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Abstract

Full Text

MECHANICS

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A KINEMATIC INTERPRETATION OF ONE SOLUTION OF THE PROBLEM OF THE MOTION OF A BODY HAVING A FIXED POINT

(Presented by Academician P. Ya. Kochina, 29 IV 1964)

Dividing the components of the angular velocity by the constant ν and introducing dimensionless quantities

$$v_i = \frac{\gamma_i}{\Gamma} \quad (i = 1, 2, 3), \quad \mu = \frac{\lambda}{B\nu}, \quad R^2 = 1 + \frac{2H}{B\nu^2} + \frac{\Gamma^2}{B^2\nu^4}, \quad a = \frac{\Gamma}{B\nu^2} > 0, \quad (1)$$

we write the solution indicated in the article ⁽¹⁾ in the form

$$p = \cos \alpha, \quad r = \sin \alpha; \quad (2)$$

$$\frac{dq}{d\tau} = -\sqrt{f(q)} \quad \left(\tau = \frac{\nu}{2} t \right); \quad (3)$$

$$v_1 = \frac{1}{2a} \left[(R^2 - 1 - a^2 + q^2) \cos \alpha - \sqrt{f(q)} \sin \alpha \right], \quad v_2 = \frac{1}{a} (\mu - q), \quad (4)$$

$$v_3 = \frac{1}{2a} \left[(R^2 - 1 - a^2 + q^2) \sin \alpha + \sqrt{f(q)} \cos \alpha \right].$$

Here

$$f(q) = 4a^2 - 4(\mu - q)^2 - (R^2 - 1 - a^2 + q^2)^2. \quad (5)$$

The moving hodograph of the angular velocity of the body is given by equations (2), (3). It is a segment of a straight line parallel to the second coordinate axis. It intersects the straight line joining the center of gravity of the body with the fixed point O . For real q the function $f(q)$ is nonnegative (see (3)); therefore the end points of the moving hodograph are determined by the roots of the

Fig. 1

Figure 1: Fig. 1

Fig. 3

Figure 2: Fig. 3

equation $f(q) = 0$, among which, in view of $f(q_0) \geq 0$, $f(\pm\infty) < 0$, there are at least two real ones.

Directing the ζ -axis of the fixed coordinate system $O\xi\eta\zeta$ along the unit vector v_i , we write the equations of the fixed hodograph ⁽²⁾:

$$\omega_\zeta = \frac{1}{2a}(R^2 - 1 - a^2 + 2\mu q - q^2), \quad \omega_\xi^2 + \omega_\eta^2 + \omega_\zeta^2 = 1 + q^2; \quad (6)$$

$$\frac{d\alpha}{dq} = \frac{2a\sqrt{f(q)}}{4a^2(1 + q^2) - (R^2 - a^2 - 1 + 2\mu q - q^2)^2}. \quad (7)$$

From (6) we conclude that the fixed hodograph lies on the surface of revolution

$$[\omega_\xi^2 + \omega_\eta^2 + (\omega_\zeta + a)^2 - R^2 - 2\mu^2]^2 = 4\mu^2(R^2 - 1 - a^2 + \mu^2 - 2a\omega_\zeta). \quad (8)$$

We shall confine ourselves here to the analysis of the case $\mu = 0$ (which includes also the Bobylev ⁽³⁾–Steklov ⁽⁴⁾ solution). In this case the surface (8) is a sphere, whose center, in consequence of (1), lies above the point of support of the body (Fig. 1). We introduce the new variable

$$u = \cos \beta = \frac{R^2 + a^2 - 1 - q^2}{2aR}. \quad (9)$$

Fig. 1

Fig. 3

Fig. 2

and denote

Fig. 2

Figure 3: Fig. 2

$$u_1 = \frac{R-1}{a}, \quad u_2 = \frac{R+1}{a} > 0, \quad u' = \frac{R^2 + a^2 - 1}{2aR} = \frac{u_1 u_2 + 1}{u_1 + u_2}; \quad (10)$$

then from (7), (3)

$$\frac{d\alpha}{du} = -\frac{1}{1-u^2} \sqrt{\frac{(u_2-u)(u-u_1)}{(u_1+u_2)(u'-u)}}; \quad (11)$$

$$\frac{du}{dt} = \sqrt{8aR(u'-u)(u_2-u)(u-u_1)}. \quad (12)$$

The geodesic curvature of the fixed hodograph

$$\chi = \frac{u_1 + u_2 - 2u}{2R\sqrt{(u_2-u)(u-u_1)}}$$

vanishes at

$$u_* = \frac{u_1 + u_2}{2} = \frac{R}{a}.$$

In this position the plane ONM is tangent to the sphere (Fig. 1). From (11), (9), (10)

$$(u_2 - u)(u - u_1)(u' - u) \geq 0, \quad -1 \leq u \leq 1,$$

$$u_2 > |u_1|, \quad |u_1| < 1, \quad u' > u_1.$$

Three cases are possible.

1°. $u_2 > 1$. In this case $-1 < u_1 \leq u \leq u' < 1 < u_2$. The line $\alpha = \alpha(u)$ is tangent to the parallel $u = u'$, while on the parallel $u = u_1$ it is tangent to a meridian. The successive positions of the moving hodograph on the fixed one are indicated in Fig. 2a for the case $u_* > u'$ and in Fig. 2b for the case $u_* < u'$.

2°. $u_2 < 1$. In this case $-1 < u_1 \leq u \leq u_2 < 1 < u'$. The line $\alpha = \alpha(u)$ is nowhere tangent to the parallels of the sphere, but approaches the parallels $u = u_1$, $u = u_2$, touching the meridians. On the parallel $u = u_*$ the geodesic curvature changes sign (Fig. 2b).

3°. $u_2 = 1$. From (10) $a = R + 1$, $u' = 1$, and equations (11), (12)

$$\frac{d\alpha}{du} = \frac{1}{1-u^2} \sqrt{\frac{u-u_1}{1+u_1}}, \quad \frac{du}{d\tau} = 2(1-u)\sqrt{2R(R+1)(u-u_1)}$$

show that $\alpha = \alpha(u)$, $u = u(\tau)$ are elementary functions, with $u \rightarrow 1$ as $\tau \rightarrow \infty$. The line $\alpha = \alpha(u)$, winding without bound around the pole of the sphere, has finite length (Fig. 3).

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