HYDRAULICS

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Abstract

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METHOD FOR DETERMINING THE HYDRODYNAMIC COEFFICIENT IN GAS FLOW IN MAIN PIPELINES

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In one of our previous works we considered the transient flow of gas in mains, occurring after the sudden closing of a valve at the terminal point. Assuming that the flow before the valve was closed, i.e., at the initial instant of time, was steady, and that at the supply point of the pipeline the pressure after the cessation of flow remains constant, we obtain the following equation for the increase of pressure with time at the terminal point $x = 0$:

$$e^{\alpha p_\text{k}} - e^{\alpha p} = \frac{64\lambda l Q^2 \alpha}{\pi^4 g^2 d^5 \beta} \sum_{m=0}^{\infty} \frac{\exp\left[-(2m+1)^2 \frac{\pi^2}{4}\frac{kt}{l^2}\right]}{(2m+1)^2}, \tag{1}$$

where $k = \pi d^3 g / 2\lambda\alpha Q_\text{av}$; p_k is the pressure at the supply point of the pipeline; p_0 is the pressure at the point of consumption; Q is the gravimetric flow rate; λ is the friction coefficient; d is the pipeline diameter; l is the pipeline length; the coefficients α and β are obtained from the relations

$$\alpha = \frac{\log(p_\text{k}/p_0)}{p_\text{k} - p_0}, \qquad \beta = \frac{(p_\text{k} + p_0)\log(p_\text{k}/p_0)}{2gR\overline{T}\,\bar{z}\,(e^{\alpha p_\text{k}} - e^{\alpha p_0})},$$

where $\overline{T}$ is the mean temperature and $\bar{z}$ is the deviation factor from Boyle-Mariotte' s law.

As was noted, in most cases encountered in practice, for very long pipelines one may, with good approximation, restrict oneself to the first term of the series. Then, according to equation (1), the pressure change with time in semilogarithmic coordinates will take place along a straight line with slope

$$i = \frac{\pi^2}{4}\frac{k}{l^2},$$

cutting off on the axis $(e^{\alpha p} - e^{\alpha p_\text{k}})$ the segment

$$A = \log \frac{64\lambda l Q^2 \alpha}{\pi^4 g^2 d^5 \beta}. \tag{2}$$

Thus, on the basis of measurements of the pressure change at the terminal point after the valve has been closed, it is possible, with the aid of equation (2), to determine the actual value of the hydrodynamic coefficient of friction.

Application of this method requires stopping the gas pipeline for a certain time, which causes a disturbance of the operating regime of the pipeline. It is therefore necessary to consider the problem of determining the hydrodynamic coefficient of friction of the pipeline without stopping the latter.

Let us consider the case of operation of a pipeline with constant flow rate at the terminal point. Using the same notation as in (1), and introducing the dimensionless quantities

$$\bar{x} = \frac{x}{l}; \qquad F_0 = \frac{kt}{l^2}; \qquad \bar{H} = \frac{H - H_0}{H_\mathrm{k} - H_0} = 1 - \frac{g^2 d^5 \pi^2}{8\lambda l Q_0^2} \left(H_\mathrm{k} - H\right), \tag{3}$$

the boundary and initial conditions can be represented in the form

$$F_0 > 0 \begin{cases} \bar{x} = 0, & (\partial \bar{H} / \partial x)_{x=0} = 1; \\ \bar{x} = 1, & \bar{H} = 1; \end{cases} \qquad F_0 = 0, \quad \bar{H} = 1. \tag{4}$$

In this case, for the change in pressure we obtain the equation

$$e^{\alpha p_k} - e^{\alpha p} = \frac{9\lambda l Q_0^2 \alpha}{h^2 d^5 \pi^2 \beta} \left\{ 1 - \frac{x}{l} - \frac{8}{\pi^2} \sum_{m=0}^{\infty} \frac{\cos(2m+1)\frac{\pi}{2}\frac{x}{l}}{(2m+1)^2} \exp\left[-(2m+1)^2 \frac{\pi^2}{4} F_0 \right] \right\}, \tag{5}$$

and for the mass flow rate

$$Q^2(x, F_0) = -\frac{g^2 d^5 \pi^2}{8\lambda l_j} \left(\frac{\partial H}{\partial x} \right) =$$

$$= Q_0^2 \left\{ 1 - \frac{4}{\pi} \sum_{m=0}^{\infty} \frac{\sin(2m+1)\frac{\pi}{2}\frac{x}{l}}{(2m+1)} \exp\left[-(2m+1)^2 \frac{\pi^2}{4} F_0 \right] \right\}, \tag{6}$$

where Q_0 is the value of the mass flow rate at the terminal point.

For $F_0 > 2$, the changes in mass flow rate obtained from equation (6) are practically negligible and, consequently, the flow may with good approximation be considered steady for times determined by the relations $t > 4\lambda l^2 Q_m \alpha/\pi^2 d^3 g$ in the turbulent regime and $t > 64\mu\alpha l^2/\pi d^2$ in the laminar regime.

On the basis of relation (3) and the definition of the function H, equation (5) for the terminal point $x = 0$ takes the form

$$e^{\alpha p} - e^{\alpha p_0} = \frac{64\lambda l Q_0^2 \alpha}{\pi^4 \beta d^5 g^2} \sum_{m=0}^{\infty} \frac{\exp\left[-(2m+1)^2 \frac{\pi^2}{4} F_0\right]}{(2m+1)^2}. \tag{7}$$

It is not difficult to see that equation (1) is identical to equation (7), with the only difference that instead of the pressure value at the terminal point p_0, corresponding to the steady-state regime, it contains the pressure value at the supply point, corresponding to the initial moment of time. Thus, in the case of operation of a pipeline at constant flow rate at the terminal point and constant pressure at the supply point, the decrease in pressure with time will occur according to the same law as in the case of an increase in pressure after closing a valve, if at the initial moment of time the flow was steady. Consequently, in semilogarithmic coordinates the pressure decrease will be represented by a straight line with a slope to the abscissa axis determined by equation (2).

It follows from the above that obtaining the pressure-settling curve without shutting down the gas pipeline, from the standpoint of determining the hydrodynamic coefficient of friction, is equivalent to obtaining this curve in the case of shutting down the gas pipeline by closing a valve.

In practice, pipeline operation occurs at a variable flow rate, provided for by the prospective consumption schedule. In this connection, in order to determine the pressure change, it is necessary to apply the superposition principle (which is possible owing to the linearity of the flow equations), expressed through Duhamel' s integral in the form

$$H_k - H = \int_0^t Q^2(t-\lambda) f'(\lambda)\, d\lambda, \tag{8}$$

or, after integration by parts, in the form

$$H_k - H = Q_0^2 f(t) + \int_0^t \frac{dQ^2(t-\lambda)}{d\lambda} f(\lambda)\, d\lambda, \tag{9}$$

where the first term represents the pressure drop corresponding to a constant flow rate, and the second is the additional pressure drop due to the change in flow rate over time.

In equation (9), it is assumed, as also in work ([1]), that $Q = Q_{\text{av}}$, where Q_{av} is the mean gravimetric flow rate.

The function $f(t)$ is determined by equation (5), and the difference $(H_\mathrm{k} - H)$ by the equation

$$H_\mathrm{k} - H = \frac{\beta}{\alpha}\left(e^{\alpha p_\mathrm{k}} - e^{\alpha p}\right) \tag{1}$$

where p is the cumulative pressure. On the other hand, in most real cases the possibility of ensuring strict observance of the boundary and initial conditions used to determine the function $f(t)$, and correspondingly the pressure drop at constant flow rate, is governed by very many factors inherent in the flow in gas mains, which cannot be influenced. For this reason, mathematical processing of measurements along the curve of pressure variation in time may introduce errors that usually exceed the permissible limit. Elimination of errors associated with nonobservance of the boundary and initial conditions would be possible if, simultaneously with the measurement of flow rate, it were possible also to measure the pressure drop corresponding to this flow rate. In present-day practice, pressure cannot be recorded in this way, since, as a result of variation in flow rate, a drop in cumulative (or total) pressure is obtained, and not the pressure corresponding to the measured flow rate.

As is known, application of the principle of superposition makes it possible to eliminate an increase or decrease in pressure in the case of pipeline operation with a variable, increasing or decreasing flow rate, without imposing conditions on the nonuniformity of the distribution of pipeline roughness, and correspondingly on the variation of the hydrodynamic coefficient of friction along the pipeline under the quadratic law of flow friction.

If it is taken into account that the weight flow rate does not become equal to zero over a certain time interval $0 < t < t_1$, then, by introducing the notation

$$\varphi_0(t) = \frac{\beta}{\alpha}\frac{\left(e^{\alpha p_\mathrm{k}} - e^{\alpha p}\right)}{Q_0^2(t)}, \qquad \varphi_1(t-\lambda) = -\frac{1}{Q_0^2}\frac{dQ^2(t-\lambda)}{d\lambda},$$

equation (9) can be reduced to the form

$$f(t) = \varphi_0(t) + \int_0^t \varphi_1(t-\lambda) f(\lambda)\, d\lambda. \tag{10}$$

Thus, the pressure transfer function $f(t)$, corresponding to a constant flow rate during operation of the pipeline with a time-variable flow rate at the point of consumption and necessary for determining the hydrodynamic coefficient of friction, is obtained from the solution of the Volterra integral equation of the second kind (10).

Since the changes in pressure p and weight flow rate are determined by means of recording instruments, we shall therefore have at our disposal the curves of the function $\varphi_1(t-\lambda)$, representing the kernel of the equation, and $\varphi_0(t)$, the cumulative pressure.

If the recorded curves can be approximated by known functions, then the solution of equation (10) can be found by the method of successive approximations.

In addition, by applying the operational method, one can transform the integral equation (10) into an algebraic equation and, with the aid of a table [2], in certain cases find the original of the transformed function. Thus, for example, if the graphs of variation of the functions $\varphi_0(t)$ and $\varphi_1(t-\lambda)$ can be approximated by linear functions of time in the form

$$\varphi_0(t) = A + Bt, \qquad \varphi_1(t-\lambda) = C + D(t-\lambda), \tag{11}$$

then for the function $f(t)$ we have

$$f(t) = k_1 e^{m_1 t} + k_2 e^{m_2 t}, \tag{12}$$

where the constants k_1, k_2, m_1, and m_2 are expressed by the relations

$$k_1 = \frac{B + aA}{a - b}; \qquad k_2 = \frac{B + bA}{a - b};$$

$$a = -\frac{C}{2} + \sqrt{\frac{C^2}{4} - D}; \qquad b = -\frac{C}{2} - \sqrt{\frac{C^2}{4} - D}. \tag{13}$$

It is not difficult to see that, assuming the kernel of the equation $\varphi_1(t-\lambda)$ to vary linearly, we obtain for the gravimetric flow rate a variation according to a parabolic law.

In the case $Q_0 = 0$, the function $f(t)$ is obtained from the integral equation

$$\varphi_0(t) = \int_0^t \varphi_1(t-\lambda) f(\lambda)\, d\lambda,$$

where

$$\varphi_0(t) = \frac{\beta}{\alpha}\left(e^{\alpha p_k} - e^{\alpha p}\right), \qquad \varphi_1(t-\lambda) = \frac{dQ^2(t-\lambda)}{d\lambda}.$$

For $Q_0 = 0$,

$$f(t) = \frac{B}{C} e^{-Dt/C}. \tag{14}$$

It should be noted that, under the influence of the initial flow rate Q_0, during the initial time interval the pressure variation occurs along a curve which, in semilogarithmic coordinates, very quickly becomes a straight line.

In the case when the functions $\varphi_0(t)$ and $\varphi_1(t-\lambda)$ cannot be approximated satisfactorily, the solution of the problem may be obtained by applying the numerical integration method indicated in [(3)].

According to the results obtained in [(3)], one can determine a mesh of nodes in an arithmetic progression in some interval $[t_0, t_n]$, corresponding to a given positive number ε, as well as an algorithm for computing the number $f_i^{(p)}$, where $p = 0, 1, \ldots, r$, so as to satisfy the condition

$$\left| f(t_i) - f_i^{(p)} \right| < 2\varepsilon$$

at all mesh nodes.

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Note: Figure translations are in progress. See original paper for figures.