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Abstract

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PHYSICS

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ON RESONANT INDUCED SCATTERING AND RADIATION IN A MEDIUM

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1. Induced Cherenkov radiation and absorption of waves in a medium ([1]) may lead to effects of acceleration of a charged particle by waves ([2–4]), and in the general case, as applied to a plasma, to effects of diffusion of particles in energy space (the "quasilinear" approximation ([5–7])). In the next order of perturbation theory there are possible effects of induced scattering, induced emission of two quanta, and induced absorption of two quanta. Since these effects depend quadratically on the number of quanta, they describe nonlinear effects of wave interaction.

Here we would like to draw attention to the fact that the use of perturbation theory to describe induced scattering and induced emission and absorption of two quanta becomes impossible if the frequency and angle of the incident radiation are close to the corresponding Cherenkov cone $\omega = \mathbf{kv}$. Owing to the possibility of spontaneous Cherenkov radiation, the scattering becomes resonant*. In considering resonant effects it is necessary to sum the perturbation-theory series ([9]), which leads to taking account of radiative corrections to the Green' s function of the charged particle. It is sufficient to take into account the mass operator, which describes the renormalization of the particle mass due to the presence of the medium ([10]) and its energy losses. All that has been said indicates that, for example, nonlinear effects in a plasma may depend on the indicated radiative corrections**.

2. To construct a theory of the effects of induced scattering, emission, and absorption, taking radiative corrections into account, let us note that the Green' s function of a charged particle with allowance for the mass operator in a medium in the approximation e^2 has the form ([10])

$$G(p) = i\,\frac{\hat{\pi} - \mu}{\pi^2 - \mu^2}, \tag{1}$$

where $\pi_\mu = p_\mu + \delta p_\mu$; $\mu = m + \delta m$, and δp_μ and δm are found in ([10]). In a homogeneous medium in the transparency region only longitudinal and transverse waves are possible, whose operators have the form***

$$\hat{A}^t_\mu = \frac{1}{\sqrt{V}} \sum_{\mathbf{k},s} e^{t,s}_\mu v^t_s \left\{ \hat{a}_{t,s}(\mathbf{k}) \exp[-i\omega^t_s(\mathbf{k})t + i\mathbf{kr}] + \hat{a}^+_{t,s}(\mathbf{k}) \exp[i\omega^t_s(\mathbf{k})t - i\mathbf{kr}] \right\}; \tag{2}$$

$$\hat{A}^l_\mu = \frac{1}{\sqrt{V}} \sum_{\mathbf{k},s} e^{l,s}_\mu v^l_s \left\{ \hat{a}_{l,s}(\mathbf{k}) \exp[-i\omega^l_s(\mathbf{k})t + i\mathbf{kr}] + \hat{a}^+_{l,s}(\mathbf{k}) \exp[i\omega^l_s(\mathbf{k})t - i\mathbf{kr}] \right\}. \tag{3}$$

* The situation is analogous to resonant scattering of light by bound electrons ([8]) or emission of two quanta ([8,9]).

** To avoid misunderstandings, let us emphasize that this is not about radiative corrections to the energy losses of particles ([11]), but about radiative corrections to the particle Green's function, i.e., about taking into account the mass operator in the first order in e^2 ([10]).

*** Quantization of transverse waves is a straightforward generalization ([12,13]) to media with spatial dispersion. In the case when there are several branches of new waves ([14]), the sum over branches $\omega_s(\mathbf{k})$ enters into (2), (3). Quantization of longitudinal waves was considered in ([15]), but an incorrect normalization was used there.

where

$$e^t_{\mu s} e^t_{\mu s'} = \delta_{s,s'}, \qquad e^{t,s}_\mu = \{e^{t,s}, 0\}; \qquad (e^{t,s}_\mu)^2 = (e^{l,s}_\mu)^2 = 1;$$

$$e^l_\mu e^t_\mu = 0; \qquad e^l_\mu = \left(1 - \frac{\mathbf{k}^2}{\omega^2}\right)^{-1/2} \left\{ \frac{\mathbf{k}}{|\mathbf{k}|}; \frac{i|\mathbf{k}|}{\omega} \right\};$$

$$v^t_s = \sqrt{4\pi} \left[\frac{\partial}{\partial \omega} \omega^2 \varepsilon^t(\omega, \mathbf{k}) \right]^{-1/2}_{\omega = \omega^t_s(\mathbf{k})}; \qquad v^l_s = \left[(\omega^2 - \mathbf{k}^2) \frac{\partial \varepsilon^l(\omega, \mathbf{k})}{\partial \omega} \right]^{-1/2} \sqrt{4\pi} \Bigg|_{\omega = \omega^l_s(\mathbf{k})},$$

where $\varepsilon^l(\omega, \mathbf{k})$ and $\varepsilon^t(\omega, \mathbf{k})$ are the longitudinal and transverse dielectric constants of the medium; $\omega^t_s(\mathbf{k})$ and $\omega^l_s(\mathbf{k})$ are the solutions of the dispersion equations $\omega^2 \varepsilon^t(\omega, \mathbf{k}) = \mathbf{k}^2$, $\varepsilon^l(\omega, \mathbf{k}) = 0$. Effects are possible involving only transverse quanta, only longitudinal ones, and with conversion of longitudinal into transverse ones*. With the aim of clarifying the principal points, we shall restrict ourselves here to the consideration of transverse waves.

Using (1), (2), (3), by the standard methods of quantum electrodynamics we obtain the differential probability of a change (per 1 sec.) of the state of a particle of momentum $\mathbf{p}$ and energy ε_p:

$$dW_p(\mathbf{k}_1, \mathbf{k}_2) = 2r_0^2 N_{\mathbf{k}1} d\mathbf{k}_1 N_{\mathbf{k}2} d\mathbf{k}_2 \left[\frac{\partial}{\partial\omega_1}\omega_1^2\varepsilon^t(\omega_1, \mathbf{k}_1)\right]^{-1} \left[\frac{\partial}{\partial\omega_2}\omega_2^2\varepsilon^t(\omega_2, \mathbf{k}_2)\right]^{-1}$$
$$\times \{[W_p^{\pm}(\mathbf{k}_1, \mathbf{k}_2)\delta(\varepsilon_p - \varepsilon_{p+\mathbf{k}1-\mathbf{k}_2} + \omega_1 - \omega_2) + 1 \rightleftarrows 2]$$
$$+ W_p^{++}(\mathbf{k}_1, \mathbf{k}_2)\delta(\varepsilon_p - \varepsilon_{p+\mathbf{k}1+\mathbf{k}_2} + \omega_1 + \omega_2)$$
$$+ W_p^{--}(\mathbf{k}_1, \mathbf{k}_2)\delta(\varepsilon_p - \varepsilon_{p-\mathbf{k}1-\mathbf{k}_2} - \omega_1 - \omega_2)\}. \quad (4)$$

Here $r_0 = e^2/m$ is the classical radius; $N_{\mathbf{k}1}, N_{\mathbf{k}2}$ are the numbers of quanta with momenta $\mathbf{k}_1$ and $\mathbf{k}_2$ ($N_{\mathbf{k}1} \gg 1$; $N_{\mathbf{k}2} \gg 1$); $\omega_1 = \omega^t(\mathbf{k}_1)$; $\omega_2 = \omega^t(\mathbf{k}_2)$. $W_p^{\pm}(\mathbf{k}_1, \mathbf{k}_2)$ describes induced scattering with absorption of $\mathbf{k}_1$ and emission of $\mathbf{k}_2$; $W_p^{++}(\mathbf{k}_1, \mathbf{k}_2)$, induced absorption of two quanta; $W_p^{--}(\mathbf{k}_1, \mathbf{k}_2)$, induced emission of two quanta. Resonance effects occur for large wavelengths ($|\mathbf{k}_1|, |\mathbf{k}_2| \ll p$). Below are exact quantum expressions of interest in problems of Compton scattering in the presence of a medium:

$$W_p^{\pm} = \frac{m^2}{\varepsilon_1\varepsilon_2}\frac{1}{|\tilde{\chi}_1|^2|\tilde{\chi}_2|^2}\left\{W_{p,0} - \frac{k_1^2}{\mathbf{k}_1^2}W_{p,1} - \frac{k_2^2}{\mathbf{k}_2^2}W_{p,2} + \frac{k_1^2k_2^2}{\mathbf{k}_1^2\mathbf{k}_2^2}W_{p,1,2}\right\}, \quad (5)$$

where

$$k_1^2 = \mathbf{k}_1^2 - \omega_1^2; \qquad k_2^2 = \mathbf{k}_2^2 - \omega_2^2; \qquad \chi_1 = \frac{2\varepsilon_p}{m^2}\left[(\mathbf{k}_1\mathbf{v} - \omega_1) + \frac{k_1^2}{2\varepsilon_p}\right],$$

$$\chi_2 = -\frac{2\varepsilon_p}{m^2}\left[(\mathbf{k}_2\mathbf{v} - \omega_2) - \frac{k_2^2}{2\varepsilon_p}\right]; \qquad \tilde{\chi} = \chi + \frac{2\Delta m}{m} - i\frac{\varepsilon_p\gamma_p}{m^2}; \qquad \mathbf{v} = \frac{\mathbf{p}}{\varepsilon_p}$$

is the particle velocity; the quantity Δm is the change of the particle mass in the medium [10], and γ_p is the probability of a change of the particle state in the approximation e^2 [10]. Here it has been taken into account that in Δm and γ_p one may retain only the classical limit ($|\mathbf{k}| \ll p$), since $\tilde{\chi}_1, \tilde{\chi}_2$ differ appreciably from χ_1, χ_2 only near resonance ($\chi \to 0$). Further,

$$W_{p,0} = 4(\chi_1 + \chi_2)^2 - 4\chi_1\chi_2(\chi_1 + \chi_2) - \frac{2(k_1^2 + k_2^2)}{m^2}[\chi_1^2 + \chi_2^2 + \chi_1\chi_2(\chi_1 + \chi_2)]$$
$$+ \frac{k_1^2k_2^2}{m^4}(\chi_1^2 + \chi_2^2) + 2\chi_1\chi_2\frac{(k_1^2 + k_2^2)^2}{m^4} - \chi_1\chi_2(\chi_1^2 + \chi_2^2); \quad (6)$$

* Let us note in passing that the effects of induced scattering and emission could in principle be used to detect new branches of radiation because of the conversion of a wave of a branch difficult to measure (for example, because of absorption) into waves of a branch accessible to measurement. The resonant character of scattering by fast particles appreciably increases the cross section of the process.

$$\begin{aligned}W_{p,1} = & -\frac{\chi_1\chi_2}{2}(\chi_1^2+\chi_2^2) - \frac{k_1^2}{m^2}(\chi_1+\chi_2)^2 + \frac{k_1^2}{m^2}\chi_1\chi_2(\chi_1+\chi_2) - \chi_1\chi_2\frac{k_1^2(k_1^2+k_2^2)}{m^4} + \\ & + \frac{k_1^2k_2^2}{2m^4}(\chi_1^2+\chi_2^2) + \frac{4\varepsilon_1\varepsilon_2}{m^2}\left[(\chi_1+\chi_2)^2\left(1-\frac{k_2^2}{2m^2}\right) + \chi_1\chi_2\frac{k_1^2}{m^2}\right] \\ & + \frac{4\varepsilon_1\omega_2}{m^2}\left[\chi_2(\chi_1+\chi_2) - \frac{k_2^2}{2m^2}\chi_2^2 + \frac{k_1^2}{2m^2}\chi_1\chi_2\right] + \frac{4\varepsilon_2\omega_2}{m^2}\left[-\chi_1(\chi_1+\chi_2) + \frac{k_2^2}{2m^2}\chi_1^2 - \frac{k_1^2}{2m^2}\chi_1\chi_2\right] \\ & + \frac{2\varepsilon_2\omega_1}{m^2}\left(\chi_1\chi_2^2 - \frac{k_2^2}{m^2}\chi_1\chi_2\right) + \frac{2\varepsilon_1\omega_1}{m^2}\left(-\chi_1^2\chi_2 + \frac{k_2^2}{m^2}\chi_1\chi_2\right) - \frac{4\chi_1\chi_2\omega_2^2}{m^2}.\end{aligned} \tag{7}$$

Here $\varepsilon_1 = \varepsilon_p$, $\varepsilon_2 = \varepsilon_{p+\mathbf{k}_1-\mathbf{k}_2}$. The quantity $W_{p,2}$ differs from $W_{p,1}$ by the replacement $1 \rightleftarrows 2$ in (7) in all quantities except χ_1, χ_2;

$$\begin{aligned}2W_{p,1,2} = & -\frac{\chi_1\chi_2}{2}(\chi_1^2+\chi_2^2) + \chi_1^2\chi_2^2 + \frac{k_1^2k_2^2}{2m^2}(\chi_2^2+\chi_1^2-2\chi_1\chi_2) + \\ & + \frac{2\varepsilon_1\varepsilon_2}{m^2}\left[4\chi_1\chi_2(\chi_1+\chi_2) - \frac{k_1^2+k_2^2}{m^2}(\chi_1+\chi_2)^2\right] \\ & + \frac{2\varepsilon_1\omega_2}{m^2}\left[2\chi_1\chi_2^2 - \frac{k_2^2}{m^2}\chi_2^2 - \frac{k_1^2}{m^2}\chi_1\chi_2\right] + \frac{2\varepsilon_2\omega_1}{m^2}\left[2\chi_1\chi_2^2 - \frac{k_1^2}{m^2}\chi_2^2 - \frac{k_2^2}{m^2}\chi_1\chi_2\right] \\ & + \frac{2\varepsilon_1\omega_1}{m^2}\left[-2\chi_1^2\chi_2 + \frac{k_1^2}{m^2}\chi_1^2 + \frac{k_2^2}{m^2}\chi_1\chi_2\right] + \frac{2\varepsilon_2\omega_2}{m^2}\left[-2\chi_2\chi_1^2 + \frac{k_2^2}{m^2}\chi_1^2 + \frac{k_1^2}{m^2}\chi_1\chi_2\right] + \\ & + \frac{8\varepsilon_1\varepsilon_2}{m^4}\left[(\varepsilon_1+\omega_1)\chi_2 + (\varepsilon_1-\omega_2)\chi_1\right]^2.\end{aligned} \tag{8}$$

For $k_1^2 = k_2^2 = 0$ we obtain the Klein-Nishina-Tamm formula (8). In the limit $|\mathbf{k}_1|, |\mathbf{k}_2| \ll p$.

$$
\begin{aligned}
W_p^{\pm} = \frac{m^2}{\varepsilon_p^2}&\left[\left(\mathbf{k}_1\mathbf{v}-\omega_1+\frac{m}{\varepsilon_p}\Delta m\right)^2+\frac{\gamma_p^2}{4}\right]^{-1}\left[\left(\mathbf{k}_2\mathbf{v}-\omega_2-\frac{m}{\varepsilon_p}\Delta m\right)^2+\frac{\gamma_p^2}{4}\right]^{-1}\times\\
&\times\Bigg\{(\mathbf{k}_1\mathbf{k}_2-\omega_1\omega_2)^2\left(v^2-\frac{1}{\varepsilon_1^t}\right)\left(v^2-\frac{1}{\varepsilon_2^t}\right)-2(\mathbf{k}_1\mathbf{k}_2-\omega_1\omega_2)(\mathbf{k}_1\mathbf{v}-\omega_1)\times\\
&\times\left[(\omega_1+\omega_2)\left(\frac{1}{\varepsilon_1^t}-1\right)\left(\frac{1}{\varepsilon_2^t}-1\right)+\frac{m^2}{\varepsilon_p^2}\omega_1\left(\frac{1}{\varepsilon_2^t}-1\right)+\frac{m^2}{\varepsilon_p^2}\omega_2\left(\frac{1}{\varepsilon_1^t}-1\right)\right]\\
&-2(\mathbf{k}_1\mathbf{k}_2-\omega_1\omega_2)(\mathbf{k}_1\mathbf{v}-\omega_1)^2\left[\left(\frac{1}{\varepsilon_1^t}-1\right)\left(\frac{1}{\varepsilon_2^t}-1\right)-\frac{m^2}{\varepsilon_p^2}\right]\\
&+(\mathbf{k}_1\mathbf{v}-\omega_1)^2\times\\
&\times\Bigg[(\omega_1+\omega_2)^2\left(\frac{1}{\varepsilon_1^t}-1\right)\left(\frac{1}{\varepsilon_2^t}-1\right)+\frac{m^2}{\varepsilon_p^2}\omega_1^2\left(\frac{1}{\varepsilon_2^t}-1\right)+\\
&\qquad+\frac{m^2}{\varepsilon_p^2}\omega_2^2\left(\frac{1}{\varepsilon_1^t}-1\right)+\omega_1^2\varepsilon_1^t\left(v^2-\frac{1}{\varepsilon_1^t}\right)+\omega_2^2\varepsilon_2^t\left(v^2-\frac{1}{\varepsilon_2^t}\right)\Bigg]+\\
&+2(\mathbf{k}_1\mathbf{v}-\omega_1)^3\left[\frac{\omega_1}{\varepsilon_2^t}\left(\frac{1}{\varepsilon_1^t}-1\right)+\frac{\omega_2}{\varepsilon_1^t}\left(\frac{1}{\varepsilon_2^t}-1\right)\right]+(\mathbf{k}_1\mathbf{v}-\omega_1)^4\left(1+\frac{1}{\varepsilon_1^t\varepsilon_2^t}\right)\Bigg\};\\
&\varepsilon_{1,2}^t=\varepsilon^t(\omega_{1,2},\mathbf{k}_{1,2}).
\end{aligned}
\tag{9}
$$

In (9) the structure of the resonant denominators is explicitly visible. Let us note that in the numerator of (5)–(9) we have neglected the small quantities $\Delta m, \gamma_p$. In (4), along with (9), there is contained the term (9) with $\mathbf{k}_1 \rightleftarrows \mathbf{k}_2$. The probability of induced emission of two quanta $W_p^-(\mathbf{k}_1,\mathbf{k}_2)$ is obtained from $W_p^+(\mathbf{k}_1,\mathbf{k}_2)$ by the replacement $\mathbf{k}_1 \to -\mathbf{k}_1$, and the probability of induced absorption of two quanta

is obtained from $W_p^{\pm}(\mathbf{k}_1,\mathbf{k}_2)$ by replacing $k_2 \to -k_2$. Let us note that in the case when $N_{\mathbf{k}1}$ (or $N_{\mathbf{k}2}$) is absent, one obtains expressions for the effects of scattering of electromagnetic waves by a charge moving in a medium (see also (17)) and oscillating under the action of the incident electromagnetic wave. In the quantum calculation, scattering with radiation of anomalous-Doppler-effect frequencies corresponds to processes of emission of two quanta, which is also clear from (18).

3. In addition to the processes considered, effects are possible, in the same order in the number of quanta, which describe corrections to induced Cherenkov radiation and to the absorption of one quantum (see (17)).

Let us draw attention to the fact that the mean change in the energy of the particle due to all the induced processes described above*, as the calculation shows, is not of a resonant character. Thus, the resonant character of induced scattering and radiation may, for example, affect the distribution function of the quanta, but not the mean characteristics of the plasma particles. The criterion for applicability of the expansion in the number of quanta for mean quantities is obtained by writing the condition that the induced processes considered here be small in comparison with the processes of induced Cherenkov radiation and absorption (1). Because of the nonresonance of the mean effect, we obtain $p^2 \ll p_{\max}^2 \simeq e^2k^4N_k(\Delta k)^3$ (i.e., roughly $p \ll eA$; A is the vector potential of the wave; Δk is the interval of wave vectors in which $N_k \neq 0$). Formulas (1) give the correct order of magnitude also for $p \sim p_{\max}$, and then formulas (1) go over into the formulas for the Fermi acceleration. The criterion for the possibility of an expansion in the number of quanta when considering changes in the distribution of quanta due to nonlinear effects is substantially more stringent. The present work indicates a possible way of resolving the difficulties with zeros of the energy denominators that might arise in the approach of (19).

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* It can be found by summing expressions of the type $W_p \Delta\varepsilon$, where W_p is the probability of the process, and $\Delta\varepsilon$ is the change in the particle energy in the process.

Note: Figure translations are in progress. See original paper for figures.

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