# THEORY OF ELASTICITY

V. L. AGAMIROV, A. S. VOL' MIR

1964

SovietRxiv

## Abstract

## Full Text

# THEORY OF ELASTICITY

**V. L. AGAMIROV, A. S. VOL' MIR**

# ON THE STABILITY OF A CYLINDRICAL SHELL UNDER LONGITUDINAL IMPACT

*(Presented by Academician Yu. N. Rabotnov on 13 February 1964)*

In recent years a number of works have been published devoted to the behavior of closed cylindrical shells under rapid loading by an axial force and normal pressure (for an exposition of these works see ($^1$)). It was shown that the nature of the dynamic buckling of a shell depends essentially on the rate of loading: as the rate increases, the snap-through phenomenon of the shell becomes increasingly prolonged; at the same time the form of wave formation changes.

In the present paper the buckling of a shell under impact loading proper is investigated. A circular cylindrical shell of mass $M_1$, fixed at one of its ends, is considered, under the assumption that a central axial impact by a load of mass $M_2$ is applied to the free end.

Experiments in which shells were subjected to this kind of loading are described in ($^2$). These experiments showed that the character of buckling of the shell is approximately the same as in the case of static loss of stability of the shell as a whole. Therefore, as the basis for solving the problem we shall take the dynamic differential equations of the nonlinear theory of shells ($^1$).

In a first approximation we shall consider separately the process of propagation of an elastic wave along the length of the shell and the phenomenon of buckling (see ($^2$)). In other words, we shall assume that the deformations caused by the impact are distributed along the length of the shell in the same way as in the case of a straight elastic rod of mass $M_1$. Solving the corresponding wave equation for $u$ by the method of characteristics, we determine the displacement of the struck end of the shell $u^0$, and then the "mean" axial stress along the length, $p = Eu^0/L$, where $L$ is the length of the shell.

Next we write the equations describing the buckling of the shell, taking into account initial imperfections in the form of the middle surface:

$$D\nabla^2\nabla^2(w - w_0) = h\left(w_{,xx}\Phi_{,yy} + w_{,yy}\Phi_{,xx} - 2w_{,xy}\Phi_{,xy} + \frac{1}{R}\Phi_{,xx} - \frac{\gamma}{g}w_{,tt}\right), \tag{1}$$

$$\frac{1}{E}\nabla^2\nabla^2\Phi = w_{,xy}^2 - w_{,xx}w_{,yy} - w_{0,xy}^2 + w_{0,xx}w_{0,yy} - \frac{1}{R}(w-w_0)_{,xx}, \tag{2}$$

where $\Phi$ is the stress function in the middle surface; $w$ and $w_0$ are the total and initial deflections; $t$ is time; $h$ is the shell thickness, $R$ the radius of curvature; the coordinate $x$ is measured along the generator, $y$ along the arc; subscripts following a comma denote differentiation with respect to the corresponding variable.

We approximate the functions $w$ and $w_0$ by means of the expressions

$$\begin{aligned} w &= f\left(\sin\alpha x\sin\beta y + \psi\sin^2\alpha x\sin^2\beta y\right), \\ w_0 &= f_0\left(\sin\alpha x\sin\beta y + \psi\sin^2\alpha x\sin^2\beta y\right); \end{aligned} \tag{3}$$

here $\alpha = m\pi/L,\ \beta = n/R$. Substituting (3) into (2), we find the function $\Phi$; into the expression for $\Phi$ we introduce the term $(-py^2/2E)$. Applying further to equation (1) the usual procedure of the Bubnov-Galerkin method, we arrive at a system of ordinary nonlinear differential equations determining the variation of the parameters $f$ and $\psi$ with time. Integration of these equations

was carried out by the Runge–Kutta method, with the aid of an electronic digital computer, under the initial conditions $f = f_0,\ f_{,t} = 0$ for $t = 0$.

Figure 1 presents the results of computations for the case $R/h = 180$ and $L/R = 2.2,\ f_0 = 10^{-3}h$. Along the abscissa is plotted the parameter $p^*/p^*$ of the force transmitted by the impacting mass to the end of the shell $(p^* = 1/\sqrt{3(1-\mu^2)}$ is the parameter of the upper static critical force), and along the ordinate–the dimensionless deflection arrow $\zeta = f/h$. Two series of curves, $II$ and $III$, correspond to the case of an infinitely large mass $M_2$ at different velocities $V$ of the load at the instant of impact ($II$—$M_2/M_1 = \infty,\ V/c = 10^{-3}$; $III$—$M_2/M_1 = \infty,\ V/c = 2\cdot10^{-3}$; here $c$ denotes the velocity of propagation of an elastic wave in the material of the shell).

Fig. 1

**Fig. 1**

Series $I$ corresponds to a finite mass $M_2$, with $M_2/M_1 = 30$ and $V/c = 2\cdot10^{-3}$. In the case of a finite mass the quantity $p^*$ first increases and then decreases; at the end of the impact $p^*$ becomes zero. Each series contains curves $\zeta(p^*)$ found for different numbers $n$ of waves around the circumference of the shell; these numbers are indicated on the graph. In addition, the parameter $\xi$, which determines the shape of the dent, was varied. The values of $n$ and $\xi$ were determined for which the front of the rapid increase in deflection corresponds to the smallest value of the pressure parameter $p^*$; the corresponding dependence $\zeta(p^*)$ is shown by a solid line. The dash-dot curve is constructed under the assumption that the force $p^*$ varies statically; it is the envelope of the series of curves constructed for different wave numbers $n$. It may be assumed that an impact on the shell causes the phenomenon of snap-through if the dynamic deflection exceeds the static value at the corresponding value of the force. Evidently, in the cases shown in the graph, snap-through of the shell must occur. As was to be expected, the critical number of waves $n$ decreases as the velocity $V$ decreases; in the case of a finite mass of the impacting load, $n$ proves to be smaller than for an infinitely large mass.

Preliminary experiments have shown that the character of wave formation in the shell under impact is different for different transverse sections. Therefore another approximate way of solving the problem is possible. It consists in selecting a certain transverse section of the shell; for it the character of the variation of the compressive force with time is determined. Taking into account that the buckling of the shell is local in character, we integrate equations (1), (2) for a narrow zone adjacent to the section under consideration, under conditionally chosen boundary conditions. In this way we determine the behavior of the shell in different zones along its length and establish in which of these zones the buckling process develops most intensively.

Received
31 I 1964

## REFERENCES

1. A. S. Vol' mir, *Stability of Elastic Systems*, Moscow, 1963.
2. A. R. Coppa, N. Y. Space Sci. Lab., Gen. Electric, 1960.

*Note: Figure translations are in progress. See original paper for figures.*

*Source: Math-Net.Ru and CyberLeninka. Machine translation. Verify with the original.*