A SEMI-MOMENTLESS THEORY OF CYLINDRICAL PLASTIC SHELLS

1964

SovietRxiv

Abstract

Full Text

THEORY OF ELASTICITY

M. Sh. Mikeladze

A SEMI-MOMENTLESS THEORY OF CYLINDRICAL PLASTIC SHELLS

(Presented by Academician A. Yu. Ishlinskii, 27 VII 1963)

Let an orthotropic cylindrical shell be subjected to the action of prescribed forces. We shall choose the unknown thickness of the shell h in such a way that the stresses at every point of the structure satisfy the yield condition. If the influence of the longitudinal bending and twisting moments (M_1 and M_{12}) on the stressed state of the shell is neglected, then the yield condition, expressed in terms of forces and moment, takes the form (see ([1]), Ch. I, § 2):

$$\frac{GH + HF + FG}{(H+F)h^2} T_1^2 + \frac{2N}{h^2} T_{12}^2 + \frac{16(H+F)}{h^4} M_2^2 = 1, \tag{1}$$

where T_1 denotes the longitudinal normal force acting along the generator of the cylinder; T_{12} is the shearing force, and M_2 is the transverse bending moment. The coefficients G, H, F, and N are known constants ([2]) characterizing the plastic properties of the shell material in different directions. With respect to the principal axes of anisotropy, it is assumed that they coincide respectively with the directions of the generator, the tangent to the arc of the contour line, and the internal normal.

Formula (1) does not contain the normal transverse force T_2. The latter is proportional to the force T_1 (see ([1]), Ch. I):

$$T_2 = \frac{H}{H+F} T_1. \tag{2}$$

Owing to equality (2), and also to the absence of the longitudinal bending and twisting moments (M_1 and M_{12}), the problem of determining the internal forces in the shell becomes statically determinate. Indeed, in this case the differential equations of equilibrium ([3]) may be written in the form:

$$\begin{aligned}\frac{H+F}{H}\frac{\partial T_2}{\partial z}+\frac{\partial T_{12}}{\partial s}+P_z=0,\\ \frac{\partial^2 M_2}{\partial s^2}+\frac{T_2}{R}+P_n=0,\\ \frac{\partial T_2}{\partial s}+\frac{\partial T_{12}}{\partial z}-\frac{1}{R}\frac{\partial M_2}{\partial s}+P_s=0,\end{aligned} \tag{3}$$

where z and s are the coordinates of a point of the middle surface of the shell, measured along the generator and the directrix, respectively; $R = R(s)$ is the radius of curvature, while P_z, P_s, P_n are the prescribed components of the surface load.

Having determined the forces T_1, T_2, T_{12} and the moment M_2 from (3) and (2), we can easily calculate the required shell thickness h by means of relation (1).

The problem is simplified for a circular cylindrical shell ($R = a$) subjected to a load symmetric with respect to the lock: $P_z(z,s) = P_z(z,-s)$, $P_n(z,s) = P_n(z,-s)$, $P_s(z,s) = P_s(z,-s)$. As a result of eliminating the variables T_{12} and M_2 and passing to the new coordinate $\theta = s/a$ ($\alpha \geq \theta \geq -\alpha$), the system of equations (3) is reduced to the following

equation of hyperbolic type for the force T_2:

$$\frac{\partial^2 T_2}{\partial \theta^2} = \frac{a^2(H+F)}{H}\frac{\partial^2 T_2}{\partial z^2} - T_2 - P, \tag{4}$$

where

$$P = aP_n + a\frac{\partial P_s}{\partial \theta} - a^2\frac{\partial P_z}{\partial z}.$$

If the ends of the shell are free, and the longitudinal edges are hinged, then the boundary conditions of the problem will have the form:

$$T_2(\theta,0) = T_2(\theta,l) = 0, \qquad \left.\frac{\partial T_2}{\partial \theta}\right|_{\theta=0} = -aP_s, \qquad T_2(0,z) = 0,$$

where l is the length of the shell.

The first two conditions are a consequence of the absence of longitudinal forces T_1 at the ends of the shell; the third is due to the symmetry of the shell and of the load acting on it with respect to the plane $\theta = 0$, in which $T_{12} = \partial T_{12}/\partial z = \partial M_2/\partial \theta = 0$. Finally, the fourth condition follows from the assumption that, in the limiting plastic equilibrium of the shell, the rate of curvature $\varkappa_2(0,z) \to \infty$. The validity of the last condition is readily verified with the aid of relations (2.8) of Chapter I of work ([1]), taking into account that the stress state of the shell under consideration belongs to case (c) of the "simplest complex stress state," while $\mu = \lambda_0/\lambda_1$ and the quadratic form Q_t, appearing in (2.8), tend to zero as $\varkappa_2(0, z) \to \infty$.

If we now assume that the expression P on the right-hand side of (4) is a function only of z $[P = P(z)]$, then the solution of this equation is naturally sought in the form of a sum:

$$T_2 = \psi(z) + \varphi(\theta, z),$$

where $\psi(z)$ is the solution of the ordinary differential equation

$$\frac{a^2(H+F)}{H}\frac{d^2\psi(z)}{dz^2} - \psi(z) - P(z) = 0,$$

satisfying the boundary conditions

$$\psi(0) = \psi(l) = 0,$$

and $\varphi(\theta, z)$ is the solution of the equation

$$\frac{\partial^2\varphi(\theta,z)}{\partial\theta^2} = \frac{a^2(H+F)}{H}\frac{\partial^2\varphi(\theta,z)}{\partial z^2} - \varphi(\theta, z)$$

under the conditions

$$\varphi(\theta, 0) = 0, \qquad \varphi(\theta, l) = 0, \qquad \left.\frac{\partial\varphi(\theta,z)}{\partial\theta}\right|_{\theta=0} = -aP_s, \qquad \varphi(0, z) = -\psi(z).$$

The last equation is solved by the method of separation of variables (see, for example, ([4])). To determine the two other unknowns M_2 and T_{12}, we use the first two equations (3) with the corresponding boundary conditions.

The investigation of several other cases encountered in engineering also leads to the boundary-value problem considered. Namely, the case of a shell whose longitudinal edges are free and whose ends rest on diaphragms rigid in their own plane, or the case of a cantilever shell fixed along one of the longitudinal edges with the other three free. In this case the coordinate angle θ should be measured from the free longitudinal edge of the shell.

Georgian Polytechnic Institute
named after V. I. Lenin

Received
21 VII 1963

CITED LITERATURE

1. M. Sh. Mikeladze, *Statics of Anisotropic Plastic Shells*, Publishing House of the Academy of Sciences of the Georgian SSR, 1963.
2. R. Hill, *The Mathematical Theory of Plasticity*, Oxford, 1950.
3. V. Z. Vlasov, *General Theory of Shells*, 1949.
4. A. N. Tikhonov, A. A. Samarskii, *Equations of Mathematical Physics*, 1951.

Note: Figure translations are in progress. See original paper for figures.