DESCENDING CONVECTIVE FLOWS

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Abstract

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DESCENDING CONVECTIVE FLOWS

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It is known that stationary convective flows can arise spontaneously in unstable layers. Flows of this kind are formed as a result of certain perturbations in the temperature or wind field; their development occurs at the expense of the energy of the vertical instability of the layer. The laws governing the change of temperature and velocity of ascending "spontaneous jets" have been investigated both theoretically ([1,2]) and experimentally ([3]). Meanwhile, in unstable layers descending convective jets may also arise (see, for example, ([4])). In the case where the instability of the layer changes with height (as occurs in the real atmosphere ([5,2])), it is evident that the laws governing the change of temperature and velocity of descending convective jets will differ from the laws obtained for ascending jets. The present work is an attempt to estimate these differences theoretically.

Let us consider an unstable layer ($\gamma > \gamma_*$), in which the vertical temperature gradient γ is greater than the dry-adiabatic or moist-adiabatic value γ_*, depending on whether we consider a cloudless or cloudy layer. The phenomenon of a stationary axisymmetric jet propagating in the indicated layer, after the usual simplifications of the theory of convection and boundary-layer theory ([6,2]), may be described (in cylindrical coordinates) by the equations of motion, heat influx, and continuity in the form

$$u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z} = \beta g\vartheta + \frac{1}{r}\frac{\partial}{\partial r}\left(K_1 r\frac{\partial w}{\partial r}\right),$$

$$u\frac{\partial \vartheta}{\partial r} + w\frac{\partial \vartheta}{\partial z} = (\gamma - \gamma_*)w + \frac{1}{r}\frac{\partial}{\partial r}\left(K_2 r\frac{\partial \vartheta}{\partial r}\right), \qquad (1)$$

$$\frac{\partial}{\partial r}(ur) + \frac{\partial}{\partial z}(wz) = 0$$

under the boundary conditions

$$u = \frac{\partial w}{\partial r} = \frac{\partial \vartheta}{\partial r} = 0 \quad \text{for } r = 0; \qquad (2)$$

$$w = \vartheta = K_1 r \frac{\partial w}{\partial r} = K_2 r \frac{\partial \vartheta}{\partial r} = 0; \quad ur \text{ bounded for } r = R(z).$$

Here u and w are the radial and vertical components of velocity; ϑ is the temperature excess in the jet; g is the acceleration of gravity; β is the temperature coefficient of density; K_1 and K_2 are the coefficients of turbulent friction and turbulent heat exchange; R is the radius of the jet, assumed known from observations ([3]).

In an approximate solution of the problem (using integral relations describing the cross-section-averaged energy balance in the jet) and seeking the solution in the form

$$w = w_0(z) f_1(r/R), \qquad \vartheta = \vartheta_0(z) f_2(r/R), \tag{3}$$

where w_0 and ϑ_0 are the vertical velocity and temperature excess on the axis of the jet, and f_1 and f_2 are the profiles of w and ϑ in the jet ($f_1(0) = f_2(0) = 1$), system (1), with (2) and (3) taken into account, is transformed into a system of two ordinary equa-

$$\frac{d}{dz}(w_0 R)^2 = a_1 \beta g \vartheta_0 R^2, \qquad \frac{d}{dz}(w_0 \vartheta_0 R^2) = a_2(\gamma - \gamma_*) w_0 R^2. \tag{4}$$

This system can be reduced to a single second-order equation

$$\frac{1}{R}\frac{d}{dz}\left\{\frac{1}{R}\frac{d}{dz}(w_0 R)^3\right\} = \frac{3}{2} a_1 a_2 \beta g (\gamma - \gamma_*) w_0 R, \tag{5}$$

where a_1 and a_2 are constants depending on the profiles f_1 and f_2.

Theoretical considerations ([5]) and observations ([3]) show that both the radius of the jet and the degree of instability of the layers under consideration can be represented as power functions of height,

$$R = b z^n, \qquad \gamma - \gamma_* = c z^{-p}, \tag{6}$$

where b, n, c, and p are constants.

If we introduce the new variables

$$v = (w_0 z^n)^3, \qquad \zeta = z^{n+1}, \tag{7}$$

then (5), taking (6) and (7) into account, takes the form

$$\frac{d^2 v}{d\zeta^2} = A \zeta^{-p/(n+1)} v^{1/3}, \tag{8}$$

where the constant

$$A = \frac{3}{2}\frac{a_1 a_2}{(n+1)^2}\beta g c. \tag{9}$$

Up to now we have considered ascending jets. However, equation (5) applies equally to both ascending and descending jets. For descending jets, measuring from some level H the negative height z_1,

$$z_1 = H - z \tag{10}$$

and taking $R = b z_1^n$, by analogous reasoning we obtain the equation

$$\frac{d^2 u}{d\xi^2} = A\left(H - \xi^{1/(n+1)}\right)^{-p} u^{1/3}, \tag{11}$$

where

$$u = (w_0 z_1^n)^3, \qquad \xi = z_1^{n+1}. \tag{12}$$

Equation (8) has the particular solution

$$w_0 = \left\{\frac{4(n+1)^2}{3\left[8(n+1)^2 - 10p(n+1) + 3p^2\right]}\right\}^{1/2} A^{1/2} z^{1-p/2}, \tag{13}$$

$$\vartheta_0 = \frac{2}{a_1 \beta g}\left\{\frac{2(n+1)^2(2-p+2n)}{3\left[8(n+1)^2 - 10p(n+1) + 3p^2\right]}\right\} A z^{1-p},$$

which, for any $n > 0$ and $p < 4/3$, describes the case of so-called spontaneous jets, i.e., jets formed in the absence of heat sources.

Descending jets may also arise spontaneously. The vertical velocity and temperature of such descending jets, on the basis of the solution of (11), are determined by the relations

$$w_0 = \frac{z_1}{H^{n+1}} \sum_{k=0}^{\infty} A_k \left(\frac{z_1}{H}\right)^k,$$

$$\vartheta_0 = \frac{2}{a_1 \beta g}\frac{z_1}{H^{2(n+1)}} \sum_{k=0}^{\infty} A_k \left(\frac{z_1}{H}\right)^k \sum_{l=0}^{\infty} (n+l+1) A_l \left(\frac{z_1}{H}\right)^l, \tag{14}$$

where the constants

Fig. 1

Figure 1: Fig. 1

$$A_m^* = A_m^* A^{1/2} H^{n+1-p/2}, \tag{15}$$

and the numerical values A_m^* are determined on the basis of the recurrence formulas

$$2A_0^{*2} = \frac{1}{3}, \tag{16}$$

$$\frac{1}{2}\sum_{k=0}^{i}\{k(n+k+1)+(i-k)(n+i-k+1)+4(n+k+1)(n+i-k+1)A_k^* A_{i-k}^*\} = \frac{(n+1)^2}{3}\frac{p(p+1)\ldots(p+i}{i!}$$

where $i = 1, 2, 3, \ldots$

Fig. 1. Dependence on the quantity p of the ratios of the velocities (1) and temperatures (2) of descending conical jets to the corresponding values of ascending jets at heights $z_1 = z = 0.1H$ (a) and at heights $z_1 = z = H$ (b)

The estimates carried out showed that if, in the unstable near-surface layer (in which, according to [5], $p = 4/3$), the profile of the descending jets is the same as that of the ascending jets [3], i.e. $n = 1/3$, then descending jets originating at height H will have, near the earth' s surface, a velocity approximately 40% smaller than the velocity of ascending jets at height H. The temperature excess of the descending jets, on the contrary, will be almost 30% greater (in absolute magnitude) than that of the ascending jets.

In clouds, according to measurements [3], the jets have a conical shape ($n = 1$). For such jets, Fig. 1 presents, as functions of the values of p, the ratios of the velocities and temperatures of descending jets to the corresponding values for ascending jets at heights $z_1 = z = 0.1H$ and $z_1 = z = H$. As can be seen from the graphs presented, at comparatively small heights (Fig. 1a) the indicated ratios for $p > 0$ are less than unity and decrease substantially with increasing p. However, the final velocities and temperatures of the descending jets (Fig. 1b), over a wide range of values of p, turn out to be greater than those of the ascending jets.* For the value $p = 1/2$ [3], obtained from observations in developing cumulus clouds, these differences (far from optimal) proved to be of the order of 10% for velocity and about 70% for temperature.

The results obtained agree with observational data [8,9]. In descending currents in thunderclouds, as one approaches the base of the cloud, the intensity and the difference of temperature from the surroundings increase–

* The data presented should be regarded as an estimate of the relative values of the velocities and temperatures of descending motions; in the calculation, 6 terms of the series (14) were used. It should also be noted that for $z \to H$ the convergence of the series slows down, especially for $p > 1$. However, in clouds the value of p, apparently, cannot exceed 4/3, which corresponds to the case of constancy of the heat flux with height ([5]).

···air, with the velocities and temperatures of the descending currents near the base of the cloud being substantially greater than those of the ascending ones.

The theoretical consideration presented above explains the cause both of these phenomena and of a number of phenomena accompanying them, in particular the presence beneath thunderclouds, at the earth' s surface, of local zones of increased pressure, a substantial lowering of the air temperature there, and also squally gusts of cold wind directed away from the cloud, which are usually observed during the passage of thunderclouds ([8,9]).

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