PHYSICS

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1964

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Abstract

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PHYSICS

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ON THE PHENOMENA OF A CRITICAL MAGNETIC FIELD AND ANOMALOUS DIFFUSION IN A WEAKLY IONIZED PLASMA

(Presented by Academician L. A. Artsimovich, 12 X 1963)

1. In a number of experiments on plasma diffusion across a magnetic field, when the magnetic-field strength is increased monotonically, starting from a certain value of it the appearance of "noise" is observed, which indicates the presence of an instability and an increase in diffusion.

In the case $\omega \ll \omega_{Hi}$ (ω is the frequency of the noise oscillations, ω_{Hi} is the ion cyclotron frequency), this critical-magnetic-field phenomenon is explained by means of an instability associated with a "branch" of oscillations whose phase velocity coincides with the drift velocity of electrons in the magnetic field due to pressure gradients— "drift" waves (see, for example, the review (^{1})). However, in the frequency interval $\omega_{Hi} \ll \omega \ll \omega_{He}$, the "drift" waves should be damped.

The investigation carried out in the present work for this frequency interval indicates the existence of a new "branch" of oscillations— "antidrift" waves, with the help of which it proves possible to estimate the magnitude of the critical magnetic-field strength, obtaining good agreement with experimental data (^{2}).

2. Let us consider a weakly ionized inhomogeneous plasma of sufficiently high density, so that frequent collisions ensure the applicability of a hydrodynamic description of the motion of the electron component (the ions will be discussed separately).

Let the density of charged particles n depend only on x, and let the magnetic field of strength H be directed along the z-axis. We consider the most interesting case, when the electron temperature is constant and much greater than the ion temperature. For the electrons we shall assume the validity of the drift approximation ($\omega \ll \omega_{He}$ and $\omega_{He}\tau_{en} \gg 1$, where ω is the oscillation frequency, ω_{He} is the electron cyclotron frequency, and τ_{en} is the time between electron-neutral collisions).

As for the ions, for the oscillation frequencies considered here ($\omega \gg \omega_{Hi}$ and $\omega_{Hi}\tau_{in} \ll 1$, where ω_{Hi} is the ion cyclotron frequency and τ_{in} is the time between ion-neutral collisions), the action of the magnetic field on them may be neglected.

We shall assume the perturbation to be potential ($\delta E_i = -ik_i\delta\varphi$, where E_i is the i-th component of the electric-field strength, k_i is the i-th component of the wave vector, φ is the potential; by δa is meant the perturbation of the quantity a) and choose it in the form $\delta a = \delta\tilde{a}\exp(ik_x x + ik_y y + ik_z z + i\omega t)$ (below, unperturbed quantities will be denoted by the subscript 0). Then, under the assumption of quasineutrality ($n_e = n_i = n,\ \delta n_e = \delta n_i = \delta n$), analogously to (^{3}), in a coordinate system in which $E_\perp^{(0)} = 0$, we have the following linearized system of equations:

$$\frac{d}{dx}(T\,\delta n) - en_0\frac{d}{dx}\delta\varphi + \frac{e}{c}n_0\delta v_{ey}B + \ln' n(x)\,T\,\delta n = 0; \tag{1}$$

$$ik_y T\,\delta n - ik_y en_0\delta\varphi - \frac{e}{c}n_0\delta v_{ex}B = 0; \tag{2}$$

$$ik_z T\,\delta n - ik_z en_0\delta\varphi = -n_0 m_e(\nu_{ei} + \nu_{en})\delta v_{ez} - n_0 m_e\nu_{ei}\delta v_{iz}; \tag{3}$$

$$i\omega\,\delta n + \operatorname{div}(\mathbf{v}_{e0}\delta n) + \operatorname{div}(\delta\mathbf{v}_e n_0) = 0; \tag{4}$$

$$i\omega m_i\delta v_{ix} + e\frac{d}{dx}\delta\varphi = 0; \tag{5}$$

$$\omega m_i\delta v_{iy} + ek_y\delta\varphi = 0; \tag{6}$$

$$\omega m_i\delta v_{iz} + ek_z\delta\varphi = 0; \tag{7}$$

$$i\omega\,\delta n + \operatorname{div}(n_0\delta\mathbf{v}_i) + \operatorname{div}(\mathbf{v}_{i0}\delta n) = 0, \tag{8}$$

where T is the electron temperature (under the assumption that $T_e \gg T_i$); e is the electron charge; c is the speed of light in vacuum; m_e, m_i are, respectively, the masses of the electron and ion; ν_{ei}, ν_{en} are the frequencies of electron-ion and electron-neutral collisions, respectively; $\mathbf{v}_e, \mathbf{v}_i$ are the directed velocities of the electrons and ions, respectively. In equations (5)–(7) we have neglected the dissipative terms in view of the condition $\omega \gg \nu_{in}$.

The system of equations (1)–(8) can be reduced to the following differential equation:

$$\frac{d^2\delta\varphi}{dx^2} + \left\{\frac{ik_z^2\frac{\omega^2}{\nu_{en}}\frac{m_i}{m_e} + \frac{\nu_{en}}{\nu_{en}+\nu_{ei}}\omega k_z^2}{i\frac{k_z^2}{\nu_{en}}\frac{T}{m_e} - \omega + \frac{\omega^2}{\omega_{H_i}}\frac{\ln' n}{k_y}} - k_y^2\right\}\delta\varphi = 0. \tag{9}$$

The problem arises of finding the eigenvalues corresponding to eigenfunctions that vanish outside the transition layer, where the density is inhomogeneous.

3. The effect of plasma inhomogeneity is given by the terms containing the factor $\ln' n$. The dispersion equation (9) for a homogeneous plasma has roots corresponding to damped solutions. It is therefore natural to seek the solution near the point of maximum density gradient.

Substituting $\ln' n = \ln' n_0(1 - x^2/R^2)$ into form (9), we obtain the dispersion equation in the form

$$\frac{d^2\delta\varphi}{dx^2} + 2\left(E - \frac{\tilde{\omega}^2 x^2}{2}\right)\delta\varphi = 0, \tag{10}$$

where

$$E = \frac{ik_z^2 \frac{\omega^2}{\nu_{en}} \frac{m_i}{m_e} + \frac{\nu_{en}}{\nu_{en}+\nu_{ei}}\omega k_z^2}{2\left(i\frac{k_z^2}{\nu_{en}}\frac{T}{m_e} - \omega + \frac{\omega^2}{\omega_{H_i}}\frac{\ln' n_0}{k_y}\right)} - \frac{k_y^2}{2}; \tag{11}$$

$$\tilde{\omega}^2 = -\frac{\frac{\omega^2}{\omega_{H_i}}\frac{\ln' n_0}{k_y}\frac{1}{R^2}\left(ik_z^2\frac{\omega^2}{\nu_{en}}\frac{m_i}{m_e} + \frac{\nu_{en}}{\nu_{en}+\nu_{ei}}\omega k_z^2\right)}{\left(i\frac{k_z^2}{\nu_{en}}\frac{T}{m_e} - \omega + \frac{\omega^2}{\omega_{H_i}}\frac{\ln' n_0}{k_y}\right)^2}. \tag{12}$$

It is easy to see that equation (10), in a system of units where $h = \mu = 1$, has the form of Schrödinger' s equation for a linear oscillator with complex frequency $\tilde{\omega}$ and energy E.

The solution of such an equation, as is known, has the form:

$$\delta\varphi_{\tilde{n}} = \left(\frac{\tilde{\omega}}{\pi}\right)^{1/4} \frac{1}{\sqrt{2^{\tilde{n}}\tilde{n}!}} e^{-\frac{\tilde{\omega}}{2}x^2} H_{\tilde{n}}(x\sqrt{\tilde{\omega}}), \tag{13}$$

where $H_{\tilde{n}}(x\sqrt{\tilde{\omega}})$ is a Hermite function, $\tilde{n} = 0, 1, 2, \ldots$

The behavior of the function $\delta\varphi_{\tilde{n}}$ at infinity depends on the sign of $\operatorname{Re}\tilde{\omega}$, which is determined from the equation

$$E_{\tilde{n}} = (\tilde{n} + 1/2)\tilde{\omega} \tag{14}$$

by substituting into it expressions (11) and (12). After simple calculations we obtain $\operatorname{Re}\tilde{\omega} > 0$. This latter circumstance indicates the local character of the perturbations, which confirms the correctness of the approach to the problem set forth above.

It is convenient to represent the frequency ω in the form

$$\omega = \omega_0 + \Delta\omega, \tag{15}$$

where $\Delta\omega/\omega \ll 1$.

Substituting expression (15) into equation (10), we find

$$\omega = \frac{k_y \omega_{H_i}}{\ln' n_0} + i\,\frac{k_z^2 \omega_{H_i}^2}{{n'}^2 n_0 \nu_{en}}\,\frac{m_i}{m_e}(1 - r_i^2(T_e){n'}^2 n_0) - \frac{\omega_{H_i}^2}{{n'}^2 n_0}\,\frac{\tilde{n}}{R}\,2\sqrt{-i\,\frac{k_z^2}{\omega_{H_i}\nu_{en}}\,\frac{\ln' n_0}{k_y}\,\frac{m_i}{m_e}}. \tag{16}$$

From (16) we find that the instability develops under the condition that $r_i^2 {n'}^2 n_0 \gtrsim 1$.* With increasing k_z and $\ln' n_0$, the increment also grows until it reaches its maximum value $\sim \omega$.

It is easy to see that the expression obtained by us for the frequency ω (16) differs fundamentally from the corresponding expressions, for example, in works ([1,4]). We shall call the obtained "branch" of oscillations antidrift waves.

4. As already mentioned, a number of experimental works have appeared on determining the magnitude of the critical magnetic-field strength B_c. Let us estimate the critical field for our model. At small H there appears a stabilizing effect associated with the growth of plasma diffusion across the magnetic field. This effect becomes substantial when

$$\nu \sim \omega > \frac{D}{\lambda_\perp^2} \sim k_y^2 r_e^2 \nu_{en}, \tag{17}$$

where D is the diffusion coefficient, and $\lambda_\perp$ is the transverse wavelength of the perturbation.

From equation (10) at $\nu \to 0$ we have

$$\omega \sim k_{yR} \omega_{H_i}. \tag{18}$$

From the condition of smallness of ion-neutral collisions ($\nu_{in} < \omega$), taking (18) into account, we obtain an estimate for k_y:

$$k_y > \frac{\nu_{in}}{\omega_{H_i} R}. \tag{19}$$

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* As for the conditions giving an unstable solution, one should mention the presence of the dissipative term in equation (3), for the solution of the homogeneous system of equations (3)—(8), neglecting the small term $i\frac{T}{m_e}k_x \ln' n$, gives ion sound:

$$\omega^2 = \frac{T}{m_i} k^2.$$

Relations (17), (18), (19) give the condition of instability

$$(R\omega_{H_i})^2 > r_e^2 \nu_{en} \nu_{in}. \tag{20}$$

From the last relation we determine the magnitude of the field strength of the critical magnetic field B_c:

$$B_c \sim \frac{c}{e} \sqrt[4]{T \frac{m_e m_i^2}{R^2} \nu_{en} \nu_{in}}\,. \tag{21}$$

For the upper frequency threshold $\nu_{en} \sim 170 \cdot 10^7$ sec^{-1} ($\nu_{en}/\nu_{in} \sim 10$, $R \sim 2$ cm, $T \sim 1$ eV), the critical-field strength is $B_c \sim 210$ gauss; for the lower frequency threshold $\nu_{en} \sim 10^7$ sec^{-1} (with the remaining parameters unchanged), $B_c \sim 30$ gauss. The calculated orders of magnitude of B_c agree with the data in Ref. [2].

The dependence of the critical-field strength on the plasma pressure has the form

$$B_c \sim k\sqrt{n_n} \tag{22}$$

(where $k \ll 1$ is a constant coefficient), i.e., it differs little from a linear one, which was also noted in Ref. [2]. (The points calculated from formula (23) fell on the corresponding curves of the graph in Ref. [2].)

In conclusion, the author expresses gratitude to A. A. Galeev and R. Z. Sagdeev for useful advice and fruitful discussion of the results of the work.

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Received
12 X 1963

CITED LITERATURE

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Note: Figure translations are in progress. See original paper for figures.

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