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1964

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Abstract

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ON THE INTERACTION OF INTENSE HIGH-FREQUENCY RADIATION WITH A PLASMA

(Presented by Academician V. I. Veksler, 16 IV 1964)

1. Of great interest is the interaction with plasma of radiation of high frequencies adjacent to the optical range, for which in the linear theory the plasma may be regarded as transparent. Let us note that at high wave intensity the plasma becomes opaque because of nonlinear effects. Within the framework of relatively weak nonlinearity, only processes with the minimum possible number of absorbed and emitted waves play a significant role (diagrams with the smallest number of external "photon" lines). Intensive interaction between waves and plasma particles occurs only in the presence of resonance, which corresponds to the conservation laws of energy and momentum for the corresponding diagrams. Thus, absorption or emission of one wave corresponds to the condition of Cherenkov resonance $\omega^t = \mathbf{k}^t\mathbf{v}^*$. From the resonance conditions one can immediately indicate possible processes of nonlinear interaction of high-frequency radiation with plasma. Resonant interaction of particles with two waves corresponds to scattering, i.e., for example, $\omega^t + \omega^l = \mathbf{k}^t\mathbf{v} + \mathbf{k}^l\mathbf{v}$. For high frequencies $\omega^t \gg \omega^l$, $|\mathbf{k}^t| \simeq \omega^t$, we have $\omega^t \simeq \mathbf{k}^l\mathbf{v}(1 - v\cos\theta)^{-1}$, $\cos\theta = \mathbf{k}^t\mathbf{v}/|\mathbf{k}^t||\mathbf{v}|$. The maximum $|\mathbf{k}^l|$ is $\omega_0|v\tau_e|$. This indicates that the interaction takes place in the presence of suprathermal beams of charged particles. For $\omega^t \sim 10^{15}$ sec^{-1}, $\omega_0 \sim 3\cdot 10^{11}$ sec^{-1}, $v\tau_e \sim 3\cdot 10^{-3}$, the velocity of the particle beam must be close to the speed of light, $(1 - v\cos\theta)^{-1} \sim 10$. The interaction of three waves corresponds to the resonance condition $\omega_1^t - \omega_2^t \pm \omega^l = (\mathbf{k}_1^t - \mathbf{k}_2^t \pm \mathbf{k}^l)\mathbf{v}$, which may correspond already to thermal velocities v even for very large values of ω^t/ω^l. Finally, for three waves coherent interaction is possible, described by the decay conditions $\omega_1^t - \omega_2^t \pm \omega^l = 0$, $\mathbf{k}_1^t - \mathbf{k}_2^t \pm \mathbf{k}^l = 0$. Let us analyze these three possibilities.

2. Let there be no beams of charged particles in the plasma. We note that there is a deep analogy between the passage through a homogeneous isotropic plasma of beams of charged particles and beams of transverse waves. Indeed, decay processes of transverse waves are analogous to Cherenkov radiation of charged particles. The conservation laws determine the angle θ between the longitudinal and transverse quantum in decay (+) and coalescence (−)**

$$x_\pm = \cos\theta_\pm = \frac{(\mathbf{k}^t\mathbf{k}^l)}{k^t k^l} = \frac{\omega^l}{k^l} \pm \frac{(k^l)^2 - (\omega^l)^2}{2k^l k^t}. \quad (1)$$

The second term in (1), describing recoil, in contrast to the Cherenkov effect may by no means be small***. Analogously to charged particles, beams of transverse waves may be unstable with respect to the pumping of longitudinal waves. The "increment" of pumping is

$$\gamma_{k^l} = \int N^t_{\mathbf{k}^t}\left[w_{\mathbf{k}^t}(\mathbf{k}^l) - w_{\mathbf{k}^t}(-\mathbf{k}^l)\right] d\mathbf{k}^t \quad (2)$$

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* Induced Cherenkov radiation and absorption of waves (1) are described by the quasi-linear approximation (2); induced processes with a large number of absorbed and emitted waves correspond to the nonlinearities of interest to us. In what follows, the index l denotes longitudinal waves, and t transverse waves, $\hbar = c = 1$.

** Radiation of waves by waves under other assumptions was considered in (8).

*** For example, if $k^l/k^t > \omega^l/k^l$, $\omega^l/k^l \ll 1$. For relativistic plasma waves $k^l/\omega^l \to 1$ when $\omega^t \gg \omega^l$, recoil is always small.

characterizes the time scale of the process. Here $N^t_{\mathbf{k}^t}$ is the number of transverse quanta; $w_{\mathbf{k}^t}(\mathbf{k}^l)$ is the probability of decay of $\mathbf{k}^t$, and $w_{\mathbf{k}^t}(-\mathbf{k}^l)$ is the probability of coalescence; $k^l \equiv \{\mathbf{k}^l, \omega^l(\mathbf{k}^l)\}$,

$$w_{\mathbf{k}^t}(\mathbf{k}^l) = \frac{e^2(\mathbf{k}^l)^2|\omega^l|}{16\pi m_e^2|\mathbf{k}^t|\,|\mathbf{k}^t - \mathbf{k}^l|}\left[1 + \frac{(\mathbf{k}^t, \mathbf{k}^t - \mathbf{k}^l)^2}{(\mathbf{k}^t)^2(\mathbf{k}^t - \mathbf{k}^l)^2}\right]\delta(|\mathbf{k}^t| - |\mathbf{k}^t - \mathbf{k}^l| - \omega^l). \quad (3)$$

Expression (3) is valid for $\omega^l/k^l \gg v_{Te} = \sqrt{T_e/m_e}$; $\omega^t \simeq |\mathbf{k}^t|$. For small recoil, (2) contains a derivative with respect to the momentum $\mathbf{k}^t$

$$\gamma_{k^l} \simeq \int w^{(0)}_{\mathbf{k}^t}\left(\mathbf{k}^l \frac{d}{d\mathbf{k}^t}\right) N^t_{\mathbf{k}^t}\, d\mathbf{k}^t; \qquad w^{(0)}_{\mathbf{k}^t} = \frac{e^2\omega^l}{8\pi m_e^2}\left(\frac{k^l}{k^t}\right)^2 \delta(\mathbf{k}^l\mathbf{v}^t_{\text{gr}} - \omega^l), \quad (4)$$

where

$$\mathbf{v}^t_{\text{gr}} = \frac{\mathbf{k}^t}{|\mathbf{k}^t|}\frac{d\omega^t}{dk^t} \simeq \frac{\mathbf{k}^t}{\sqrt{(k^t)^2 + \omega_0^2}}; \qquad \omega_0^2 = \frac{4\pi n e^2}{m_e}.$$

For almost monochromatic radiation with $v^l_\phi = \omega^l/k^l \ll 1$, we obtain

$$\gamma_{k^l} \simeq \omega^l \frac{\pi^2}{2} \frac{e^2}{mc^2} \left(\frac{c}{\omega^t}\right)^2 \left(\frac{k^l}{k^t}\right)^2 \frac{d}{d\theta^2} \frac{W^t(\theta^2)}{mc^2}; \qquad W^t = \int W(\theta^2)\, d\theta^2 = \int \frac{|\mathbf{k}^t|\, N^t_{\mathbf{k}^t}\, d\mathbf{k}^t}{(2\pi)^3}. \tag{5}$$

Thus, the increment in the present case is proportional to the angular derivative and is the larger the better collimated the beam of transverse waves is.* For large

$$\left(\frac{k^l}{2k^t}\right)^2 \frac{1}{W^t(\theta^2)} \frac{dW^t(\theta^2)}{d\theta^2},$$

however, (5) is inapplicable, since the expansion loses its meaning. For a well-collimated beam, the maximum increment corresponds to setting to zero the probability of coalescence in (2). This gives an estimate of the maximum increment arising when $\mathbf{k}^l$ is parallel to $\mathbf{k}^t$ and $k^l \simeq 2k^t$,

$$\gamma^{\max}_{k^l} \simeq \omega^t \frac{W^t}{nmc^2}. \tag{6}$$

Formula (6), applicable for $\gamma^l_{\mathbf{k}} \ll \omega^l$, $W^t \ll nm_e v^2_{Te} \left(\omega^t/\omega^l\right)^2$, indicates the possibility of the occurrence of large increments (of order ω^l). We note that, when a large value of the energy of longitudinal waves W^l arises, a substantial nonlinear interaction of longitudinal waves with longitudinal ones may occur, leading to a decrease of k^l [3,4].

3. The analogy with beams of charged particles becomes still more complete if one considers the change in the distribution of relatively weak transverse waves in their interaction with longitudinal ones. In the case of small recoil the equation obtained is

$$\frac{\partial N^t_{\mathbf{k}^t}}{\partial t} = \frac{\partial}{\partial k^t_i} D_{ij} \frac{\partial N^t_{\mathbf{k}^t}}{\partial k^t_j}; \qquad D_{ij} = \int k^l_i k^l_j N^l_{k^l} w^{(0)}_{k^l}\, dk^l \tag{7}$$

analogously to quasilinear equation (2). The change in the spectrum of transverse waves in a turbulent plasma has an order of magnitude

$$\frac{\Delta\omega^t}{\omega^t} \simeq \left(\frac{e^2}{m_e} \frac{t}{\omega^t} \frac{W^l}{m_e c^2}\right)^{1/2} \left(\frac{k^l}{k^t}\right)^{3/2}. \tag{8}$$

and can, in principle, be used for diagnostics of plasma noise and modulation of beams of transverse waves. We note that at large energies of transverse waves there becomes significant a nonlinear transfer of the energy of transverse waves from higher frequencies to lower ones, with a characteristic increment of order

$$\gamma_{\mathbf{k}^t} \simeq \omega^l \left(\frac{\omega^t}{\Delta\omega^t}\right) \frac{e^2}{m_e(\omega^t)^2} \left(\frac{k^l}{k^t}\right)^2 \frac{W^t}{m_e c^2}; \tag{9}$$

* The increment proves to be proportional to the derivative with respect to momenta, in complete analogy with the case of a beam of particles, only in the generation of relativistic plasma waves.

and nonlinear generation of longitudinal waves

$$\frac{\partial N^l_{\mathbf{k}^l}}{\partial t} = \int N^t_{\mathbf{k}^t} \left[N^t_{\mathbf{k}^t-\mathbf{k}^l} w_{\mathbf{k}^t}(k^l) + N^t_{\mathbf{k}^t+\mathbf{k}^l} w_{\mathbf{k}^t}(-k^l)\right] dk^t, \tag{10}$$

which is of order

$$W^l \simeq \pi^2(\omega^l t) \frac{e^2}{m^2(\omega^t)^2} \frac{W_1^t W_2^t}{\Delta\theta_1 \Delta\theta_2} \left(\frac{\omega^l}{\omega^t}\right) \left(\frac{k^l}{k^t}\right)^4, \tag{11}$$

where $\Delta\theta$ is the angular divergence of the beam. Note that, according to (11), longitudinal waves can be generated both by one and by two beams of transverse waves. The latter case has the advantage of making it possible to create waves with prescribed phase velocities. If ω^l/k^l is close to v_{Te}, the Landau absorption ([5]) of the generated waves will increase the mean thermal energy of the plasma particles.

In the case in which the decay conditions are not fulfilled for the given $\mathbf{k}_1^t, \omega_1^t$ and $\mathbf{k}^l, \omega^l; \mathbf{k}_2^t, \omega_2^t$, it is sufficient that they be approximately fulfilled in order for the conditions for three-plasmon scattering to be satisfied. If

$$\left|\frac{\omega_2^t - \omega_1^t \pm \omega^l}{|\mathbf{k}_2^t - \mathbf{k}_1^t \pm \mathbf{k}^l|}\right| \lesssim v_{Te},$$

processes of induced three-plasmon scattering can generate longitudinal waves with an increment of order

$$\gamma_{\mathbf{k}^l} \simeq \gamma_{1\mathbf{k}^l} \frac{W^t}{n m_e v_{Te}^2}, \tag{12}$$

which may be considerable.

4. In the presence of two beams of transverse waves let us consider the case in which the frequency difference $\omega_2^t - \omega_1^t$ and the difference of the wave vectors $|\mathbf{k}_2^t - \mathbf{k}_1^t|$ correspond to the absorption region of longitudinal waves, $|\omega_2^t - \omega_1^t| \ll |\mathbf{k}_2^t - \mathbf{k}_1^t| v_{Te}$. Here it is expedient to speak of nonlinear damping of the transverse waves of each of the beams. To calculate the decrement

it is necessary to take into account only the forced fields E^l, quadratic in the fields of the two beams,

$$\frac{\mathbf{k}^l}{k^l} E^l_{\omega^l \mathbf{k}^l} = -\frac{4\pi i e^3}{\varepsilon^l(\omega^l, \mathbf{k}^l)\, k^l m^3} \int \frac{d\mathbf{v}}{\omega^l - \mathbf{k}^l \mathbf{v}} \left(\mathbf{F}_1 \frac{\partial}{\partial \mathbf{v}}\right) \frac{1}{\omega_2^t - \mathbf{k}_2^t \mathbf{v}} \left(\mathbf{F}_2 \frac{\partial}{\partial \mathbf{v}}\right) f(\mathbf{v})\, d\Lambda, \tag{13}$$

$$\mathbf{F}_i = \mathbf{E}_i^t \left(1 - \frac{\mathbf{k}_i^t \mathbf{v}}{\omega_i^t}\right) + \frac{\mathbf{v}(\mathbf{k}_i^t \mathbf{E}_i^t)}{\omega_i^t};$$

$$d\Lambda = d\omega_1^t d\omega_2^t d\mathbf{k}_1^t d\mathbf{k}_2^t\, \delta(\omega^l - \omega_1^t + \omega_2^t)\, \delta(\mathbf{k}^l - \mathbf{k}_1^t + \mathbf{k}_2^t).$$

Using the approximation $\omega^l \ll |\mathbf{k}^l|\langle v \rangle$, substituting (13) into the nonlinear current $\mathbf{j}^t$, quadratic in E^l, E^t, and averaging over phases, we obtain an effective nonlinear permittivity $\varepsilon^t_{\text{eff}}$, which makes it possible to find the desired decrement, whose optimal value corresponds to $|\mathbf{k}_2^t - \mathbf{k}_1^t| \ll \omega_0 / v_{Te}$, with

$$\gamma^t_{\text{eff}} \simeq \omega_2^t \operatorname{Im} \varepsilon^t_{\text{eff}} \simeq \omega_2^t \frac{e^2}{m_e (\omega_2^t)^2} \frac{W^t}{mc^2} \left(\frac{\omega^l}{\omega^t}\right)^2 \frac{|\omega_1^t - \omega_2^t|}{|\mathbf{k}_1^t - \mathbf{k}_2^t|} \left\langle \frac{1}{v} \right\rangle. \tag{14}$$

Owing to the presence of the last two small factors in (14), γ^t_{eff} is substantially smaller than (9), and still more so than (6).*

A generation of plasma sound by transverse waves is close to the effect considered above, if $T_e \gg T_i$. In this case it is necessary that $|\mathbf{k}_2^t - \mathbf{k}_1^t| v_{Ti} \ll |\omega_2^t - \omega_1^t| \ll |\mathbf{k}_2^t - \mathbf{k}_1^t| v_{Te}$. Without dwelling in detail on questions of pumping and generation of sound by beams of transverse waves,

* The qualitative explanation of this is that, although the arising fields E^l lie in the absorption region, the absolute values of E^l are not large.

modulations by sound waves of transverse waves, etc., the analysis of which is analogous to Secs. 2 and 3, we give the value for the probability of decay of transverse waves into sound

$$w_{\mathbf{k}^t}(k^s) = \frac{e^2 \omega^s \omega_0^2}{16\pi |\mathbf{k}^t|\, |\mathbf{k}^t - \mathbf{k}^s|} \left\langle \frac{1}{v^2} \right\rangle \left(1 + \frac{(\mathbf{k}^t, \mathbf{k}^t - \mathbf{k}^s)^2}{(k^t)^2 (\mathbf{k}^t - \mathbf{k}^s)^2}\right) \delta(|\mathbf{k}^t| - |\mathbf{k}^t - \mathbf{k}^s| - \omega^s), \tag{15}$$

where $\omega^s = k^s v_s$; $v_s = \sqrt{m_e/m_i}\, v_{Te}$.

5. Let us next consider a plasma in the presence of beams of charged particles. The effects of induced scattering of transverse waves on a beam with conversion into longitudinal waves make the interaction of high-frequency radiation with the plasma effective. We note that particle beams may be unstable with respect to transverse waves, losing in this case a substantial fraction of their energy, even if direct generation of longitudinal waves is impossible. If the longitudinal waves in a turbulent plasma are directed along the particle beam, then the increment for the excitation of transverse waves is [6]

$$\gamma^t_{\mathbf{k}^t} = -\frac{32\pi^3 e^4 \omega_0^2}{m_e^2 (k^t)^3} \sin^2 2\theta \int \varphi(\mathbf{v})\, d\mathbf{v}\ \left.\frac{dN^l_{\mathbf{k}^l}}{dv^l_\varphi}\right|_{v^l_\varphi = {}^t v_0/k^t}, \tag{16}$$

where $\varphi(\mathbf{v})$ is the one-dimensional distribution function of the beam particles; θ is the angle between $\mathbf{k}^t$ and $\mathbf{k}^l$; $v \ll 1$. Generation of transverse waves is effective in bounded systems. The analysis shows that the maximum possible energy of the transverse waves that arise may be estimated by the formula

$$W^t_{\max} \simeq \frac{m_e n}{6\pi} \left(\frac{\omega^t}{\omega^l}\right) v_0 \left(\frac{v_0}{l\omega_0}\right)^{1/2}, \tag{17}$$

where v_0 is the beam velocity and l is the size of the system.

In the presence of external radiation, effects of beam acceleration may arise. If, after the development of instabilities of longitudinal waves, the state of the beam corresponds to a plateau up to $v = v_0$ [7], then absorption of transverse waves occurs for particles with $v > v_0$, $\partial\varphi/\partial v < 0$, which are effectively accelerated in a characteristic time of order

$$\tau \simeq \left(\frac{n}{n_1}\right)^2 \left(\frac{\omega^t}{\omega^l}\right)^2 \frac{m_e n_1 (\Delta v_0/v_0)^2}{64\pi^4 \omega^l W^t(0)} \ln \frac{\omega^l}{\omega^t} \frac{m n_1 v_0^2}{W^l(0)}, \tag{18}$$

where n_1 is the beam density. In a time of order τ all the energy of the transverse waves is transferred to the beam particles. Analogous relations hold for relativistic beams. The frequency of the transverse waves that accelerate the beam particles or are generated by them increases with increasing particle energy.

The authors express their deep gratitude to V. I. Veksler, M. S. Rabinovich, and Ya. B. Fainberg for their interest in the work and for discussion of the results.

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Received
16 IV 1964

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Note: Figure translations are in progress. See original paper for figures.