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Fig. 1

Figure 1: Fig. 1

Abstract

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INVESTIGATION OF THE SPATIAL BEHAVIOR OF COMPOSITE STRUCTURES

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To evaluate the load-bearing capacity and deformability of structures composed of several elements (Fig. 1), investigations are needed that make it possible to refine the analysis of such structures by taking into account their spatial behavior and the influence of the compliance of elastic connections joining the individual component elements.

Fig. 1

The spatial behavior of the structure as a whole can be ensured if the component elements are connected to one another by four types of connections: C-1, C-2, C-3, and C-4. We shall consider composite structures with elastic-compliant joints uniformly distributed along the length of the seams.

We shall base the solution on the following hypotheses:

1. The deformation of transverse compression (tension) in the direction of the axis s is zero for all plates (faces) of the component elements.
2. All types of connections joining the individual elements are elastic.

The longitudinal and tangential displacements of a point of the middle surface of a component element (in the general case, a prismatic shell) are determined by the formulas (see ([1])):

$$u(z;s)=\sum_{i=1}^{m_j}U_i(z)\varphi_i(s),\quad \text{where } i=1,2,\dots,m_j;$$

$$v(z;s) = \sum_{k=1}^{n_j} V_k(z)\psi_k(s), \quad \text{where } k = 1, 2, \dots, n_j, \tag{1}$$

where the functions $U_i(z)$ and $V_k(z)$, depending only on z, are the unknowns, while the functions $\varphi_i(s)$ and $\psi_k(s)$ are to be chosen in advance; m_j and n_j are the numbers of degrees of freedom, respectively, for the longitudinal and transverse displacements of an elementary strip cut out by the sections $z = \text{const}$ and $z + dz = \text{const}$ from the j-th element. A transverse strip selected from a structure consisting of α elements has a number of degrees of freedom equal to

$$m_1 + m_2 + \dots + m_j + \dots + m_\alpha = \sum_{j=1}^{\alpha} m_j$$

with respect to longitudinal displacements, and

$$n_1 + n_2 + \dots + n_j + \dots + n_\alpha = \sum_{j=1}^{\alpha} n_j$$

with respect to transverse displacements. Thus, the transverse strip will have in all

$$A = \sum_{j=1}^{\alpha} m_j + \sum_{j=1}^{\alpha} n_j$$

degree of freedom, and, consequently, the number of functions $\varphi(s)$ and $\psi(s)$ will also be equal to A.

In the general case of a composite structure, formulas (1) are written in the form:

$$u(z;s) = \sum_{j=1}^{\alpha}\sum_{i=1}^{m_j} U_{ij}(z)\varphi_{ij}(s), \qquad v(z;s) = \sum_{j=1}^{\alpha}\sum_{k=1}^{n_j} V_{kj}(z)\psi^*_{kj}(s). \tag{2}$$

The functions $\varphi_{ij}(s)$ $(\psi_{ki}(s))$ will be taken, in the state ij (ki), to be different from zero only for the i-th bar of the frame cut out of the j-th element.

On the basis of Hooke' s law, the normal and shear stresses in the cross section of any element are written in the form

$$\sigma(z;s) = E\,\partial u/\partial z, \qquad \tau(z;s) = G\,(\partial u/\partial s + \partial v/\partial z). \tag{3}$$

Taking (2) into account, we obtain

$$\sigma(z;s) = E \sum_{j=1}^{\alpha} \sum_{i=1}^{m_j} U'_{ij}(z)\varphi_{ij}(s),$$

$$\tau(z;s) = G \left[\sum_{j=1}^{\alpha} \sum_{i=1}^{m_j} U_{ij}(z)\varphi'_{ij}(s) + \sum_{j=1}^{\alpha} \sum_{k=1}^{n_j} V'_{kj}(z)\psi_{kj}(s) \right]. \tag{4}$$

Let us compose the equilibrium equations for an elementary transverse strip of width dz of the composite structure in the form of the work of all forces acting on it on possible displacements (Lagrange' s principle), for which we take the previously chosen functions $\varphi_{ry}(s)$ and $\psi_{gp}(s)$:

$$\int_F \frac{\partial \sigma}{\partial z}\varphi_{ry}(s)\,dF - \sum_t T\left[\varphi^j_{ry}(s) - \varphi^{j+1}_{ry}(s)\right] - \int_F \tau\varphi'_{ry}(s)\,dF + \int_L p\varphi_{ry}(s)\,ds = 0;$$

$$\int_F \frac{\partial \tau}{\partial z}\psi_{gp}(s)\,dF - \sum_t N_x[\psi^j_{gp}(s) - \psi^{j+1}_{gp}(s)] - \sum_t N_y[\psi^j_{gp}(s) - \psi^{j+1}_{gp}(s)]$$
$$- \sum_t m[f^j_{gp}(s) - f^{j+1}_{gp}(s)] + \sum_{j=1}^{\alpha} \frac{\partial H_j}{\partial z} f_{gp}(s) - \sum_{j=1}^{\alpha}\sum_{k=1}^{n_j} V_{kj} \int_L \frac{M_{kj}M_{gp}}{EI}\,ds + \int_L q\psi_{gp}(s)\,ds = 0, \tag{5}$$

where

$$r = 1, 2, \dots, m_j; \qquad g = 1, 2, \dots, n_j; \qquad y = 1, 2, \dots, \alpha; \qquad p = 1, 2, \dots, \alpha;$$

$$j = 1, 2, \dots, \alpha;$$

$$T(z;s) = \sum_{j=1}^{\alpha} \sum_{i=1}^{m_j} \beta_t U_{ij}(z)\left[\varphi^j_{ij}(s) - \varphi^{j+1}_{ij}(s)\right] \tag{6}$$

are the forces in the ties (C-1) of longitudinal shear of the t-th seam; $[\varphi^j_{ij}(s) - \varphi^{j+1}_{ij}(s)]$ is the magnitude of the mutual longitudinal displacement of two connected elements j and $j+1$ at the location of this seam from the unit state $U^*_{ij}(z) = 1$, and β_t is the stiffness coefficient of the longitudinal-shear ties of the t-th seam;

$$N_x(z;s) = \sum_{j=1}^{\alpha}\sum_{k=1}^{n_j} \gamma_{xt} V_{kj}(z)\,[\psi_{kj}^{j}(s) - \psi_{kj}^{j+1}(s)],$$

$$N_y(z;s) = \sum_{j=1}^{\alpha}\sum_{k=1}^{n_j} \gamma_{yt} V_{kj}(z)\,[\psi_{kj}^{j}(s) - \psi_{kj}^{j+1}(s)] \tag{7}$$

are the forces in the ties (C-2 and C-3) of transverse displacement of the t-th seam; $[\psi_{kj}^{j}(s) - \psi_{kj}^{j+1}(s)]$ is the magnitude of the mutual transverse displacement of two connected elements at the location of this seam from the unit state $V_{kj}^{*}(z) = 1$, and γ_{xt} and γ_{yt} are the stiffness coefficients of the transverse-displacement ties in the-offsetof

in the directions of the x-axis and the y-axis, respectively, of the t-th seam;

$$m(z;s) = \sum_{j=1}^{\alpha}\sum_{k=1}^{n_j} \varphi_t V'_{kj}(z)\left[f_{kj}^{j}(s) - f_{kj}^{j+1}(s)\right] \tag{8}$$

forces in the angular connections (C-4); $f_{kj}(s)$ is a function determining the angle of rotation of the cross section of the individual elements of the structure only in the direction of the s-axis; $\left[f_{kj}^{j}(s) - f_{kj}^{j+1}(s)\right]$ is the magnitude of the mutual angle of rotation of the cross sections of two connected elements at the location of the given seam from the unit state $V_{kj}^{*}(z) = 1$; the function $f_{kj}(s)$ is completely determined by the choice of the magnitude and form of the function $\psi_{kj}(s)$; φ_t is the stiffness coefficient of the angular connections of the t-th seam;

$$H_j(z) = \sum_{j=1}^{\alpha}\sum_{k=1}^{n_j} K V'_{kj}(z) \tag{9}$$

the twisting moment in an individual element of the structure; $V_{kj}(z)$ are the functions of twisting of an individual plate (face) of the j-th component element; K is the coefficient of the shape of the transverse section of the face.

Substituting expressions (4), (6)–(9) into (5), we obtain a system A of linear differential equations with respect to the sought generalized $\sum_{j=1}^{\alpha} m_j$ longitudinal $U_{ij}(z)$ and $\sum_{j=1}^{\alpha} n_j$ transverse $V_{kj}(z)$ displacements:

$$\begin{aligned}&\sum\sum Ea_{ij,ry}U''_{ij}(z)-\sum\sum\beta_t\bar{a}_{ij,ry}U_{ij}(z)-\sum\sum Gb_{ij,ry}U_{ij}(z)\\&\qquad-\sum\sum Gc_{kj,ry}V'_{kj}(z)+p_{ry}=0,\\&\sum\sum Gc_{ij,gp}U'_{ij}(z)+\sum\sum Gr_{kj,gp}V''_{kj}(z)-\sum\sum\chi_{xt}\bar{r}_{kj,gp}V_{kj}(z)\\&\qquad-\sum\sum\chi_{yt}\bar{r}_{kj,go}V_{kj}(z)-\sum\sum\varphi_t\bar{d}_{kj,gp}V_{kj}(z)+\sum\sum t_{go}V'_{kj}(z)\\&\qquad-\sum\sum s_{kj,gp}V_{kj}(z)+q_{gp}=0,\end{aligned}\tag{10}$$

where

$$a_{ij,ry}=\int_F\varphi_{ij}(s)\varphi_{ry}(s)\,dF;\qquad\bar{a}_{ij,ry}=\sum_t\left[\varphi^j_{ij}(s)-\varphi^{j+1}_{ij}(s)\right]\left[\varphi^j_{ry}(s)-\varphi^{j+1}_{ry}(s)\right];$$

$$b_{ij,ry}=\int_F\varphi'_{ij}(s)\varphi'_{ry}(s)\,dF;\qquad c_{kj,ry}=\int_F\psi_{kj}(s)\varphi'_{ry}(s)\,dF;$$

$$c_{ij,gp}=\int_F\varphi'_{ij}(s)\psi_{gp}(s)\,dF;$$

$$r_{kj,gp}=\int_F\psi_{kj}(s)\psi_{gp}(s)\,dF;\qquad\bar{r}_{kj,gp}=\sum_t\left[\psi^j_{kj}(s)-\psi^{j+1}_{ki}(s)\right]\left[\psi^j_{gp}(s)-\psi^{j+1}_{gp}(s)\right];$$

$$\bar{d}_{kj,gp}=\sum_t\left[f^j_{kj}(s)-f^{j+1}_{kj}(s)\right]\left[f^j_{gp}(s)-f^{j+1}_{gp}(s)\right];$$

$$t_{gp}=\sum_{j=1}^{\alpha}Kf_{gp}(s);\qquad s_{kj,gp}=\int_L\frac{M_{kj}M_{gp}}{EI}\,ds.\tag{11}$$

Formulas (11) are of a general character and make it possible to compute the coefficients of equations (10) for a composite structure of arbitrary outline in the transverse section, with an arbitrary method of approximating the sought displacements $u(z;s)$, $v(z;s)$ with respect to the variable s.

The quantities $p_{ry}(z)$ and $q_{gp}(z)$, corresponding to the free terms of the equations (10), are known functions of z, and for given forces $p(z;s)$ and $q(z;s)$ are computed by the formulas

$$p_{ry}(z)=\int_L p\varphi_{ry}(s)\,ds,\qquad q_{g\rho}(z)=\int_L q\psi_{g\rho}(s)\,ds.\tag{12}$$

Fig. 2

Figure 2: Fig. 2

Let us apply the equations obtained to the solution of a specific problem concerning the bending of a box structure (Fig. 2). We represent the displacements of an arbitrary point of any plate of the box by the expressions

$$u(z;s) = U_{11}(z)\varphi_{11}(s)+U_{12}(z)\varphi_{12}(s)+U_{13}(z)\varphi_{13}(s), \qquad v(z;s) = V_{11}(z)\psi_{11}(s),$$

where the functions $\varphi_{ij}(s)$ and $\psi_{kj}(s)$ are chosen in the form

$$\varphi_{11}(s) = y'(s); \qquad \varphi_{12}(s) = y(s); \qquad \varphi_{13}(s) = y(s); \qquad \psi_{11}(s) = y'(s).$$

Fig. 2

Having computed the coefficients by formulas (11), we obtain for the problem under consideration the following system of equations (for simplicity of notation it is assumed that $U_{11}(z) = U_{11}$, $U_{12}(z) = U_{12}$, $U_{13}(z) = U_{13}$, $V_{11}(z) = V_{11}$):

$$\begin{aligned} &a''_{11}U''_{11} - a_{11}U_{11} + a_{12}U_{12} + a_{13}U_{13} - a'_{14}V'_{11} + p_{11} = 0,\\ &a_{21}U_{11} + a''_{22}U''_{12} - a_{22}U_{12} + 0 + 0 + p_{12} = 0,\\ &a_{31}U_{11} + 0 + a''_{33}U''_{13} - a_{33}U_{13} + 0 + p_{13} = 0,\\ &a'_{44}U'_{11} + 0 + 0 + a''_{44}V''_{11} + q_{11} = 0. \end{aligned} \tag{13}$$

In the equations of the system

$$a''_{11} = 2EF_2(h^2/3 - hh_c + h_c^2) = 2EI_{x_2}; \qquad a_{11} = 2F_2G + 2\beta[(h-h_c)^2 + h_c^2];$$

$$a_{12} = a_{21} = 2\beta h_c^2; \qquad a_{13} = a_{31} = 2\beta(h-h_c)^2; \qquad a'_{14} = a'_{41} = 2F_2G;$$

$$a''_{22} = EF_1h_c^2 = EI_{x_1}; \qquad a_{22} = 2\beta h_c^2; \qquad a''_{33} = EF_3(h-h_c)^2 = EI_{x_3};$$

$$a_{33} = 2\beta(h-h_c)^2; \qquad a''_{44} = 2F_2G,$$

where E and G are, respectively, the moduli of elasticity of the first and second kind of the material of the plates; h_c is the distance of the neutral axis of the box section from the axis of the upper plate; $F_1 = a\delta_1$, $F_2 = h\delta_2$, $F_3 = a\delta_3$ are the cross-sectional areas, respectively, of the upper, side, and lower plates.

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Note: Figure translations are in progress. See original paper for figures.

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