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Abstract

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ON TRANSFORMATIONS OF CERTAIN TYPES OF SCHEMES CONNECTED WITH FINITE AUTOMATA

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In the synthesis of automata, in practice one uses various kinds of transformations of their diagrams (see, for example, ([1])). In the present note we consider equivalent transformations of diagrams by replacing their subschemes; it is proved that there does not exist a finite number of rules of this type that make it possible to obtain from each diagram any diagram equivalent to it, even if substitution is allowed when applying the rules. An analogous result is obtained for one class of schemes describing systems of regular events ([2]).

1. -schemes. We shall call an **-scheme*** in an alphabet B a network ([5]) with directed edges and any number of poles, in which letters of the alphabet B are assigned to certain edges. If π is some (possibly self-intersecting) path in an -scheme and Q is the word consisting of the letters assigned to the edges of this path (in the corresponding order; if no edge of π has letters assigned to it, then Q is the empty word), then we shall say that the path π is a path of the kind (has the form) Q. If α, β are vertices of an -scheme S, then we shall denote by $M^S_{\alpha\beta}$ the set of all words Q such that in S there is a path of the kind Q with beginning at α and end at β.

Two -schemes S and T, between the poles of which a one-to-one correspondence has been established, are called **equivalent** if, for any two poles α, β of the scheme S and the corresponding poles α', β' of the scheme T, $M^S_{\alpha\beta} = M^T_{\alpha'\beta'}$. It is not difficult to show that every -scheme is transformed into an equivalent one if some of its subschemes** is replaced by an equivalent one.

Let Φ be an -scheme in the alphabet $\{x_1, \dots, x_k\}$, and let $R_1, \dots, R_k$ be two-pole -schemes (in any alphabet), in each of which one of the poles is declared the input and the other the output. We shall denote by $\Phi_{x_1 \to R_1, \dots, x_k \to R_k}$ the -scheme obtained from Φ by replacing each edge to which the letter x_i is assigned by the scheme R_i (the input of R_i is identified with the beginning of the edge, the output with its end; the poles of Φ are preserved). We shall say that $\Phi_{x_1 \to R_1, \dots, x_k \to R_k}$ is the result of substitution into Φ of the schemes $R_1, \dots, R_k$ in place of the variables $x_1, \dots, x_k$, respectively.

We shall call a two-pole -scheme R **regular** if it has no (directed) paths from the output to the input and has no nonempty paths from the input to the

Fig. 1

Figure 1: Fig. 1

input or from the output to the output. If Φ, Ψ are equivalent -schemes in the alphabet $\{x_1, \dots, x_k\}$, and $R_1, \dots, R_k$ are regular -schemes, then, as is not difficult to show, the -schemes
$\Phi_{x_1 \to R_1, \dots, x_k \to R_k}$ and $\Psi_{x_1 \to R_1, \dots, x_k \to R_k}$ are also equivalent.

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* The choice of the term -scheme is suggested by the close analogy between -schemes and finite sources ([3],[4]).

** The definition of a subscheme is analogous to that given in ([6]).

Let a, β be vertices of a scheme S, N a number, and Q a word. We shall say that the scheme S has the property $\mathcal{E}^{\alpha,\beta}_{N,Q}$ if in S there exists a (possibly self-intersecting) cycle of the form Q^k, where k has no prime divisors greater than N, and moreover in S there are paths from α to some vertex of this cycle and from it to β.

Lemma. Let Φ, Ψ be equivalent I-schemes in the alphabet $\{x_1, \dots, x_k\}$; let N be the number of vertices of the scheme Ψ; let Q be a nonempty word; let S be an I-scheme with poles α, β (and, possibly, other poles); and let $R_1, \dots, R_k$ be regular schemes. Suppose S contains a subscheme $\Phi_{x_1 \to R_1, \dots, x_k \to R_k}$, replacing which by $\Psi_{x_1 \to R_1, \dots, x_k \to R_k}$ transforms S into an I-scheme T (necessarily equivalent to S). Then, if S has the property $\mathcal{E}^{\alpha,\beta}_{N,Q}$, then T also has this property.

Let Φ and Ψ be equivalent I-schemes in the alphabet $\{x_1, x_2, \dots\}$. We shall say that they define a **rule scheme** of equivalent transformation of I-schemes. If $R_1, R_2, \dots$ are arbitrary regular I-schemes, then the pair of I-schemes $S = \Phi_{x_1 \to R_1, \dots}$, $T = \Psi_{x_1 \to R_1, \dots}$ is called a **realization** of the given rule scheme and defines a **rule** for transforming I-schemes, according to which in any I-scheme U one may replace a subscheme isomorphic to S by T, and conversely.

Fig. 1

Theorem 1. Let B be an arbitrary alphabet. There does not exist a finite set of rule schemes such that any two equivalent I-schemes in the alphabet B can be transformed one into the other by applying rules that are realizations of schemes from the given set.

The theorem follows from the lemma: if N is a prime number, then the equivalent I-schemes shown in Fig. 1 (the poles of the I-schemes are indicated by circles), by virtue of the lemma, cannot be transformed one into the other by means of realizations of rule schemes that are defined by I-schemes Φ, Ψ with $\leq N-1$ vertices. From the lemma it is also not difficult to derive the theorems of V. N. Red' ko ([7],[8]) on the nonaxiomatizability ([9]) of the algebra of events.*

Fig. 2

Figure 2: Fig. 2

Remark. The following generalization of rule schemes is possible: a rule scheme is given by a pair of "schemes" Φ and Ψ, containing (marked by letters) elements of two types: elements of the first type are two-terminal; regular I-schemes may be substituted for them; elements of the second type may have any number of poles, and arbitrary I-schemes may be substituted in their place; here it is required that, under any such substitution, the "schemes" Φ and Ψ turn into a pair of equivalent I-schemes. With this "generalized" concept of a rule scheme, the theorem of this section remains valid; the method of proof is the same. In Section 2, when considering transformations of automaton diagrams, precisely this generalized approach is used.

2. Automaton diagrams. We shall consider the set $K_{A,B}$ of (finite, everywhere-defined) Mealy automata ([2]) with input alphabet $A = \{a_1, \ldots, a_m\}$ and output alphabet $B = \{b_1, \ldots, b_n\}$, $n \geq 2$. Let $A \times B$ be an alphabet of mn letters (a_i, b_j), $1 \leq i \leq m$, $1 \leq j \leq n$. The state diagrams of automata from $K_{A,B}$ will be regarded as I-schemes in $A \times B$ with one pole–the initial state; we shall call such schemes A-diagrams. If two automata realize the same mapping ([2]) of the set of input words into the set of output words, then the corresponding A-diagrams will be called A-equivalent. Let S, T be two I-schemes

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* In an alphabet containing at least two letters; for a one-letter alphabet the proof can be carried out by a similar method.

in $A \times B$. If there exist two A-diagrams D and D', obtained one from the other by replacing the subscheme S by T, then the pair (S, T) is called **regular**; if any such A-diagrams D and D' are A-equivalent, then S and T are called **A-equivalent** (this concept agrees with the previously introduced A-equivalence of A-diagrams). The properties of regularity of a pair and of A-equivalence are algorithmically recognizable.

Fig. 2

We shall assume that a pair of A-equivalent I-schemes S, T specifies a **rule** of equivalent transformation of A-diagrams, according to which one may pass from a diagram D to a diagram D', if D' is obtained from D by replacing the subscheme S by T (or conversely). Let us now introduce schemes of rules. Let Φ, Ψ be two schemes (together with a correspondence between their poles), constructed from elements of two types: oriented two-pole elements, denoted by the letters $x_1, x_2, \ldots$, and elements with an arbitrary number of poles $(0, 1, 2, 3, \ldots)$, denoted by $X_1, X_2, \ldots$; we agree that elements denoted by the same letter must have the same number of poles; the poles of the elements will be numbered.

Let $R_1, R_2, ...$ be regular two-pole I-schemes in $A \times B$, and let $S_1, S_2, ...$ be I-schemes in $A \times B$ (with numbered poles), where S_i has as many poles as the elements labeled X_i have. Then I-schemes S and T are defined, which are the result of substituting the I-schemes $R_1, R_2, \dots, S_1, S_2, ...$ in place of the elements $x_1, x_2, \dots, X_1, X_2, ...$, respectively (the substitution in place of the elements X_i is carried out with account taken of the numbering of the poles). Suppose that the schemes Φ and Ψ are such that, under any substitution of the described kind, the obtained I-schemes S and T are A-equivalent if and only if they form a regular pair. In this case we shall say that the pair Φ, Ψ specifies a **scheme of rules**; the rule specified by the pair S, T will be called a **realization** of the rule scheme under consideration.

As an example, Fig. 2a shows a scheme of rules, and Fig. 2b shows one of its realizations.

Theorem 2. *There does not exist a finite set of rule schemes whose realizations make it possible to transform any A-diagram from $K_{A,B}$ into any A-diagram equivalent to it.*

The analogous theorem is true for Moore automata.

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Note: Figure translations are in progress. See original paper for figures.