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The content of the present note is determined by the following three questions.

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Abstract

Full Text

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INFINITE GROUPS WITH CERTAIN PRESCRIBED PROPERTIES OF THE SYSTEMS OF THEIR INFINITE SUBGROUPS

(Presented by Academician A. I. Mal'cev, 11 VI 1964)

The content of the present note is determined by the following three questions.

For which groups is the minimality condition for abelian subgroups equivalent to the minimality condition for nonabelian subgroups?

For which infinite groups is the normalizer condition equivalent to the requirement that proper infinite subgroups not coincide with their normalizers?

For which infinite groups is the condition of complementability of all subgroups equivalent to the requirement of complementability of infinite subgroups?

1. **Theorem 1.** If an infinite nonabelian group \mathfrak{G} , possessing a normal system with finite factors, and, in particular, an infinite nonabelian locally soluble group \mathfrak{G} , has no infinite descending chains of nonabelian subgroups (the minimality condition for nonabelian subgroups), then it is a finite extension of an abelian group satisfying the minimality condition for subgroups (an extremal group).

Corollary 1. If an infinite nonabelian group \mathfrak{G} , possessing a normal system with finite factors (and, in particular, an infinite locally soluble nonabelian group \mathfrak{G}), has an infinite descending chain of abelian subgroups (has elements of infinite order), then it also has an infinite descending chain of nonabelian subgroups.

Since every locally soluble group \mathfrak{G} that has no infinite descending chains of abelian subgroups is extremal (see ⁽¹⁾), it follows that:

Corollary 2. For locally soluble nonabelian groups, the minimality condition for abelian subgroups is equivalent to the minimality condition for nonabelian subgroups.

Since every locally finite group \mathfrak{G} possessing a normal system with finite factors and satisfying the minimality condition for abelian subgroups is extremal (see ⁽²⁾), it follows that:

Corollary 3. For locally finite nonabelian groups possessing a normal system with finite factors, the minimality condition for abelian subgroups is equivalent to the minimality condition for nonabelian subgroups.

Corollary 4. If an infinite nonabelian group possessing a normal system with finite factors contains no infinite nonabelian subgroup distinct from it, then it is extremal.

With the aid of this proposition one can prove that if an infinite locally finite nonabelian group contains no infinite nonabelian subgroup distinct from it, then it is extremal.

It is clear that not every extremal nonabelian group is such a group all of whose proper infinite subgroups are abelian.

Theorem 2. If an infinite locally finite nonabelian group G contains no infinite nonabelian subgroup distinct from it, then its maximal complete abelian normal divisor A has finite index in it, is primary, and contains no proper infinite subgroups invariant in G .

From the condition of Theorem 2 it is not difficult to see that if the center Z of the group G is finite, then G/A is a primary cyclic group. If its center Z is infinite, then it follows from the theorem that A is a quasicyclic group and that $A \subset Z$. In this case the factor group G/A is a finite group having no proper nonabelian subgroups.

In both cases the group G is readily described by means of generators and defining relations.

2. Theorem 3. Every infinite locally finite group G in which all proper infinite subgroups are distinct from their normalizers in G either satisfies the normalizer condition, or contains, for some p , an invariant extremal Sylow p -subgroup P with a nontrivial finite nilpotent factor group G/P and has finite center.

It follows that, for infinite locally finite groups not satisfying the minimal condition for abelian subgroups (in particular, containing no quasicyclic subgroups), the normalizer condition is equivalent to the requirement that proper infinite subgroups do not coincide with their normalizers.

3. Theorem 4. Every infinite group G possessing a normal system with finite factors and in which all infinite subgroups are complemented is locally soluble. If the group G contains no quasicyclic subgroups, then under these conditions all its subgroups are complemented; but if it contains a quasicyclic subgroup, then under these conditions the latter has finite index in it.

Thus, for infinite groups possessing normal systems with finite factors and having no quasicyclic subgroups, the condition that all subgroups be complemented is equivalent to the condition that all infinite subgroups be complemented.

Theorem 5. A locally nilpotent infinite group G in which all infinite subgroups are complemented is abelian if all its 2-subgroups are abelian. In this case it decomposes either into the direct product of infinitely many cyclic groups of prime orders, or into the direct product of finitely many cyclic groups of prime orders and one quasicyclic group. If at least one of its 2-subgroups is nonabelian, then it decomposes into the direct product of finitely many cyclic groups of prime

orders and a nonabelian 2-group, which decomposes into the semidirect product of a quasicyclic 2-group and a cyclic group of order two.

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Note: Figure translations are in progress. See original paper for figures.

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