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Abstract

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ON THE ADIABATIC TEMPERATURE GRADIENT IN THE EARTH’ S MANTLE

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The question of the adiabatic temperature gradient is one of the fundamental ones for mantle physics. Usually, when determining the adiabatic gradient, the Earth’s mantle is assumed to be chemically homogeneous and single-phase. Such a natural consideration, necessary at the outset, leads to an adiabatic gradient of less than 1 deg/km in the upper layers and less than 0.5 deg/km in the lower parts of the mantle. (The data are taken from (^{1}), where a bibliography on the question under consideration is given.)

Such small values of the adiabatic gradient lead to a number of difficulties when considering the state of matter in the outer zones of the mantle and the temperature distribution in them. For zone B the difficulties are as follows.

As is known, in zone B, at a depth of 50-200 km, there is a low-velocity layer. Most authors consider this to be an effect of thermal origin; namely, they assume that the decrease of velocity in this layer due to the rise of temperature predominates over its increase due to the rise of pressure. The required gradients are 7-10 deg/km when the usual temperature dependence of the elastic moduli and density is taken into account, and 1-1.5 deg/km under the assumption of relaxation effects in this layer (^{2}). On the other hand, Slichter (^{3}), in considering the question of the thermal history of the Earth, drew attention to the fact that even at viscosities $\eta \lesssim 10^{22}$ poise, convective heat transfer begins to predominate over transfer due to the usual mechanisms of thermal conductivity. Further, according to (^{4}), it may be assumed that the low-velocity layer lies in the region of a low-viscosity layer; moreover, with all the uncertainties in the question under consideration, such large viscosities can hardly be expected in this region. In this connection there arises the problem of finding new physical causes that hinder convection in the low-velocity layer. First of all, the question arises as to what influence a small change in concentration will have on the adiabatic gradient. This question is investigated below.

For the derivation, let us consider, as usual (^{5}), the process of exchange of two neighboring elements of the medium $V(r)$ and $V(r+\zeta)$, which we shall regard as occurring adiabatically, i.e., without a change of entropy S. By r we denote the height, which in the particular case of the Earth coincides with the radius, and ζ is a small quantity—the distance between the volume elements under consideration.

Along with adiabaticity, we shall assume that the concentration in the volume elements exchanging places is preserved. This latter assumption is based on estimates of diffusion lengths in the Earth' s mantle (^{4}), which proved to be very small. For equilibrium stability it is necessary that the force arising in such an exchange should tend to return the lower element to its initial position:

$$V(p', S, c') - V(p', S, c) > 0. \tag{1}$$

Expanding this difference in powers of the small quantities

$$S' - S = \frac{dS}{dr}\zeta, \qquad c' - c = \frac{dc}{dr}\zeta,$$

we obtain

$$\left(\frac{\partial V}{\partial S}\right)_{p,c}\frac{dS}{dr} + \left(\frac{\partial V}{\partial c}\right)_{p,S}\frac{dc}{dr} > 0. \tag{2}$$

Next it will be necessary to make use of thermodynamic identities for a solution. Let us write the differential of the energy E of our system, referred to unit mass, in the form

$$dE = T\,dS - p\,dV + \mu\,dc. \tag{3}$$

Here the chemical potential of the solution μ and the mass concentration of the first component c have the form

$$\mu = \frac{\mu_1}{m_1} - \frac{\mu_2}{m_2}, \qquad c = n_1 m_1, \qquad n_1 m_1 + n_2 m_2 = 1, \tag{4}$$

where n_1, n_2 are the numbers of molecules of the first and second kinds in one gram of substance, and m_1 and m_2 are the weights of the corresponding molecules. Using the thermodynamic identities that follow from (3), as well as the condition of hydrostatic equilibrium

$$-\frac{dp}{dl} = \frac{dp}{dr} = -\rho g = -\frac{g}{V}, \tag{5}$$

where l is depth, g is the acceleration of gravity, and p is pressure, we transform (2) to the desired form

$$\frac{dT}{dl} < \tau_c, \tag{6}$$

$$\begin{aligned}\tau_c &= \frac{T\alpha g}{c_p} + \left[\frac{T}{c_p}\left\{\frac{1}{m_2}\left(\frac{\partial \mu_2}{\partial T}\right)_{p,c} - \frac{1}{m_1}\left(\frac{\partial \mu_1}{\partial T}\right)_{p,c}\right\} + \frac{1}{V\alpha}\left(\frac{\partial V}{\partial c}\right)_{p,S}\right]\frac{dc}{dr} \\ &= \frac{T\alpha g}{c_p} + \left[\frac{T}{c_p}\left(\frac{s_1^*}{m_1} - \frac{s_2^*}{m_2}\right) + \frac{1}{V\alpha}\left(\frac{\partial V}{\partial c}\right)_{p,S}\right]\frac{dc}{dr}.\end{aligned} \tag{7}$$

In (7), c_p is the heat capacity at constant pressure, referred to unit mass, $\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_{p,c}$ is the coefficient of thermal expansion, s_1^* is the entropy of the pure substance in equilibrium with the solution, referred to one molecule; s_2^* has an analogous meaning.

In order to use (7) in practice, it is necessary to estimate the additional term proportional to the concentration gradient. Let the volume per particle of the first kind be v_1, and that of the second v_2. Then the volume of unit mass V (under the assumption of additivity of volumes) is

$$V = n_1 v_1 + n_2 v_2 = cV_1 + (1-c)V_2, \qquad V_1 = \frac{v_1}{m_1}, \qquad V_2 = \frac{v_2}{m_2}, \tag{8}$$

$$\left(\frac{\partial V}{\partial c}\right)_{p,S} = (V_1 - V_2) = \frac{1}{\rho_1} - \frac{1}{\rho_2} = \frac{\Delta\rho}{\rho_1\rho_2} \sim \frac{\Delta\rho}{\rho^2}. \tag{9}$$

We shall estimate the entropy terms in (7) in two ways. First, using the definition $S_1^* = s_1^*/m_1$ and $S_2^* = s_2^*/m_2$ as the entropies of the pure phases 1 and 2 in equilibrium with the solution, we shall use for S^* the Debye expression for the limiting case of high temperatures

$$S^* = \frac{3R}{M}\left(\frac{4}{3} - \ln\frac{\theta}{T}\right), \tag{10}$$

where R is the gas constant, θ is the Debye temperature, and M is the mean atomic weight. For silicates $M_1, M_2 \sim 21$, therefore $|M_1 - M_2| \ll M_1$.

$M_2 \sim M$ and

$$\frac{T}{c_p}(S_1^* - S_2^*) \approx \frac{3RT}{c_p M}\ln\frac{\theta_2}{\theta_1}. \tag{11}$$

Substituting (9), (11) into (7), we obtain

$$\tau_c \approx \frac{T\alpha g}{c_p} + \left[\frac{3RT}{c_p M}\ln\frac{\theta_2}{\theta_1} + \frac{\Delta\rho}{\rho\alpha}\right]\frac{dc}{dr}. \tag{12}$$

We assume that substance 2 is heavier than substance 1, $\Delta\rho > 0$, and that with increasing depth the substance is enriched in the second phase, $dc/dr > 0$. Further estimates can be made by putting[1]

$$(\theta_2 \sim \theta_1), \quad \left|\ln\frac{\theta_2}{\theta_1}\right| \ll 1, \quad R \sim 8.3\cdot 10^7, \quad c_p \sim 1.3\cdot 10^7, \quad M \sim 21,$$

$$T \sim 2\cdot 10^3, \quad \frac{\Delta\rho}{\rho} \sim 0.2, \quad \alpha \sim 5\cdot 10^{-5}, \quad \frac{3RT}{c_pM}\ln\frac{\theta_2}{\theta_1} \sim 1.8\cdot 10^3 \ln\frac{\theta_2}{\theta_1},$$

$$\frac{\Delta\rho}{\rho\alpha} \sim 4\cdot 10^3. \tag{13}$$

As a result,

$$\tau_c \sim \frac{T\alpha g}{c_p} + \frac{\Delta\rho}{\rho\alpha}\frac{dc}{dr}. \tag{14}$$

Let us determine the scale that the quantity dc/dr must have in order to double the adiabatic gradient:

$$\left(\frac{dc}{dr}\right)_0 = \frac{\rho\alpha^2 Tg}{\Delta\rho\, c_p} \sim 1.9\cdot 10^{-9}\ \text{cm}^{-1} = 1.9\cdot 10^{-2}/10^2\ \text{km} \quad (g \sim 10^3). \tag{15}$$

We see that a change in concentration by 2% over a distance of 100 km is sufficient to double the value of the adiabatic gradient in the low-velocity zone. In order to raise it to ~ 7.5 deg/km, a change in concentration by $\sim 10\%$ over a distance of 100 km is necessary. The entropy term in (7) can be estimated under the assumption of a weak solution $n_2 \ll n_1$, $x = n_2/n_1 \ll 1$. Also assuming the Einstein model of a solid, i.e., taking all atomic-vibration frequencies equal to the Einstein frequency ω_e and $kT > \hbar\omega_\mathrm{e}$, we obtain for the entropy term in (7)

$$\frac{3RT}{c_pM}\left\{\frac{\ln x}{3\nu} + \ln\frac{\bar\omega}{\omega_\mathrm{e}} + \frac{4}{3} - \ln\frac{\theta_1}{T}\right\}, \tag{16}$$

where ν is the number of atoms in the molecule, and $\bar\omega$ is the changed Einstein frequency associated with the impurity atom of substance 2. It follows from (16) that the estimate of the entropy term given in (13) is retained.

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Note: Figure translations are in progress. See original paper for figures.