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Abstract

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The Influence of "Drift" Waves on Plasma Diffusion in a Magnetic Field

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Recent theoretical studies of the stability of the equilibrium of an inhomogeneous plasma in a strong magnetic field ([1,2]) show that, at least in magnetic traps with a large Larmor radius ($r_i/r \ll 1$, r_i is the ion Larmor radius, r is the transverse size of the magnetic trap), there exist instabilities with wavelengths of the order of, or smaller than, r_i, which apparently have a "universal" character.* It is important to know to what extent the pulsations arising as a result of this instability worsen the magnetic confinement of the plasma. Here one should distinguish two cases: 1)

$$\frac{p^2}{H^2/8\pi} = \beta < \frac{m}{M}$$

and 2)

$$\frac{m}{M} < \beta \ll 1.$$

In the first case there exists an aperiodic-type instability ([1]) with growth increment

$$\nu \sim k_0 v_i$$

(where $k_0 \equiv \frac{1}{n_0}\frac{dn_0}{dx}$, and v_i is the ion thermal velocity), with $\nu > k_z v_i$. Therefore the anomalous diffusion coefficient D associated with this instability can be estimated from simple dimensional relations:

$$D \sim \frac{\nu}{k_x^2}.$$

Substituting here the value of the increment ν and the minimum value of k_x, which proves to be of order $1/r_i$ ([1]), one easily obtains

Fig. 1

Figure 1: Fig. 1

$$D \sim \frac{r_i}{r}\frac{cT}{eH}.$$

Let us consider in more detail the second limiting case, which is of greatest interest for applications. In this case there exist only instabilities with respect to the excitation of "drift" waves. Following ([2,3]), one may write

$$\omega \simeq \frac{\omega_n}{k_y r_i}\left(1 + i\sqrt{\frac{m}{M}}\frac{\omega_n}{|k_z|v_i}\right). \tag{1}$$

Here $\omega_n = k_y v_n,\ v_n = k_0 r_i v_i,\ n_0 = n_0(x),\ T_0(x) = \text{const}$. Such a dependence is retained for $\omega_n/v_a < |k_z| < \omega_n/v_i$. For large $|k_z|$ the "drift" waves become damped:

$$\omega \simeq \omega_n\left[1 - i\exp\left(-\frac{\omega_n^2}{k_z^2 v_i^2}\right)\right]. \tag{2}$$

(The contribution of "Landau damping" from resonant ions, $\omega/k_z \simeq v_z$, becomes appreciable.) For $k_z < \omega_n/v_a$ the instability is also absent. In what follows it is convenient to represent $\nu\ (\omega/k_z)$ graphically (see Fig. 1).

* These instabilities are absent in sufficiently "short" systems, where $L \lesssim 10r$ (L is the "effective" length of the trap along the field line).

Let us note that in deriving (1) (see (3)) perturbations were considered which oscillate within the region of inhomogeneity and decay as $x \to \infty$. In other words, finite solutions in the quasiclassical approximation were considered. In this case the dependence of the perturbed quantities on x has the form $e^{ik_x x}$, where k_x is determined by the "dispersion" equation:

Fig. 1

$$\frac{2n}{T} - \sum\left[\frac{\omega}{T} - \frac{k_y}{m_j\Omega_j}\frac{d}{dx}\right]A(\theta_j)\frac{n_0(x)}{\sqrt{2\pi T}}\int_{-\infty}^{\infty}\frac{\exp(-m_j v^2/2T)\,dv}{\omega - k_z v} = 0, \tag{3}$$

where $\Omega_j = eH/m_{jc}, \quad A(\theta_j) = e^{-\theta_j^2/2}I_0(\theta_j^2/2), \quad \theta_j = k_\perp r_j, \quad k_\perp^2 = k_y^2 + k_x^2.$

It follows from (1) that the growth increment of the waves is less than ω and $k_z v_i$. Therefore, in order to take into account the averaged effect of "drift" waves on "diffusion," it is natural to apply the quasilinear method ([4]). If the wavelengths of the turbulent pulsations λ_n exceed the Larmor radius of the particles, one may use the drift kinetic equation.

Following ([4]), let us split the distribution function $f(\mathbf{r}, \mathbf{v}, t)$ into a "slowly varying" part $f_0(x, \mathbf{v}, t)$ and a "rapidly oscillating" part $f_\sim(\mathbf{r}, \mathbf{v}, t)$. The equation for $f_\sim$ has the same form as in the linear theory*:

$$f_\sim = \frac{c\frac{E_y}{H}\frac{\partial f_0}{\partial x} + \frac{k_z}{k_y}\frac{e}{m}E_y\frac{\partial f_0}{\partial v_z}}{-i\omega + ik_{zv}z}, \tag{4}$$

where

$$E_y = \sum E_k e^{i(\omega t - \mathbf{kr})} + \text{c.c.},$$

and in the equation for f_0 it is necessary to retain the averaged second-order term:

$$\frac{df_0}{dt} \simeq \sum_k \left\{\frac{c}{H}\frac{\partial}{\partial x} + \frac{e}{m}\frac{k_z}{k_y}\frac{\partial}{\partial v_z}\right\}|E_k|^2\,\mathrm{Im}(\omega - k_{zv}z)^{-1}\left\{\frac{c}{H}\frac{\partial}{\partial x} + \frac{e}{m}\frac{k_z}{k_y}\frac{\partial}{\partial v_z}\right\}f_0. \tag{5}$$

The term

$$\frac{\partial}{\partial x}\left\{c^2\frac{|E_k|^2}{H^2}\,\mathrm{Im}(\omega - k_{zv}z)^{-1}\frac{\partial f_0}{\partial x}\right\} \tag{6}$$

in equation (5) gives anomalous diffusion. It can be shown that, if no assumptions are made concerning the ratio r_i/λ_n, the term describing anomalous diffusion has the form

$$\frac{\partial}{\partial x}\left\{c^2\frac{|E_k|^2}{H^2}J_0^2(\theta)\,\mathrm{Im}(\omega - k_{zv}z)^{-1}\frac{\partial f_0}{\partial x}\right\}, \tag{7}$$

where $\theta = k_\perp v_\perp/\Omega_i$.

As is seen from (7), the rate of anomalous diffusion is determined by the spectrum

$$|E|^2 = |E|^2\left(\frac{\omega}{k_z}\right),$$

* In the "drift" waves considered by us, the electric fields may be regarded as potential, i.e. $\mathrm{rot}\,\mathbf{E} = 0$. This relation is used below.

which is established because of the competition of two factors: 1) the "pumping" of "drift" waves owing to the instability; 2) nonlinear effects of the interaction between different "harmonics." Since there is no regular method for finding

Fig. 2

Figure 2: Fig. 2

the turbulence spectrum in problems of this kind, we shall estimate the order of magnitude of the pulsations on the basis of the following visual physical considerations.

In Fig. 2, in **k**-space, the instability region is shown (region I is hatched), as obtained from the linear theory. If the turbulent "background" is represented as a superposition of different "harmonics"—scales **k**, then the process by which the spectrum is established in the "language" of Fig. 2 proceeds as follows. As a result of the instability, the amplitudes of the pulsations with **k** lying inside region I begin to grow. As the amplitude increases, the nonlinear interaction between the "scales" within region I is switched on, leading, in particular, to the "birth" of fluctuations with **k** lying in region II, where there is strong damping (see (2)). Thus, when the two processes become comparable, a quasi-stationary picture is established.

Fig. 2

In the kinetic equation for the ions, the nonlinear term has the form $\frac{e}{M}E_\perp\frac{\partial f_1}{\partial v}$. For an estimate of the amplitude of the pulsations, let us compare it with the linear term $\partial f_1/\partial t \sim \nu f_1$. Putting $\partial f_1/\partial v \sim f_1/v_i$, we obtain

$$\nu f_1 \sim \frac{e}{M}E_\perp\frac{f_1}{v_i}; \qquad E_\perp \sim \frac{Mv_i}{e}\,\nu. \tag{8}$$

From (1) it is seen that the largest contribution to the diffusion may be expected from waves with $kr_i \sim (M\beta/m)^{1/2}$. Using (1), (7), (8), one can estimate the diffusion coefficient*:

$$D_x \approx \sum c^2\frac{|E_k|^2}{H^2}\,\frac{1}{kr_i}\,\frac{\nu}{\omega^2} \approx \frac{M^2c^2v_i^2}{e^2H^2}\,\frac{1}{kr_i}\,\frac{v_i}{r} \approx \left(\frac{m}{M\beta}\right)^{1/2}\frac{r_i}{r}\,\frac{cT}{eH}. \tag{9}$$

Until now we have assumed that the velocity distribution of the electrons is close to Maxwellian. Therefore all the expressions written above are valid only in the case when collisions, though rare, have time to restore the Maxwellian distribution, which is disturbed by the diffusion of resonant electrons (for which $v \approx \omega/k_z$). In the opposite limiting case (when the diffusion of resonant particles, determined by the term describing collisions of electrons with waves $St_b(f)$, substantially affects the increment) a curious situation arises, in which it becomes unnecessary to estimate the amplitudes of the pulsations. For this case

$$St_b(f) \gg St_c(f), \tag{10}$$

where $St_c = (f)$ is the term describing Coulomb collisions. The diffusion coefficient then contains the already strongly reduced value of the increment $\bar{\nu}$:

$$\bar{D}_x \approx \sum c^2 \frac{|E_k|^2}{H^2} \frac{\bar{\nu}}{\omega^2} \frac{1}{kr_i}. \tag{11}$$

To determine the increment, taking (10) into account, let us represent the distribution function in the resonant region in the form $f = f^{(0)} + f^{(1)}$, where $f^{(0)}$ satisf–

* Naturally, for $\nu \sim \omega$ the expression for the diffusion coefficient (7) is not exact; therefore it is used only to estimate the order of magnitude.

satisfy the equation:

$$St_b(f^{(0)}) = 0. \tag{12}$$

The solution of this equation gives a steady "plateau" in the space (x, v_z), so that

$$\nu^{(0)}\{f^{(0)}\} = \frac{\omega}{n_0} \left\{ -v_e^2 \frac{\partial f^{(0)}}{\partial v_z} + \frac{cT}{eH} \frac{k_y}{k_z} \frac{\partial f^{(0)}}{\partial x} \right\}_{v_z \simeq \omega/k_z}. \tag{13}$$

Finding $f^{(1)}$ from the equation

$$\left[St_b(f^{(1)}) + St_c(f^{(0)}) \right]_{v_z \simeq \omega/k_z} = 0 \tag{14}$$

and taking into account that the quantity $f^{(0)}$ itself (but not its derivatives with respect to v_z!) differs little from a Maxwellian, we obtain

$$\bar{\nu} \simeq \frac{\nu_e v_e^2}{\bar{D}_{v_z}}, \tag{15}$$

where ν_e is the frequency of electron collisions, $D_{v_z} \simeq e^2 |E_z|^2 / m^2 \omega$; v_e^2 is the electron thermal velocity. From (15) and (13) it is easy to obtain the anomalous diffusion coefficient

$$\bar{D}_x \simeq v_e^2 r^2 \frac{v_a}{v_e}. \tag{16}$$

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Note: Figure translations are in progress. See original paper for figures.