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Abstract

Full Text

Astronomy

I. N. Minin

On the Nonstationary Glow of a Semi-Infinite Medium

(Presented by Academician V. A. Ambartsumian, 16 X 1963)

Let us consider the nonstationary glow of a homogeneous semi-infinite medium in which isotropic scattering of light occurs with a quantum “survival” probability, in an elementary scattering event, equal to λ . Denote by t_1 and t_2 the mean time spent by a quantum in the absorbed state and on the path between two successive scatterings, respectively, and assume that these quantities are independent of coordinates and time. We shall assume that the probability of emission of a quantum after some time interval following absorption decreases exponentially with increase of the indicated interval. Instead of the time t , introduce the variable $u = \frac{t}{t_1 + t_2}$, and also the notation

$$\beta_1 = \frac{t_1}{t_1 + t_2}, \quad \beta_2 = \frac{t_2}{t_1 + t_2}.$$

The author ⁽¹⁾ proposed a method for obtaining solutions of various problems in the theory of nonstationary diffusion of radiation under the condition that the optical properties of the medium do not change with time. The method is based on finding the Laplace transform, with respect to time, of any characteristic of the radiation field directly from the corresponding characteristic in the stationary case. For this purpose, in the solution of the stationary problem one should replace the parameter λ by $\lambda/(1 + \beta_1 s)(1 + \beta_2 s)$, and instead of the optical depth τ introduce the quantity $\tau(1 + \beta_2 s)$, where s is the parameter of the Laplace transform. Inversion of the transform thus obtained solves the problem.

Here we shall apply the indicated method to finding the intensity of radiation emerging from a semi-infinite medium for two cases of source arrangement. In doing so, we shall assume that the medium is subjected to an instantaneous action of the sources. For any other dependence of the source power on time, the solution is easily obtained from the one found by integration.

1. Diffuse reflection of light. Let the medium be illuminated by parallel rays producing an illumination, on a surface perpendicular to them, equal to πS . Denote by $I(\eta, \zeta)$ the intensity of the reflected light, where ζ is the cosine

of the angle of incidence of the rays, and η is the cosine of the angle of reflection. The solution of the problem in the stationary case was obtained by V. A. Ambartsumian ⁽²⁾ and has the form

$$I(\eta, \zeta) = \frac{\lambda}{4} S \zeta \frac{\varphi(\eta)\varphi(\zeta)}{\eta + \zeta}, \quad (1)$$

where the function $\varphi(\eta)$ is determined by the equation

$$\varphi(\eta) = 1 + \frac{\lambda}{2} \eta \varphi(\eta) \int_0^1 \frac{\varphi(\zeta)}{\eta + \zeta} d\zeta. \quad (2)$$

Let us now turn to consideration of the nonstationary case. Let $I(u, \eta, \zeta)$ be the intensity of radiation at the instant of dimensionless time u .

Then, introducing the Laplace transform

$$\bar{I}(s, \eta, \zeta) = \int_0^\infty e^{-su} I(u, \eta, \zeta) du, \quad (3)$$

and also putting $S(u) = \delta(u)$, where $\delta(u)$ is the Dirac function, we find

$$\bar{I}(s, \eta, \zeta) = \frac{\lambda}{4(1 + \beta_1 s)(1 + \beta_2 s)} \zeta \frac{\bar{\varphi}(s, \eta)\bar{\varphi}(s, \zeta)}{\eta + \zeta}. \quad (4)$$

In relation (4), $\bar{\varphi}(s, \eta)$ is determined by the equation

$$\bar{\varphi}(s, \eta) = 1 + \frac{\lambda}{2(1 + \beta_1 s)(1 + \beta_2 s)} \eta \bar{\varphi}(s, \eta) \int_0^1 \frac{\bar{\varphi}(s, \zeta)}{\eta + \zeta} d\zeta, \quad (5)$$

where

$$\bar{\varphi}(s, \eta) = \int_0^\infty e^{-su} \varphi(u, \eta) du. \quad (6)$$

Instead of the function $\varphi(u, \eta)$, let us introduce a new function $\omega(u, \eta)$ as follows:

$$\bar{\omega}(s, \eta) = \frac{\bar{\varphi}(s, \eta)}{(1 + \beta_1 s)(1 + \beta_2 s)}. \quad (7)$$

Then from relation (4) we shall have

$$\bar{I}(s, \eta, \zeta) = \frac{\lambda}{4} \frac{\zeta}{\eta + \zeta} (1 + s + \beta_1 \beta_2 s^2) \bar{\omega}(s, \eta) \bar{\omega}(s, \zeta), \quad (8)$$

and from equation (5) we obtain

$$\bar{\omega}(s, \eta) = \frac{1}{(1 + \beta_1 s)(1 + \beta_2 s)} + \frac{\lambda}{2} \eta \bar{\omega}(s, \eta) \int_0^1 \frac{\bar{\omega}(s, \zeta)}{\eta + \zeta} d\zeta. \quad (9)$$

Using the known rules of operational calculus, from (8) and (9) we find

$$I(u, \eta, \zeta) = \frac{\lambda}{4} \frac{\zeta}{\eta + \zeta} \left[a(u, \eta, \zeta) - \frac{\partial a(u, \eta, \zeta)}{\partial u} + \beta_1 \beta_2 \frac{\partial^2 a(u, \eta, \zeta)}{\partial u^2} \right], \quad (10)$$

where

$$a(u, \eta, \zeta) = \int_0^u \omega(u', \eta) \omega(u - u', \zeta) du', \quad (11)$$

and the function $\omega(u, \eta)$ is determined by the equation

$$\omega(u, \eta) = \frac{e^{-u/\beta_1} - e^{-u/\beta_2}}{\beta_1 - \beta_2} + \frac{\lambda}{2} \eta \int_0^u \left[\omega(u', \eta) \int_0^1 \frac{\omega(u - u', \zeta)}{\eta + \zeta} d\zeta \right] du'. \quad (12)$$

The solutions of the problem considered for the special cases $\beta_1 = 0$, $\beta_2 = 1$ and $\beta_2 = 0$, $\beta_1 = 1$ coincide. In these cases, instead of (10) we have

$$I(u, \eta, \zeta) = \frac{\lambda}{4} \frac{\zeta}{\eta + \zeta} \left[a(u, \eta, \zeta) + \frac{\partial a(u, \eta, \zeta)}{\partial u} \right], \quad (13)$$

and, to determine the function $\omega(u, \eta)$, instead of equation (12) we find

$$\omega(u, \eta) = e^{-u} + \frac{\lambda}{2} \eta \int_0^u \left[\omega(u', \eta) \int_0^1 \frac{\omega(u - u', \zeta)}{\eta + \zeta} d\zeta \right] du'. \quad (14)$$

Let us note that in the general case, from equation (12) it follows that $\omega(0, \eta) = 0$. In the special cases $\beta_1 = 0$, $\beta_2 = 1$ and $\beta_2 = 0$, $\beta_1 = 1$, from (14) we have $\omega(0, \eta) = 1$. The formula (13) obtained by us, after passing in it from the function $\omega(u, \eta)$

to the function $\varphi(u, \eta)$ takes the form

$$I(u, \eta, \zeta) = \frac{\lambda}{4} \frac{\xi}{\eta + \xi} \int_0^u e^{-(u-u')} \left[\int_0^{u'} \varphi(u'', \zeta) \varphi(u' - u'', \eta) du'' \right] du'. \quad (15)$$

In passing from (13) to (15) the relation

$$\varphi(u, \eta) = \omega(u, \eta) + \frac{\partial \omega(u, \eta)}{\partial u} + \delta(u), \quad (16)$$

which follows from (7), was used, taking into account that $\omega(0, \eta) = 1$. To determine the function $\varphi(u, \eta)$ from (5), we find

$$\varphi(u, \eta) = \delta(u) + \frac{\lambda}{2} \eta \int_0^1 \left[\int_0^u e^{-(u-u')} \left(\int_0^{u'} \varphi(u'', \xi) \varphi(u' - u'', \eta) du'' \right) du' \right] \frac{d\xi}{\eta + \xi}. \quad (17)$$

The solution of the problem, represented by formula (15) and equation (17), coincides with that obtained in the work of Ueno ⁽³⁾ for the case $\beta_1 = 0$, $\beta_2 = 1$.

The function $\omega(u, \eta)$ introduced by us has a simple probabilistic meaning. As V. V. Sobolev ⁽⁴⁾ showed, in the stationary case the relation

$$p(0, \eta) = \frac{\lambda}{4\pi} \varphi(\eta), \quad (18)$$

holds, where $p(\tau, \eta)$ is the probability that a light quantum absorbed at optical depth τ will emerge from the medium at an angle $\arccos \eta$ within a unit solid angle. Let us introduce $p(u, \tau, \eta)$, the probability that a light quantum absorbed at optical depth τ at the time $u = 0$ will emerge from the medium at an angle $\arccos \eta$ within a unit solid angle at time u . According to what was said above, we may write

$$\bar{p}(s, 0, \eta) = \frac{\lambda}{4\pi} (1 + \beta_2 s) \bar{\omega}(s, \eta), \quad (19)$$

whence it follows that

$$p(u, 0, \eta) = \frac{\lambda}{4\pi} \left[\omega(u, \eta) + \beta_2 \frac{\partial \omega(u, \eta)}{\partial u} \right]. \quad (20)$$

We note that in the case $\beta_1 = 0$, $\beta_2 = 1$, instead of (20) one will have

$$p(u, 0, \eta) = \frac{\lambda}{4\pi} \left[\delta(u) + \omega(u, \eta) + \frac{\partial \omega(u, \eta)}{\partial u} \right], \quad (21)$$

since in this case $\omega(0, \eta)$ is not equal to zero, but $\omega(0, \eta) = 1$.

2. Luminescence of the medium for a uniform distribution of sources.

In the stationary case the solution of the problem obtained by V. A. Ambartsumian ⁽⁵⁾ has the form

$$I(\eta) = B_0 \frac{\varphi(\eta)}{\sqrt{1-\lambda}}, \quad (22)$$

where $B_0 d\tau$ is the amount of energy emitted by sources located in an elementary volume with cross section 1 cm^2 and optical thickness $d\tau$ in 1 sec.; $I(\eta)$ is the intensity of the radiation emerging from the medium in a direction making an angle $\arccos \eta$ with the normal to the boundary.

In the nonstationary case, for $B_0(u) = \delta(u)$, to determine $I(u, \eta)$ we find

$$\bar{I}(s, \eta) = (1 + \beta_1 s) \sqrt{\frac{(1 + \beta_1 s)(1 + \beta_2 s)}{(1 + \beta_1 s)(1 + \beta_2 s) - \lambda}} \bar{\omega}(s, \eta). \quad (23)$$

From (23) it follows that

$$I(u, \eta) = b(u, \eta) + \beta_1 \frac{\partial b(u, \eta)}{\partial u}, \quad (24)$$

where

$$b(u, \eta) = \omega(u, \eta) + \int_0^u f(u') \omega(u - u', \eta) du', \quad (25)$$

and the function $f(u)$ is determined by the relation

$$\bar{f}(s) = \sqrt{\frac{(1 + \beta_1 s)(1 + \beta_2 s)}{(1 + \beta_1 s)(1 + \beta_2 s) - \lambda}} - 1. \quad (26)$$

In the special case $\beta_2 = 0$, $\beta_1 = 1$, instead of formula (24) one should use the formula

$$I(u, \eta) = \delta(u) + b(u, \eta) + \frac{\partial b(u, \eta)}{\partial u}, \quad (27)$$

since in this case $b(0, \eta) = \omega(0, \eta) = 1$.

Let us now find the inversion of $\bar{f}(s)$, restricting ourselves, for simplicity, to particular cases. Let $\beta_1 = 0$, $\beta_2 = 1$ or $\beta_2 = 0$, $\beta_1 = 1$. Then

$$\bar{f}(s) = \sqrt{\frac{s+1}{s+1-\lambda}} - 1, \quad (28)$$

whence it follows that

$$f(u) = \frac{\lambda}{2} e^{-(1-\lambda/2)u} \left[I_0\left(\frac{\lambda u}{2}\right) + I_1\left(\frac{\lambda u}{2}\right) \right], \quad (29)$$

where I_0 and I_1 are Bessel functions of imaginary argument. If $\beta_1 = \beta_2 = 1/2$, then

$$\bar{f}(s) = \frac{1 + s/2}{\sqrt{(1 + s/2)^2 - \lambda}} - 1, \quad (30)$$

which gives

$$f(u) = 2\sqrt{\lambda} e^{-2u} I_1(2u\sqrt{\lambda}). \quad (31)$$

In conclusion, we note that by the method employed, some other problems on the luminescence of a semi-infinite medium can also be solved. In this case the solution will include the function $\omega(u, \eta)$, determined by equation (12).

Leningrad State University
named after A. A. Zhdanov

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CITED LITERATURE

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Note: Figure translations are in progress. See original paper for figures.

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