAMICAL SYSTEMS

(Presented by Academician B. N. Petrov on 22 IV 1964)

1. The range of questions encompassed by the theory of invariance of automatic-control systems ([1–3]) includes the problem of insensitivity of a system to changes in its parameters, or the problem of parametric invariance. With the study of this question in mind, consider the system of linear differential equations

$$\dot{x} = Ax \tag{1}$$

with initial conditions $x(0) = x_0$ and the scalar function

$$y = c'x. \tag{2}$$

Here x is an n-dimensional vector, $x = x(t)$, and A is a constant $n \times n$ matrix. Along with the original system, consider the system

$$\dot{x} = (A + B)x, \tag{3}$$

where B is a variation of the matrix A, and find the conditions under which the function y is unchanged on the solutions of (1) and (3), i.e.

$$c'e^{(A+B)t}x_0 = c'e^{At}x_0, \tag{4}$$

or

$$c'\left[E + (A + B)t + \frac{(A + B)^2}{2!}t^2 + \cdots\right]x_0 = c'\left[E + At + \frac{A^2t^2}{2!} + \cdots\right]x_0. \tag{5}$$

Since x_0 is an arbitrary vector from the space $R^n = \{x : x = (x_1, \dots, x_n)\}$, (5) holds if and only if the row

$$c'\left\{[(A + B)t - At] + \left[\frac{(A + B)^2}{2!}t^2 - \frac{A^2t^2}{2!}\right] + \cdots\right\}$$

SovietRxiv

is equal to zero. Hence we obtain:

$$c'B = 0, \quad c'AB = 0, \dots, \quad c'A^{mB} = 0, \dots \tag{6}$$

Of the equalities (6), only the first r are independent, where r is the maximum number of linearly independent vectors in the set c', $c'A, \dots, c'A^n$. Thus, finally, we have the necessary and sufficient conditions for parametric invariance

$$c'B = 0, \quad c'AB = 0, \dots, \quad c'A^{r-1}B = 0. \tag{7}$$

We note that these conditions, as sufficient ones, can be obtained from the results of L. I. Rozonoer ([3]).

2. We now extend conditions (7) to the case of a nonstationary matrix $B = B(t)$; $B(t)$ is a matrix of continuous functions.

The general solution of system (3) in this case is represented in the form of the matricant ([4])

$$\Omega_0^t x_0 = \left(E + \int_0^t [A + B(\tau)]\, d\tau + \int_0^t [A + B(\tau)] \int_0^\tau [A + B(\xi)]\, d\xi\, d\tau + \cdots \right) x_0, \tag{8}$$

and condition (4) is replaced by the condition

$$c'(\Omega_0^t - e^{At})x_0 = 0, \quad t \geqslant 0. \tag{9}$$

The necessary and sufficient conditions for parametric invariance are obtained in the form

$$c'B(t) = 0, \quad c'AB(t) = 0, \dots, \quad c'A^{r-1}B(t) = 0, \quad |t \geqslant 0. \tag{7'}$$

The sufficiency of conditions (7′) follows obviously from direct substitution of (7′) into (9), taking (8) into account.

To prove necessity, suppose that equality (9) holds, but at some instant $t = \theta$ one of the equalities (7′) is not satisfied:

$$c'A^{iB}(\theta) \neq 0,$$

where i takes one of the values $0, 1, \dots, r-1$.

Let m be the smallest of such numbers. By the continuity of $c'A^{mB}(t)$, it differs from zero also on some interval $[\theta_1, \theta_2]$. Then

SovietRxiv

$$\int_{\theta_1}^{t} c' A^m \frac{\tau^m}{m!} B(\tau)\, d\tau \neq 0, \quad t \in [\theta_1, \theta_2]. \tag{10}$$

But, up to terms of higher order of smallness,

$$c'(\Omega_0^t - e^{At})x_0 = \left(\int_{\theta_1}^{t} c' A^{mB}(\tau) \frac{\tau^m}{m!} \right) \Omega_0^{\theta_1} x_0;$$

therefore (10) contradicts condition (9), which proves the necessity of (7′).

3. Let us clarify the structure of the matrix B satisfying conditions (7). It follows from (7) that the vector b can be taken as a column of the matrix B if and only if it is orthogonal to each of the vectors

$$c', \ c'A, \dots, \ c'A^{r-1}, \tag{11}$$

i.e., if it belongs to the orthogonal complement in R^n to the subspace M^r spanned by the vectors of the set (11). Denote this orthogonal complement by $R^n - M^r$. Then the vector b is represented in the form

$$b = \sum_{s=1}^{n-r} \gamma_s f_s, \tag{12}$$

where

$$f = [f_1, \ f_2, \dots, \ f_{n-r}] \tag{13}$$

is a basis in $R^n - M^r$, which is constructed by known methods ([5]).

4. For illustration and in order to clarify the physical meaning of the conditions of parametric invariance, consider a second-order system.

Let

$$c = \begin{bmatrix} 1 \\ 1 \end{bmatrix}. \tag{14}$$

If we introduce the notation of the varied values of the coefficients

$$d_{ij} = a_{ij} + b_{ij}, \tag{15}$$

then the conditions of parametric invariance (7) can be reduced to the form

SovietRxiv

$$d_{11} + d_{21} = d_{12} + d_{22} = \text{const.} \tag{16}$$

The system is realized in the form of a structure with two channels for forming the invariant quantity y. In attempting to construct a single-channel system with the vector

$$c = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{17}$$

the conditions of parametric invariance lead to the trivial degenerate scheme of an open-loop system.

5. Thus, the problem of parametric invariance for the linear system (1), under stationary and nonstationary variations B of the matrix A, has a solution in the form of the system of vector equalities (7) or (7′). These equalities are solved with respect to B by means of the procedure described above. Consideration of the physical realizability of the conditions of parametric invariance emphasizes the significance of the principle of two-channel structure put forward by B. N. Petrov [1,2].

Institute
of Automation and Telemechanics

Received
16 IV 1964

CITED LITERATURE

[1] B. N. Petrov, *Proceedings of the Second All-Union Conference on the Theory of Automatic Control*, 2, Publishing House of the Academy of Sciences of the USSR, 1955. [2] B. N. Petrov, G. M. Ulanov, *Collection: Scientific and Technical Problems of Automation of Electric Drives*, Publishing House of the Academy of Sciences of the USSR, 1957. [3] L. I. Rozonoer, *Automation and Telemechanics*, No. 6 (1963); No. 7 (1963). [4] F. R. Gantmacher, *Theory of Matrices*, Moscow, 1954. [5] D. K. Faddeev, V. N. Faddeeva, *Computational Methods of Linear Algebra*, Moscow, 1963.

Note: Figure translations are in progress. See original paper for figures.

Source: Math-Net.Ru and CyberLeninka. Machine translation. Verify with the original.