Corresponding Member of the Academy of Sciences of the USSR G. G. CHERNYI

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Abstract

Full Text

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HYPERSONIC FLOW PAST WINGS AT LARGE ANGLES OF ATTACK

In hypersonic flow past wings at a large angle of attack, the layer of compressed gas between the shock wave and the surface of the wing on the windward side is relatively thin. To describe the motion of the gas in this layer, with the aim of studying certain features of the flow past wings, one may apply the method of integral relations in its simplest form, assuming that in the compressed layer the components of the gas velocity in planes tangent to the wing element, the pressure p, and the density ρ are constant along the normal to the wing, and that the component of velocity normal to the wing in the layer may be neglected. Such a simplification of the equations is analogous to the "shallow-water" approximation in hydrodynamics; this analogy was pointed out by Hayes and Probstein (1).

We shall restrict ourselves to the consideration of plane wings. The choice of a coordinate system in a plane coinciding with the plane of the wing may be different; introduce polar coordinates r and θ, measuring the angle θ from the direction of the projection of the velocity of the oncoming flow onto the plane of the wing (if the flow is perpendicular to this plane, then the direction $\theta = 0$ may be chosen arbitrarily). The equations of conservation of mass, momentum (in the directions of increasing r and θ) and energy will then be written in the form

$$\frac{\partial}{\partial r}\rho u h + \frac{\partial}{r\partial\theta}\rho v h + \frac{\rho u h}{r} - \rho_1 V_n = 0,$$

$$\rho h\left(u\frac{\partial u}{\partial r} + v\frac{\partial u}{r\partial\theta}\right) + \frac{\partial}{\partial r}h(p - p_1) - \frac{\rho v^2 h}{r} - \rho_1(V_r - u)V_r = 0, \qquad (1)$$

$$\rho h\left(u\frac{\partial v}{\partial r} + v\frac{\partial v}{r\partial\theta}\right) + \frac{\partial}{r\partial\theta}h(p - p_1) + \frac{\rho u v h}{r} - \rho_1(V_\theta - v)V_n = 0,$$

$$\frac{u^2 + v^2}{2} + \frac{\gamma}{\gamma - 1}\frac{p}{\rho} = \frac{V^2}{2} + \frac{\gamma}{\gamma - 1}\frac{p_1}{\rho_1},$$

where

$$V_n = -V_\nu + V_r \frac{\partial h}{\partial r} + V_\theta \frac{\partial h}{r\partial\theta}; \qquad V_\nu = -V \sin\alpha, \qquad V_r = V\cos\alpha\cos\theta;$$

$$V_\theta = -V\cos\alpha\sin\theta;$$

h is the thickness of the compressed-gas layer; u, v are the velocity components in it in the directions r and θ; p_1, ρ_1, and V are the pressure, density, and speed of the oncoming flow; α is the angle of attack; γ is the ratio of specific heats.

To close the system of equations (1), one further relation is required. In the choice of this relation, a certain arbitrariness is permissible. We shall take it that the quantities p and ρ entering equations (1) are related by the condition

$$p = \sigma^\gamma \rho^\gamma; \tag{2}$$

the constant σ may be determined from the conditions at the shock wave.

Let us consider two simplest examples.

1. Normal flow past a circular wing.

In this case $v = \partial/\partial\theta = V_r = V_\theta = 0$. System (1) is easily integrated. For simplicity we shall also take $p_1 = 0$, i.e., we shall regard the Mach number as very large. The velocity distribution is then found in the form

$$\frac{r}{r_{\text{cr}}} = \frac{\lambda}{\left(\frac{\gamma+1}{4\gamma+2} + \frac{3\gamma+1}{4\gamma+2}\lambda^2\right)^{(2\gamma+1)/(3\gamma+1)}}$$

(here $\lambda = u/u_{\text{cr}}$). The shape of the shock wave is determined by the relation

$$\frac{h}{r_{\text{cr}}} = \frac{\frac{1}{2}\left(\frac{\gamma-1}{\gamma+1}\right)^{1/2}\left[\frac{4\gamma}{(\gamma+1)^2}\right]^{1/(\gamma-1)}}{\left(1 - \frac{\gamma-1}{\gamma+1}\lambda^2\right)^{1/(\gamma-1)}\left(\frac{\gamma+1}{4\gamma+2} + \frac{3\gamma+1}{4\gamma+2}\lambda^2\right)^{(2\gamma+1)(3\gamma+1)}}.$$

In particular, the greatest distance of the wave from the body is equal to

$$\frac{h_0}{r_{\text{cr}}} = \frac{\sqrt{\varepsilon}}{2}(1-\varepsilon^2)^{(1-\varepsilon)/2\varepsilon}(3+\varepsilon)^{(3+\varepsilon)/(4+2\varepsilon)},$$

where $\varepsilon = (\gamma-1)/(\gamma+1)$. Figure 1 shows the dependence of λ on r/r_{cr} for $\gamma = 1.4$; this dependence remains practically unchanged down to $\gamma = 1$, when it takes the form

Fig. 1

Figure 1: Fig. 1

Fig. 2

Figure 2: Fig. 2

$$\frac{r}{r_{\text{cr}}} = \lambda \left(\frac{1}{3} + \frac{2}{3}\lambda^2 \right)^{-3/4}.$$

2. Flow past a flat wing of infinite span without slip.

The equations of such a flow are readily obtained directly, but one may also use the system (1)–(2).

Fig. 1

Fig. 2

For this purpose we put in it $v = 0$, $r \to \infty$, $\theta = 0$, as a result of which, denoting $\partial/\partial r = \partial/\partial x$, we obtain:

$$\frac{\partial}{\partial x}\rho u h - \rho_1 V \left(\sin\alpha + \cos\alpha \frac{\partial h}{\partial x} \right) = 0,$$

$$\rho u h \frac{\partial u}{\partial x} + \frac{\partial}{\partial x} h(p - p_1) - \rho_1 V (V\cos\alpha - u) \left(\sin\alpha + \cos\alpha \frac{\partial h}{\partial x} \right) = 0, \tag{3}$$

$$\frac{u^2}{2} + \frac{\gamma}{\gamma - 1}\frac{p}{\rho} = \frac{V^2}{2} + \frac{\gamma}{\gamma - 1}\frac{p_1}{\rho_1}, \qquad p = \sigma^\gamma \rho^\gamma.$$

Let us integrate the first two equations (3); this gives

$$\rho u h = \rho_1 V [(x - x_0)\sin\alpha + (h - h_0)\cos\alpha],$$

$$\rho u h (u - V\cos\alpha) + h(p - p_1) = (p_0 - p_1) h_0. \tag{4}$$

In the case of not very large angles of attack, the shock is attached and $x_0 = h_0 = 0$; in this case the system gives the known exact solution. For angles of attack at which the shock is detached, x_0 is the coordinate of the critical point where $u = 0$ (x is measured from the leading edge of the wing); the subscript 0 denotes the values of the quantities being determined at this point.

Formulas (4), together with the last two formulas (3), make it possible to calculate all flow parameters easily. In particular, the velocity distribution along the wing and the shape of the shock wave are determined by the relations:

$$\frac{h}{h_0}=\frac{1-p_1/p_0}{\left(1-\frac{\gamma-1}{\gamma+1}\lambda^2\right)^{1/(\gamma-1)}\left(1+\lambda^2-\frac{2\gamma}{\gamma+1}\frac{V}{V_{\text{cr}}}\lambda\sin\alpha\right)-p_1/p_0},$$

$$\frac{x-x_0}{h_0}\cos\alpha=\sin\alpha+\frac{h}{h_0}\left[\frac{\rho_0 V_{\text{cr}}}{\rho_1 V}\lambda\left(1-\frac{\gamma-1}{\gamma+1}\lambda^2\right)^{1/(\gamma-1)}-\sin\alpha\right].$$

The constants x_0 and h_0 are determined from the conditions that the gas velocity is equal to the local speed of sound at both edges of the wing:

$$\frac{h_0}{l}=\frac{\left[(1-\beta)^2-\left(\frac{\gamma}{\gamma+1}\frac{V}{V_{\text{cr}}}\sin\alpha\right)^2\right]\cos\alpha}{\left(1-\frac{p_1}{p_0}\right)\left[\frac{\rho_0 V_{\text{cr}}}{\rho_1 V}(1-\beta)-\left(\frac{\gamma+1}{2}\right)^{1/(\gamma-1)}\frac{\gamma}{\gamma+1}\frac{V}{V_{\text{cr}}}\sin^2\alpha\right]},$$

$$x_0/l=$$

$$\frac{\left(1-\beta-\frac{\gamma}{\gamma+1}\frac{V}{V_{\text{cr}}}\sin\alpha\right)\left\{\left(1-\frac{p_1}{p_0}\right)\frac{\rho_0 V_{\text{cr}}}{2\rho_1 V}+\left[\frac{1}{2}\left(\frac{\gamma+1}{2}\right)^{1/(\gamma-1)}-1-\frac{\gamma}{\gamma+1}\frac{V}{V_{\text{cr}}}\sin\alpha\right]\sin\alpha\right\}}{\left(1-\frac{p_1}{p_0}\right)\left[\frac{\rho_0 V_{\text{cr}}}{\rho_1 V}(1-\beta)-\left(\frac{\gamma+1}{2}\right)^{1/(\gamma-1)}\frac{\gamma}{\gamma+1}\frac{V}{V_{\text{cr}}}\sin^2\alpha\right]}.$$

Here l is the chord length of the wing,

$$\beta=\frac{p_1}{2p_0}\left(\frac{\gamma+1}{2}\right)^{1/(\gamma-1)},$$

and the condition

$$1-\beta-\frac{\gamma}{\gamma+1}\frac{V}{V_{\text{cr}}}\sin\alpha=0$$

corresponds to attachment of the shock. The values of p_0/p_1 and ρ_0/ρ_1 are easily found if one notes that

$$\left.\frac{dh}{dx}\right|_{x=x_0}=\frac{\sin\alpha\cos\alpha}{\left(1-\frac{p_1}{p_0}\right)\frac{\gamma+1}{2\gamma}\frac{\rho_0 V_{\text{cr}}^2}{\rho_1 V^2}-\cos^2\alpha}.$$

Fig. 3

Figure 3: Fig. 3

Fig. 4

Figure 4: Fig. 4

Figure 2 gives the dependence of the coordinate of the critical point on the angle of attack for $\gamma = \frac{7}{5},\ \frac{11}{9},\ \frac{21}{19}$ at $M = \infty$; Fig. 3 gives the dependence of the normal-force coefficient on the angle of attack for $\gamma = \frac{7}{5},\ M = \infty$.

It should be noted that the problems of this and the preceding sections were previously solved by the numerical method of integral relations in the first and second approximations (in the second approximation the layer of compressed gas is divided into two strips) by A. P. Bazhinyan ([2,3]).

Fig. 3

Fig. 4

3. Flow past a triangular wing.

At angles of attack close to $\pi/2$, the shock in front of the wing is detached; behind it the flow spreads from the critical point in all directions, flowing around the edges of the wing outward, so that near the forward end of the wing the flow in the compressed layer is directed forward. In a certain neighborhood of the forward point, when the angle of the forward end of the wing is sufficiently small, the flow has a conical character. As the angle of attack decreases, the critical point shifts toward the forward end of the wing and the region of conical flow near it diminishes. At a certain angle of attack, depending on V/V_{cr}, on the value of the angle $2\theta_0$ at the forward end of the wing, and on γ, the critical point disappears and the flow in the compressed layer becomes directed toward the trailing edge of the wing. The streamlines of the averaged motion approach the edges of the wing from the midline of the wing; a further decrease in the angle of attack

leads to the fact that the streamlines approach the edges only outside a certain angle; inside the angle the streamlines approach the centerline of the wing. This angle increases as the wing angle of attack decreases. When the inequality

$$\operatorname{tg} \alpha / \sin \theta_0 \leqslant \operatorname{tg} \delta_{\max},$$

is satisfied, where $\delta_{\max}$ is the maximum angle of flow deflection in the shock for the Mach number equal to $M_1 \sqrt{1 - \cos^2 \alpha \cos^2 \theta_0}$, the shock becomes attached along the leading edges, so that the flow enters inside the wing contour. In both of the latter cases the flow near the front end of the wing, or even in the entire region behind the shock, again acquires a conical character. For conical flows u, v, p, ρ depend only on θ, while $h = rH(\theta)$. The differential equations (1) then take the form

$$2\rho u H + \frac{d}{d\theta}\rho v H - \rho_1 V_n = 0,$$

$$\rho v H \frac{du}{d\theta} + (p - p_1)H - \rho v^2 H - \rho_1 (V_r - u) V_n = 0, \tag{5}$$

$$\rho v H \frac{dv}{d\theta} + \frac{d}{d\theta}(p - p_1)H + \rho u v H - \rho_1 (V_\theta - v) V_n = 0,$$

where $V_n = -V_v + V_{rH} + V_\theta \, dH/d\theta$.

This system has the trivial solution (translational flow)

$$u = V_0 \cos(\theta - \beta), \qquad v = -V_0 \sin(\theta - \beta), \qquad H = A \sin(\theta_0 - \theta),$$

$$\rho = \rho_0, \qquad p = p_0,$$

where $V_0, A, \beta, \rho_0, p_0$ are constants determined from the equations.

It can be shown that for sufficiently large α the solution of the system has a singularity at some θ, and

$$v^2 + \frac{\gamma - 1}{\gamma + 1} u^2 = V_{\text{cr}}^2,$$

i.e., the velocity component normal to the ray is equal to the speed of sound; these values of θ correspond to the leading edges of the wing when it is flowed about with a detached wave. The solution corresponding to translational flow gives the conditions for calculating the flow with a shock attached along the edges.

A conical flow with a detached wave exists only for sufficiently small angles of the front end of the wing (depending on the angle of attack and on the Mach number); in this case there are two flow regimes. For larger angles no conical flow exists.

For the case $\theta_0^2 \ll 1$, introducing the variables

$$u = V \cos\alpha + V\theta_0 \sin\alpha \cdot \bar{u}, \qquad v = V \sin\alpha \cdot \bar{v}, \qquad p = \rho_1 V^2 \sin^2\alpha \cdot \bar{p},$$

$$\rho = \rho_1 \bar{\rho}, \qquad H = \theta_0 \bar{H}, \qquad \theta = \theta_0 \xi,$$

we find, in accordance with the similarity law (4), that $\bar{u}, \bar{v}, \bar{p}, \bar{\rho}, \bar{H}$ for $p_1 = 0$ do not depend on α and θ_0. The velocity distribution for $p_1 = 0$ is determined by the formulas

$$\frac{v}{\sqrt{\varepsilon} V \sin\alpha} = \lambda_v = \frac{1}{\xi} - \sqrt{\frac{1}{\xi^2} - 1},$$

$$\frac{u - V\cos\alpha}{\sqrt{\varepsilon} V \theta_0 \sin\alpha} = \lambda_u = \frac{\gamma - 1}{\gamma} - \frac{1}{\xi} \arcsin\xi - \sqrt{1 - \xi^2}. \qquad (6)$$

Graphs of the functions λ_v and $\lambda_u - (\gamma - 1)/\gamma$ (which are also independent of γ) are given in Fig. 4. It is obvious that, when the angle of attack is decreased to the values $\operatorname{tg}\alpha = \gamma(\gamma + 1)\theta_0$, the critical point on the wing disappears. With a further decrease of the angle of attack to the values $\operatorname{tg}\alpha = \theta_0/\sqrt{\gamma^2 - 1}$, the shock becomes attached along the leading edges of the wing. Solution (6) must then be replaced by another.

Scientific Research Institute of Mechanics
Moscow State University
named after M. V. Lomonosov

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Note: Figure translations are in progress. See original paper for figures.