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Abstract

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ON THE INSTABILITY OF A GAS DISCHARGE IN A MAGNETIC FIELD WITHOUT LONGITUDINAL CURRENT

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As is known ([1,2]), a glow discharge in the presence of a constant current along an external magnetic field is unstable with respect to twisting. However, in a number of experiments ([3–6]), enhanced diffusion of plasma was observed in a currentless discharge with a magnetic field.

We shall restrict ourselves to considering experiments in which the diffusion of the plasma was currentless. In the experiments of Jele ([5]), the gas was ionized by a high-frequency field. In this work a critical magnetic field H_c was found at which intense noise appeared and the diffusion across the magnetic field increased. The critical field decreased inversely proportionally to the tube diameter and increased slowly with the pressure of the neutral gas. Enhanced diffusion at sufficiently large values of H was also observed by Golant ([6]). In these experiments the electron temperature was lower than in ([5]), since the measurements were made some time after the high-frequency field had been switched off.

We shall show that the observed anomalies in the behavior of the plasma could have been caused by the appearance of a drift-dissipative instability, whose existence was pointed out in ([7]). We shall consider the motion of the electrons by means of the kinetic equation in the drift approximation

$$\frac{\partial f}{\partial t} + v_z \frac{\partial f}{\partial z} + \frac{c}{H^2}(([\mathbf{H}\nabla\varphi]\nabla)f + \frac{e}{m}\frac{\partial \varphi}{\partial z}\frac{\partial f}{\partial v_z} = \mathrm{St}(f), \qquad (1)$$

where collisions with atoms of the neutral gas are taken into account by means of the collision term

$$\mathrm{St}(f) = -\tau_e^{-1}(f - f_0) + \tau_e^{-1}\frac{f_0}{n_0}\int (f - f_0)\, dv_z.$$

Here $f(x, v_z)$ is the electron distribution function,

$$f_0 = (m/2\pi T)^{1/2} n_0(x) e^{-mv_z^2/2T},$$

φ is the electric-field potential, τ_e is the time between collisions of electrons with neutrals, and $\Omega_e = eH/mc$ is the cyclotron frequency. The OZ axis is directed along the magnetic field, OX along the initial density gradient. We consider oscillations with phase velocity greater than the ion thermal velocity; therefore the hydrodynamic approximation is applicable to the ions, and the ion pressure may be neglected. Such a restriction is natural, since usually in a weakly ionized plasma $T_e > T_i$. Thus, we write the equation of motion for the ions in the form

$$\frac{d\mathbf{v}_i}{dt} = -\frac{e}{M}\nabla\varphi + \frac{e}{Mc}[\mathbf{v}_i\mathbf{H}] - \frac{\mathbf{v}_i}{\tau_i}. \tag{2}$$

Allowance for ion inertia proves to be very important.

In the experiments ([5,6]) the discharge was studied in dielectric—glass—tubes, and therefore the current to the wall must vanish. As is known (see, for example, ([2,7])), in this case an electric field arises

$$\frac{d\varphi_0}{dx} = \frac{T_e}{e}\left[1 + (\Omega\tau)_i(\Omega\tau)_e\right]^{-1} n_0^{-1}\frac{dn_0}{dx}.$$

Investigating the stability, we shall restrict ourselves

quasiclassical approximation, choosing perturbations of the stationary quantities in the form $e^{-i\omega t + i\mathbf{kx}}$, and we shall assume that $k_y > k_x > \chi = n_0^{-1} dn_0/dx$. In this approximation the details of the dependence of n_0 and φ_0 on x are inessential, and the transition to plane geometry is also justified.

From equation (1), linearized with respect to small perturbations f_1 and φ_1, for the perturbation of the electron density $n_{1e} = \int f_1\, dv_z$ it is not difficult to find

$$\frac{n_{1e}}{n_0\varphi_1} = \left[\frac{e}{T}(1+Y) + i\frac{c}{H}\frac{k_y\chi}{-i\omega + \tau_i^{-1}}\,Y\right]\left[1 + (1 - i\omega\tau_e^{-1})Y\right]^{-1}. \tag{3}$$

Here

$$Y = i\sqrt{\pi}\, z e^{-z^2}\left(1 + \frac{2i}{\sqrt{\pi}}\int_0^z e^{t^2}\,dt\right), \qquad z = \frac{\tau_e^{-1} - i\omega}{k_z}\sqrt{\frac{m}{2T}}.$$

In the hydrodynamic approximation, i.e., for $\sqrt{T/m}\,\tau_e > k_z^{-1}$, $\omega < \tau_e^{-1}$, when $z \gg 1$, $Y \to (1 + 1/2z^2)$, for n_{1e} we obtain:

$$\frac{n_{1e}}{n_0\varphi_1} = \left[b_e k_z^2 + i\frac{b_e}{(\Omega\tau)_e}k_y\chi\right]\left[-i\omega + D_e k_z^2\right]^{-1}. \tag{4}$$

Here $D_e = T\tau_e/m$, $b_e = e\tau_e/m$ are the diffusion and mobility coefficients of the electrons. Using (2) and the continuity equation, it is easy to find the perturbation of the ion density:

$$\frac{n_{1i}}{n_0\varphi_1} = \frac{c}{H}\left[-\frac{k_y\chi}{\omega}\Omega_i^2 + \frac{k_y^2}{i\omega}\Omega_i(-i\omega+\tau_i^{-1})\right]\left[\Omega_i^2 + (-i\omega+\tau_i^{-1})^2\right]^{-1}. \quad (5)$$

We shall investigate the stability in the following standard way. Equating, from the condition of quasineutrality, the perturbations of the electron and ion densities, we obtain the dispersion equation for the frequency. In finding the boundary of the instability region we set $\operatorname{Im}\omega = 0$. Equating separately to zero the real and imaginary parts of the dispersion equation, we obtain conditions for the frequency and the plasma parameters under which instability sets in.

Long-wavelength ion-acoustic oscillations with $\sqrt{T/m}\,\tau_e > k_z^{-1}$ and frequency $\tau_e^{-1} > \omega > \Omega_i$ are described by the system of equations of the hydrodynamic approximation:

$$\frac{n_{1e}}{n_0\varphi_1} = \left[b_e k_z^2 + i\frac{b_e}{(\Omega\tau)_e}k_y\chi\right]\left[-i\omega + D_e k_z^2\right]^{-1},$$

$$\frac{n_{1i}}{n_0\varphi_1} = \frac{e}{M}k^2\left[\omega^2 + i\omega\tau_i^{-1}\right]^{-1}. \quad (6)$$

Its investigation shows that, for $\xi = \sqrt{T/M}\,\chi\tau_i > (\Omega\tau)_i$, oscillations with $\omega \simeq -k_y\sqrt{T/M}$, $k_y \gtrsim \chi(\Omega\tau)_i^{-1}$, $k_z^2 \lesssim k_y^2 b_i/b_e$ begin to grow.

For $(\Omega\tau)_i > 1$, in addition to ion-acoustic oscillations, drift oscillations may grow, whose phase velocity is of the order of the drift velocity $v^+ = \dfrac{T}{M}\dfrac{\chi}{\Omega_i}$, and whose frequency $\omega < \Omega_i$. They are described by the system

$$\frac{n_{1e}}{n_0\varphi_1} = \left[b_e k_z^2 + i\frac{b_e}{(\Omega\tau)_e}k_y\chi\right]\left[-i\omega + D_e k_z^2\right]^{-1},$$

$$\frac{n_{1i}}{n_0\varphi_1} = \frac{c}{H}\left[-\frac{k_y\chi}{\omega} + \frac{k_y^2}{i\omega}\frac{-i\omega+\tau_i^{-1}}{\Omega_i}\right]^{-1}. \quad (7)$$

From (7) we find that, as ξ increases, i.e., as the pressure of the neutral gas decreases, the first oscillations to be amplified are those with $k_y \lesssim \varkappa(\Omega\tau)_i$, $k_z^2 \simeq k_y^2(b_i/b_e)(\Omega\tau)_i^{-2}$; here $\xi_c \simeq 1$, $\omega \simeq k_y v^+$. The upper boundary in ξ is determined by the condition $\xi < (\Omega\tau)_i^2(b_e/b_i)^{1/2}$.

In considering short-wavelength oscillations with $k_z > \tau_e^{-1}(T/M)^{-1/2}$, the hydrodynamic approximation is inapplicable.

If $\tau_e^{-1} > \omega > \Omega_i$, then from (3), (5) we obtain

$$\frac{n_{1e}}{n_0\varphi_1} = \left\{\frac{e}{T}\left[1 + i\frac{\omega}{k_z v_T}(X_0 - 2z_0)\right] + i\frac{c}{H}\frac{k_y \varkappa}{k_z v_T}\left(X_0 + 2i\frac{\omega}{k_z v_T}\right)\right\} \times \left[1 - i\frac{\omega\tau_e^{-1}}{k_z^2 v_T^2}\right]^{-1}, \tag{8}$$

$$\frac{n_{1i}}{n_0\varphi_1} = \frac{e}{M}\frac{k^2}{\omega^2 + i\omega\tau_i^{-1}}.$$

Here $z_0 = \operatorname{Re} z \lesssim 1,\ X_0 = -iY(z_0)z_0^{-1}[1 + Y(z_0)]^{-1}$.

At the boundary of the stability region, when $\operatorname{Im}\omega = 0$, from the dispersion equation we obtain:

$$\omega^2(X_0 - 2z_0) + \omega k_{yv}^+ + X_0 + \frac{k_{zv}T}{\tau_i}\left[1 + \frac{b_i}{b_e}\frac{k_y^2}{k_z^2}\right] = 0,$$

$$(\omega^2 + \tau_i^{-2})\left(1 - \frac{k_{yv}^+\omega}{k_z^2 v_T^2}\right) + \tau_i^{-2}\frac{b_i}{b_e}\frac{k_y^2}{k_z^2} = \frac{T}{M}k^2. \tag{9}$$

From (9) it is clear that instability is possible only for $|\omega| \lesssim k_{yv}^+$, and the oscillations most easily amplified are those with $k_z^2/k_y^2 \gtrsim b_i/b_e$, at the largest possible k_y. The instability under consideration belongs to the class of drift instabilities [2,7], since for its existence it is necessary that the electrons be able to be carried outward by the drift in crossed fields—the constant magnetic field and the electric field of the wave. Therefore the wavelength must be greater than $r_e = v_T\Omega_e^{-1}$, which determines $k_{y\,\max} \simeq r_e^{-1}$. From (9) we obtain the boundary of the instability region in ξ, lying somewhat below that found in the hydrodynamic approximation, namely, for $\xi > (\Omega\tau)_i^{1/2}(b_i/b_e)^{1/4}$.

Up to this point, instabilities of dissipative character have been considered; for their existence there must be a phase shift between the perturbations of density and potential, produced by the irreversible process of electron diffusion. However, for $\omega > \tau_e^{-1}$, analogous amplification can be caused by effects of interaction with the wave of resonant electrons, for which $v_z = \omega/k_z$. For oscillations with $z < 1$, $\omega > \tau_e^{-1}$, Ω_i, we have from (3), (5):

$$\frac{n_{1e}}{n_0\varphi_1} = \frac{e}{T}\left[1 + i\sqrt{\pi}\frac{\omega}{k_{zv}T}\left(1 + \frac{k_{yv}^+}{\omega}\right)\right],$$

$$\frac{n_{1i}}{n_0\varphi_1} = \frac{e}{M}\frac{k^2}{\omega^2}. \tag{10}$$

Fig. 1

Figure 1: Fig. 1

It is easy to see that ion-acoustic oscillations are amplified with $\omega = -k_y\sqrt{T/M}$, $k_y > (T/M)^{-1/2}\tau_e^{-1}$, $k_z > k_y(m/M)^{1/2}$, and the instability condition $\xi > (\Omega\tau)_i$ coincides with that found hydrodynamically.

Using the same considerations as in finding $k_{y\,\max}$ in (9), we obtain that instability is possible if $\varkappa < r_e^{-1}$. This determines the pri-

approximately the upper boundary of the instability under consideration, $\xi < (M/m)^{1/2}(\Omega\tau)_i$. The initial electric field usually leads to stabilization ([2,7]). It is not difficult to show that when the electron drift velocity in this field $v_{0y} = \dfrac{T}{m}\dfrac{\varkappa}{\Omega_e}\,[1 + (\Omega\tau)_i(\Omega\tau)_e]^{-1}$ becomes, in order of magnitude, comparable with the Larmor drift velocity $v^+ = \dfrac{T}{m}\dfrac{\varkappa}{\Omega_e}$, the development of the instability ceases. Therefore instability is possible only for $(\Omega\tau_i) \gtrsim (b_i/b_e)^{1/2} \simeq 0.1$.

The complete region of instability is shown in Fig. 1 (hatched). It consists of three overlapping regions: 1—the drift region with $\omega < \Omega_i$ (this region was mentioned in the review article by B. B. Kadomtsev ([8])). At very large values of H, when $\Omega_i < \tau_e^{-1}$, the drift instability passes into a collisionless one ([9]). 2—the ion-acoustic region with $\omega > \Omega_i$, in which for $\omega < \tau_e^{-1}$ the excitation is due to the diffusion mechanism, and for $\omega > \tau_e^{-1}$ to resonant electrons with $v_z = \omega/k_z$. 3—a small additional region where ion-acoustic oscillations are unstable, for which the electron kinetics with allowance for collisions is essential. The maximum increment in the central part of the instability region is of the order of τ_e^{-1}.

Fig. 1

In Fig. 1 the crosses indicate approximate values of the parameters $\eta = (\Omega\tau)_i/\xi$, ξ in the experiments ([5]) with a discharge in hydrogen. We note that, as follows from the results of one cycle of measurements, the values of H_c given in ([5]) correspond to the maximum amplitude of the oscillations. If one assumes that the instability observed by Geller was caused by excitation of an ion sound, then the dependence $H_c = \text{const} \cdot d^{-1}$ recorded by him receives a natural explanation. Indeed, only such a combination enters equations (6), (10). The enhanced diffusion observed by Golant ([6]) is probably also connected with the instability under consideration, since the experimental data, marked in Fig. 1 by line segments, fall within the instability region.

Since the length of the discharge tubes in ([5,6]) greatly exceeded the diameter, losses to the ends could be neglected and the choice of perturbations in the form of plane waves along Oz is fully justified. Direct application of the formulas obtained to the experiments ([3,4]), although it gives encouraging results, is not legitimate, since losses to the ends in these experiments could substantially affect the development of the instability. Thus, for example, if electrons and ions escape to the ends with different velocities, this effect may play the role of dissipation, promoting the development of instability.

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Note: Figure translations are in progress. See original paper for figures.