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Abstract

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THE EFFECT OF COLLISIONS ON THE DRIFT INSTABILITY OF A PLASMA FOR FINITE ION LARMOR RADIUS

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The progress achieved recently in the study of oscillations of an inhomogeneous plasma (see, for example, the review ([1])) is to a large extent connected with the introduction into this field of a convenient method for solving the kinetic equation for plasma charges—the method of integration along trajectories. However, the application of this method has so far, as a rule, been limited to those cases in which collisions between particles are unimportant. This is explained to a considerable degree by the complexity of the collision term in the kinetic equation.

In the present paper we wish to draw attention to the fact that the use of a model collision term in the form of Bhatnagar—Gross—Krook ([2]) makes it possible to apply the method of integration along trajectories also to such problems on oscillations of an inhomogeneous plasma in which collisions between particles play an important role.

The linearized kinetic equation for the perturbed distribution function f_α of a certain species of charged particles (electrons, ions) has, as is known, the form:

$$\frac{\partial f_\alpha}{\partial t} + (\mathbf{v}\nabla) f_\alpha + \left\{ [\mathbf{v}\,\vec{\Omega}_\alpha] + \frac{e_\alpha}{m_\alpha}\left(\mathbf{E} + \frac{[\mathbf{v}\times\mathbf{B}]}{c}\right)\right\} \frac{\partial f_\alpha^0}{\partial \mathbf{v}} = St_\alpha. \tag{1}$$

Instead of the exact expression for the collision term we shall take the following approximate one (cf. with ([2])):

$$St_\alpha = -\sum_\beta \nu_{\alpha\beta} \left\{ f_\alpha - \left[(\eta_\alpha) + \frac{m_\alpha}{T_\alpha}(\mathbf{v}\mathbf{q}^\alpha) \right] f_\alpha^0 \right\}. \tag{2}$$

Here the quantities η and $\mathbf{q}$ have the meaning of a dimensionless perturbation of the density and of the mean perturbed particle velocity,

$$\eta_\alpha = \frac{1}{n^0}\int f_\alpha \, d\mathbf{v}, \qquad \mathbf{q}^\alpha = \frac{1}{n^0}\int \mathbf{v}^\alpha f_\alpha \, d\mathbf{v}; \tag{3}$$

f_α^0 is the equilibrium distribution function; n^0 is the density; T is the temperature; $\nu_{\alpha\beta}$ is the collision frequency of particles of species α with particles of species β.

The collision term of the form (2) satisfies the conservation laws for the number of particles and for momentum. Expression (2) corresponds to the so-called "isothermal model," i.e., approximately, one that does not take temperature perturbations into account. As a result, the hydrodynamic equations following from (1) do not describe such effects as thermal conductivity; nor are there included here collision effects associated with the detailed mechanism of collisions, for example such as the effect of anisotropy of the friction force, which follow from the kinetic equation with the exact form of the collision term.

On the other hand, the insensitivity of expression (2) to the details of the collision mechanism makes it possible to describe, in a unified (although approximate) way, both collisions between charged particles and their collisions with neutrals.

Applying the usual procedure of the method of integration along trajectories ([1]), one can find an expression for the perturbed distribution function, and at the same time expressions for the perturbed currents and density induced in an inhomogeneous plasma. Substituting the latter into Maxwell' s equations, we obtain a certain system of equations for the components of the electric field of the wave.

Thus, the problem of oscillations of an inhomogeneous plasma can be solved by methods similar to the collisionless case ([1]), if one confines oneself to a qualitative treatment of collisions on the basis of a model collision term of type (2).

We present a number of results for the problem of the drift instability of an inhomogeneous plasma obtained in this way. In the quasiclassical approximation, from (1), (2), and Maxwell' s equations for drift perturbations of a low-pressure plasma ($\beta \ll m_e/m_i$), there follows the dispersion equation (relating the frequency ω and the wave vector $\mathbf{k}$ of the oscillations)

$$1-\frac{\omega^*}{\omega}+\left(1+\frac{\omega^*}{\omega}\right)\left(1-2i\,\frac{\nu_{ei}\omega}{k_\parallel^2 v_{T_e}^2}\right)\left[z\left(1+\frac{5}{4}iz\,\frac{\nu_i}{\omega}\right)-\frac{Tk_\parallel^2}{M_i\omega^2}\left(1-i\,\frac{\nu_i}{\omega}z\right)\right]=0, \tag{4}$$

where $\omega^* = k_x v_e^0$; $z = \dfrac{Tk_\perp^2}{M_i\omega_{H_i}^2}$; $v_e^0 = \dfrac{T}{m\omega_{H_e}}\dfrac{d}{dy}\ln n^0$ is the Larmor drift velocity of the electrons; $T_e = T_i = T$; $v_{T_e} = \sqrt{2T/m}$. The perturbation was chosen in the form $\sim e^{-i\omega t + i\mathbf{kr}}$.

Here we regard the Larmor radius of the particles as finite (although small) in comparison with the wavelength ($z \ll 1$), and the mean free path of the particles as smaller than the longitudinal wavelength, $\nu/k_\parallel v_T > 1$. If ion–ion collisions are neglected, but electron collisions with ions are retained, then from this follows the well-known result for the collisional drift instability ([3])

$$\omega = \frac{1}{2}\left[-\omega^* - i\omega_s + \sqrt{(\omega^* + i\omega_s)^2 + 4i\omega^*\omega_s}\right], \tag{5}$$

where

$$\omega_s = \left(\frac{k_\parallel}{k_\perp}\right)^2 \frac{M_i}{m_e}\frac{\omega_{H_i}^2}{\nu_{ei}}.$$

From (4) it can be obtained that, as the wavelength of the perturbations decreases (as the parameter z increases), the role of ion–ion collisions increases. This leads to a decrease in the increment of the drift instability and to stabilization of the latter.

This result can also be obtained from hydrodynamics; however, the proposed method makes it possible to proceed into the region of wavelengths comparable with the Larmor radius of the particles, i.e., a region inaccessible to magnetohydrodynamic treatment. Thus, for example, at $z \gg 1$ (the wavelength is small in comparison with the ion Larmor radius) the collisional drift instability is described by the equation

$$\omega = -i\nu_i + \frac{\omega^*}{\sqrt{2\pi z}}\frac{1}{1 - i \cdot 2\,\dfrac{\nu_{ei}\omega^*}{k_\parallel^2 v_{T_e}^2}}. \tag{6}$$

As in the case $z \ll 1$, the friction of electrons against ions (the term with ν_{ei}), as is seen from this expression, leads to excitation of oscillations, while the ion viscosity (the term with ν_i) contributes to their damping.

Collisional drift instability, as follows from nonlinear estimates of the work ([3]), can lead to anomalously large plasma diffusion. It is therefore of interest to determine under what conditions this instability can arise. The stability boundaries of interest to us can be found with the aid of equation (4) in the region $z \ll 1$ and (6) for $z \gg 1$. To this end we assume the oscillation frequency to be real and separate the real and imaginary terms. Eliminating the oscillation frequency from the two equations thus obtained, we find a certain relation between the plasma parameters and the wavelengths, which we shall call the equation for the stability boundary. Solving this equation with respect to the quantity $\sigma = k_\parallel^*/(\partial \ln n^0/\partial y)$, which has the meaning of the ratio of the transverse size of the experimental apparatus to its length ($k_\parallel^*$ corresponds to the shortest wave unstable along the length of the device), we obtain:

Fig. 1

Fig. 1

Figure 1: Fig. 1

$$\sigma = z\frac{1-\frac{9}{2}\sqrt{m_e/m_i}\,\alpha_i^2 z^2}{1+\frac{9}{2}\sqrt{m_e/m_i}\,\alpha_i^2 z^2}, \qquad z \ll 1 \quad (\sigma > 0), \tag{7}$$

$$\sigma = \frac{1}{2}\left(\frac{m_e}{m_i}\right)^{1/4}\left(\frac{z}{2\pi}\right)^{1/4}\left(1-\alpha_i^2\left(\frac{m_e}{m_i}\right)^{1/2}\sqrt{2\pi z}\right)^{1/2}, \qquad z \gg 1, \tag{8}$$

$$\alpha_i = \nu_i/k_\parallel v_{Ti}.$$

Stability boundaries in the plane z, $\sigma = k_\parallel^*/(\partial \ln n^0/\partial y)$ are shown in Fig. 1. For comparison, the boundary for collisionless drift instability (arising as a result of resonant interaction of particles with the wave) from work ([4]) is also given there. Thus, for sufficiently frequent collisions $\nu/k_\parallel v_{Te} > 1$, short-wavelength perturbations ($z \gg 1$) are not amplified (in contrast to the collisionless case, where unstable oscillations are possible up to $z \simeq 100$). This is the result of the stabilizing influence of ion viscosity, which is very significant in short-wavelength perturbations.

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Note: Figure translations are in progress. See original paper for figures.