SovietRxiv

Soviet-era science, translated into English

M. V. VELICHKO, L. V. RADUSHKEVICH

1964

SovietRxiv

View the original and related papers at https://sovietrxiv.org/items/ru-196401.18289

Source: Math-Net.Ru and CyberLeninka. Machine translation. Verify with the original.

SovietRxiv

Abstract

Full Text

PHYSICAL CHEMISTRY

M. V. VELICHKO, L. V. RADUSHKEVICH

PROPERTIES OF A HYDRODYNAMIC FLOW PAST VERY SMALL STATIONARY OBSTACLES, AND THE EFFICIENCY OF CAPTURE OF AEROSOL PARTICLES

(Presented by Academician M. M. Dubinin, 8 VIII 1963)

The possibility of applying the Navier–Stokes hydrodynamic equations to solving problems of flow past obstacles whose characteristic size R is comparable with the mean free path λ of gas molecules depends on the value $M(\lambda/R)$, where M is the Mach number, equal to the ratio of the flow velocity to the speed of sound. As shown in ([1]), p. 310, ordinary hydrodynamics remains valid for $M \ll 1$, even if $\lambda/R \cong 1$. In this case, however, it is necessary to introduce special boundary conditions. Slightly modifying the method proposed by Fuchs ([2]), we have calculated the slip velocity for the cases of gas flow past a sphere and a cylinder whose radii are close to the value of λ. The solutions obtained have been used by us to calculate the efficiency of capture of aerosol particles from a flow on a stationary cylinder.

We shall assume that ordinary hydrodynamics is applicable at distances r from the obstacle greater than $a = R + \beta\lambda$, with $\beta \cong 1$, while at distances in the interval $R \ll r \ll a$ the process is carried out by free molecular flow. Matching the differential hydrodynamic and molecular flows on a surface element dS, we determine the parameters entering the expressions for the flow field of a viscous fluid.

In the general case, the friction force applied to a surface element dS with radius of curvature a is directed along the flow and is equal to

$$dF_a = (p_{rr}\cos\theta - p_{r\theta}\sin\theta)\, dS, \tag{1}$$

where the components of the hydrodynamic momentum are expressed in polar coordinates by the known relations

$$p_{rr} = -p + 2\mu\frac{\partial v_r}{\partial r}; \qquad p_{r\theta} = \mu\left(\frac{1}{r}\frac{\partial v_r}{\partial\theta} + \frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r}\right); \tag{2}$$

SovietRxiv

here v_r and v_θ are the velocity components, μ is the gas viscosity, and p is the total pressure.

Assuming a Maxwellian distribution of gas-molecule velocities, let us write the expression for the force acting on a surface element dS' of radius R, from molecules incident upon it and diffusely reflected with accommodation under free molecular flow. In doing so we use Epstein' s results (([3]); ([1]), p.283); then

$$dF_{\mathrm{m}} = \frac{1}{2} Nmc\,(\overline{v_r}\delta\cos\theta - \frac{1}{2} v_\theta \sin\theta)\,dS'. \tag{3}$$

Here $\delta = 1 + \pi/4$. Taking into account that $\mu = 0.499\,Nmc\lambda \cong \frac{1}{2} Nmc\lambda$, and equating (1) and (3), we obtain a series of identities with respect to functions of θ, from which we find the constants of the hydrodynamic equations for the flow.

1. Stationary sphere. The velocity components for a viscous flow past a sphere of radius R at small Reynolds numbers are expressed—

are of the form

$$v_r = u_0 \cos\theta \left(1 - \frac{A}{r} + \frac{B}{r^3}\right),$$

$$v_\theta = -u_0 \sin\theta \left(1 - \frac{A}{2r} + \frac{B}{2r^3}\right). \tag{4}$$

The coefficients A and B are determined by the boundary conditions. In ordinary hydrodynamics, for the motion of a sphere in a viscous liquid, it is known that

$$A = {}^3/_2 R; \qquad B = {}^1/_2 R^3. \tag{4'}$$

In the problem under consideration, with the use of (1), (2), and (3), A and B take other values, which we find as follows. Since it is known from (4) that $p = -\mu \dfrac{u_0 A}{r^2} \cos\vartheta$, from (2) and (4) we obtain

$$p_{rr} = 3\mu u_0 \cos\theta \left(\frac{A}{r^2} - 2\frac{B}{r^4}\right); \qquad p_{r\theta} = -3\mu u_0 \sin\theta \cdot \frac{B}{r^4}.$$

Substituting these expressions into (1) and taking into account that $dS = 2\pi a^2 \sin\theta\, d\theta$, we have

$$dF_a = 3\mu u_0 \left[\left(A - 2\frac{B}{a^2}\right)\cos^2\theta + \frac{B}{a^2}\sin^2\theta\right] 2\pi \sin\theta\, d\theta.$$

SovietRxiv

Introducing v_r and v_θ from (4) into relation (3), we find, for $dS' = 2\pi R^2 \sin\theta\, d\theta$,

$$dF_{\mathrm{M}} = {}^1/_2 Nmcu_0 \left[\delta\left(1 - \frac{A}{r} + \frac{B}{r^3}\right)\cos^2\theta + \right.$$

$$\left. + {}^1/_2\left(1 - \frac{A}{2r} + \frac{B}{2r^3}\right)\sin^2\theta\right] 2\pi R^2 \sin\theta\, d\theta.$$

Equating dF_a and dF_{M}, we obtain identities with respect to θ, from which, taking into account that $\mu \cong {}^1/_2 Nmc\lambda$, it follows that

$$A = \frac{1}{G}\, 3(1+\beta\xi)\left[\delta + 4\xi(1+\beta\xi)(\delta+1)\right] R; \tag{5}$$

$$B = \frac{1}{G}(1+\beta\xi)^3[\delta + 6\xi(1+\beta\xi)]\, R^3, \tag{6}$$

where

$$G = 36\xi^2(1+\beta\xi)^2 + 12\xi\delta(1+\beta\xi) + 9\xi(1+\beta\xi) + 2\delta, \tag{7}$$

with $\xi = \lambda/R$. Relations (5), (6), and (7) give the solution of the problem. In the limiting case, for a large sphere radius R and small λ, we have $\xi \to 0$, and then (5) and (6) give the classical values of A and B from (4′), and $G = 2\delta$. Integrating the expression dF_a over the sphere of radius a, we obtain the formula for the resistance

$$F = 4\pi\mu u_0 A.$$

Let us compare this relation with Millikan' s formula for a small sphere:

$$F = 6\pi\mu R u_0\left[1 + P\lambda/R + Q(\lambda/R)e^{-bR/\lambda}\right]^{-1}.$$

Here $P = 1.246$; $Q = 0.42$; $b = 0.87$; $\lambda = 0.635 \cdot 10^{-5}$ cm. Put $R = \lambda$. Then the coefficient A is transformed, for $\xi = 1$, into the form

$$A = \frac{3R(1+\beta)[\delta + 4(1+\beta)(\delta+1)]}{36(1+\beta)^2 + 12\delta(1+\beta) + 9(1+\beta) + 2\delta}.$$

From this we obtain the value $\beta \cong 0.42$, i.e., the two flows merge at a distance $\beta\lambda = 2.69 \cdot 10^{-6}$ cm from a sphere of radius $R = 0.653 \cdot 10^{-5}$ cm.

2. A stationary cylinder. Carrying out an analogous derivation for a cylinder of radius R with unit length, we use Lamb' s solution ([4]) for small Re in the form

SovietRxiv

$$v_r = \frac{A_0}{r} - \frac{A_1 \cos\theta}{r^2} + u_0 \cos\theta - C_0 \left(\frac{1}{2kr} + \frac{1}{2}\cos\theta - \frac{1}{2}\cos\theta \ln z \right),$$

$$v_\theta = -\sin\theta \left(u_0 + \frac{A_1}{r^2} + \frac{C_0}{2} \ln z \right), \tag{8}$$

where $z = \frac{1}{2}\gamma kr$. In ordinary hydrodynamics the values of the coefficients of these equations are expressed as

$$C_0 = \frac{4u_0}{1 - 2\ln z}; \qquad A_0 = \frac{4\nu}{1 - 2\ln z}; \qquad A_1 = -\frac{u_0}{1 - 2\ln z} R^2,$$

where ν is the kinematic viscosity. It is known that

$$p = -\rho u_0 \frac{A_0}{r} \cos\theta - \rho u_0 \frac{A_1}{r^2} \cos 2\theta + \ldots$$

Therefore, according to (2), we find:

$$p_{rr} = 2\mu \left[\cos\theta \left(\frac{\rho u_0}{2\mu} \frac{A_0}{r} + \frac{2A_1}{r^3} + \frac{C_0}{2r} \right) + \left(\frac{C_0}{2kr^2} - \frac{A_0}{r^2} \right) \right] - \rho u_0 A_1 \frac{\cos 2\theta}{r^2};$$

$$p_{r\theta} = \mu \frac{4A_1}{r^3} \sin\theta.$$

Here $k = u_0/2\nu$.

Introducing these expressions into formula (1) and taking into account that $dS = a\,d\theta$, we find dF_a. Substituting again the expressions for v_r and v_θ for the cylinder into formula (3), taking into account that $dS' = R\,d\theta$, we obtain dF_m. As before, equating dF_a and dF_m, and from the resulting identity finding the coefficients with the adopted boundary conditions, we then have

$$C_0 = \frac{4u_0}{\varkappa - 2\ln z}; \qquad A_0 = \frac{4\nu}{\varkappa - 2\ln z}; \qquad A_1 = -\frac{u_0}{\varkappa - 2\ln z} \omega a^2;$$

$$\omega = \frac{2\lambda/R + \delta/2}{28\lambda/R + \delta/2 + \lambda/R}; \qquad \varkappa = (8\lambda/R + 1)\omega.$$

Introducing these expressions into formulas (8); then putting $C = u_0(\varkappa - 2\ln z)^{-1}$, we finally find

$$\begin{aligned} v_r &= C\cos\theta \left(\omega a^2/r^2 + \varkappa - 2 + 2\ln r/a \right), \\ v_\theta &= C\sin\theta \left(\omega a^2/r^2 - \varkappa - 2\ln r/a \right). \end{aligned} \tag{9}$$

SovietRxiv

These relations are valid for distances from the cylinder greater than $\beta\lambda$. In the limiting case, when $\lambda/R \to 0$, we have $\varkappa = 1$, $\omega = 1$, and then the expressions for C_0, A_0, and A_1 pass into the usual relations written above.

3. Efficiency of deposition of aerosol particles on a cylindrical obstacle

Consider the deposition of aerosol particles from a flow onto a cylinder arranged perpendicular to the flow, the particle radius r_{ch} being smaller than the cylinder radius R, but under the condition that R is close to λ. Let the particle move along a streamline. From formulas (9) we easily find the stream function

$$\Psi = \frac{u_0 \sin\theta}{\varkappa - 2\ln z}\left[(\varkappa - 2)r + \omega\frac{a^2}{r} + 2r\ln\frac{r}{a}\right]. \tag{10}$$

Following Langmuir[5], we approximately find the efficiency η of deposition due to contact and diffusion. Let x_0 be the thickness of the layer at $\theta = \pi/2$, from which the particles are removed from the flow as a result of one of the two mechanisms. In the case of contact $x_0 = r_{\text{ch}}$. Then, by (6), from (9) and (10) we have, per unit length of cylinder,

$$\eta = \frac{2}{u_0 \cdot 2R}\int_a^{a+r_{\text{ch}}} \left.\frac{d\Psi}{dr}\right|_{\theta=\pi/2} dr = -\frac{1}{u_0 R}\int_a^{a+r_{\text{ch}}} v_\theta\big|_{\theta=\pi/2}\, dr =$$

$$= \frac{1+\beta\lambda/R}{\varkappa - 2\ln z}\left[(\varkappa - 2)q + \frac{\omega}{q} + 2q\ln q - (\varkappa + \omega - 2)\right], \tag{11}$$

where $q = 1 + r_{\text{ch}}/a$.

We find the deposition efficiency in diffusion by introducing the absolute displacement $\bar{x} + \beta\lambda$ of a particle in Brownian motion over the time t:

$$\beta\lambda + \bar{x} = \left(\frac{4}{\pi}Dt\right)^{1/2}. \tag{12}$$

We determine this time by considering the motion of a particle near the cylinder between the points $\theta_1 = \frac{1}{6}\pi$ and $\theta_2 = \frac{5}{6}\pi$ along the streamline passing at $\theta_0 = \pi/2$ at a distance $x_0 + \beta\lambda$ from the surface of the cylinder. Integrating the expression for v_θ, we find:

$$t = \frac{\bar{r}}{v_{\theta_0}}\ln\left(\operatorname{tg}\frac{\pi}{12}\Big/\operatorname{tg}\frac{5}{12}\pi\right), \tag{13}$$

where $\bar{r} = \bar{x} + a$. We define the quantity $\bar{x}$ as the mean distance of the particle from the cylinder of radius a as it moves along the streamline from θ_1 to θ_2, if

SovietRxiv

at θ_0 we set $x = x_0$ and $r_0 = x_0 + a$. Averaging Ψ over θ, we find the relation between x_0 and $\bar{x}$ in the form

$$\frac{(\chi - 2)r_0 + \omega a^2/r_0 + 2r_0 \ln r_0/a}{(\chi - 2)\bar{r} + \omega a^2/\bar{r} + 2\bar{r} \ln \bar{r}/a} = \overline{\sin\theta}, \tag{14}$$

we eliminate $\bar{x}$ and t from equations (12), (13), and (14) and compute x_0 for the specified values of R, r , u_0, and β.

As an example we take: $R = 2 \cdot 10^{-5}$ cm; $r = 2 \cdot 10^{-6}$ cm; $D = 3.8 \cdot 10^{-5}$ cm^2/sec and $u_0 = 2$ cm/sec. Assuming, on the basis of the preceding, $\beta = 0.42$, we find $\bar{x} = 1.18x_0$; $x_0 = 3.39(R + \beta\lambda)$; $\chi - 2\ln z = 18$; $\chi = 2.35$; $\omega = 0.63$. The deposition efficiency due to diffusion is obtained in this case as $\eta = 0.75$; for the contact mechanism we find $\eta = 0.35$.

Institute of Physical Chemistry
Academy of Sciences of the USSR

Received
27 VII 1963

REFERENCES

1. H. S. Tsien, *J. Aeron. Sci.*, **13**, No. 12 (1946); collection *Gas Dynamics*, IL, 1950.

2. . . Fuchs, *Zs. phys. Chem.*, A, **171**, 199 (1934).

3. P. S. Epstein, *Phys. Rev.*, **23**, 710 (1924); collection *Gas Dynamics*, IL, 1950.

4. G. Lamb, *Hydrodynamics*, Ch. XI, §§ 335, 343, 1947.

5. I. Langmuir, *The Collected Works*, **10**, 1961, p. 354.

Note: Figure translations are in progress. See original paper for figures.

Source: Math-Net.Ru and CyberLeninka. Machine translation. Verify with the original.