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Abstract

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ON RADIATION DAMPING OF FREE OSCILLATIONS

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As is known, quantum calculations of the damping of free oscillations of an electron in an inhomogeneous magnetic field, carried out by A. A. Sokolov and I. M. Ternov [1], showed the absence of any damping other than the usual adiabatic damping. This contradicts the classical calculation of A. A. Kolomenskii and A. N. Lebedev [2], who were the first to obtain formulas for damping decrements. It is shown below (for the nonrelativistic case) that the classical and quantum theories of damping are in complete agreement.

To simplify the proof, let us consider plane motion in a magnetic field with weak inhomogeneity

$$H_z(R) = H + gR^2; \quad n = -\frac{R}{H}\frac{\partial H_z}{\partial R} = -\frac{2gR^2}{H} \ll 1. \tag{1}$$

In the Schrödinger equation one may neglect the electron spin, which is immaterial here. The wave function in the first approximation in n has the form

$$\psi_{l,s}(y,\varphi) = \psi^0_{l,s} + \frac{\delta}{2l}\Big\{(3l+4s+4)\sqrt{(l+s+1)(s+1)}\,\psi^0_{l,s+1} -$$

$$-(3l+4s)\sqrt{(l+s)s}\,\psi^0_{l,s-1} + \frac{1}{2}\sqrt{s(s-1)(l+s)(l+s+1)}\,\psi^0_{l,s-2} -$$

$$-\frac{1}{2}\sqrt{(s+1)(s+2)(l+s+1)(l+s+2)}\,\psi^0_{l,s+2}\Big\}; \tag{2}$$

$$\delta = \frac{g\hbar l}{eH^2} \simeq -\frac{n}{4}; \qquad y = \frac{eH}{2c\hbar}R^2; \tag{3}$$

$$\psi^0_{l,s} = \frac{1}{\sqrt{2\pi}}\sqrt{\frac{(l+s)!}{s!(l!)^2}}\,e^{il\varphi - y/2}y^{l/2}F(-s,l+1,y); \tag{4}$$

$F(-s, l+1, y)$ is a degenerate hypergeometric function. The energy and the squared amplitude of the radial oscillations in the state with quantum numbers l and s are, respectively,

$$E_{l,s} = \frac{eH}{mc}\left(l + s + \frac{1}{2}\right)\left[1 + \frac{\delta}{l}\left(l + 3s + \frac{3}{l^2} + O\left(\frac{s}{l}\right)\right)\right]; \tag{5}$$

$$A^2_{l,s} = \overline{R^2} - \overline{R}^2 = \frac{c\hbar}{eH}(1 - 4\delta)s = \frac{c\hbar}{eH}(1 + n)s. \tag{6}$$

Formula (6) agrees with the corresponding expression of work [1] and corresponds to classical adiabatic damping, for which $s = \text{const}$:

$$A^2 \sim \frac{1}{H_z(R)\sqrt{1-n}} \simeq \frac{1}{H(1+2\delta)(1+4\delta)^{1/2}} \sim 1 - 4\delta \simeq 1 + n. \tag{7}$$

Under dipole radiation, both l and s change; however, the change of l is completely compensated by the adiabatic damping associated with the accelerat-

by the electric field if the energy and the radius of the equilibrium particle, which depend only on l, remain unchanged on average. The additional radiation damping is associated only with the change of s during emission:

$$\left(\frac{d\overline{A^2}}{dt}\right)_{\text{rad}} = \overline{A^2}\frac{1}{s}\frac{d\overline{s}}{dt} \equiv -\gamma_R \overline{A^2}. \tag{8}$$

The probability of dipole radiation ([3]) is equal to

$$w_{1,\pm1} = \frac{3}{2}\frac{e^2}{\hbar c}\frac{\omega_{12}^3}{c^2}\left|Re_{21}^{\pm i\varphi}\right|^2; \qquad \hbar\omega_{21} = E_2 - E_1. \tag{9}$$

With the aid of the wave functions (2), in the first order in n we obtain

$$\gamma_{R\,\text{q}} = \frac{2}{3}\frac{e^4H^2}{m^3c^5}n^2. \tag{10}$$

Allowance in (2) and (6) for terms of higher order gives in (10) terms of order n^3, n^4, etc.

Expression (10) coincides completely with the results of calculations by means of the classical energy, which we have carried out anew, since the nonrelativistic case had not been considered by anyone. Generalizing expression (4) for the four-dimensional radiation-reaction force to the case of curvilinear coordinates, we obtained the following general expressions for the classical damping decrements of radial and phase oscillations:

$$\gamma_{R\,\mathrm{cl}} = \frac{2e^4H^2\varepsilon}{3m^4c^7}\,\frac{n\left(1-\dfrac{m^2c^4}{\varepsilon^2}\right)+n^2\dfrac{m^2c^4}{\varepsilon^2}}{1-n}; \tag{11}$$

$$\gamma_{\varphi\,\mathrm{cl}} = \frac{2e^4H^2\varepsilon}{3m^4c^7}\,\frac{3-\dfrac{m^2c^4}{\varepsilon^2}-2n\left(2-\dfrac{m^2c^4}{\varepsilon^2}\right)-n\dfrac{m^2c^4}{\varepsilon^2}}{1-n}. \tag{12}$$

For $\varepsilon \simeq mc^2$ and $n \ll 1$, (11) goes over into (10).

For the ultrarelativistic case we also carried out a quasiclassical calculation, considering jumps of the oscillation amplitude at the moments of emission. It turns out that, if the radial r-oscillations are each time counted from the instantaneous equilibrium orbit, which changes discontinuously upon emission of a quantum, then in the equation of the radial oscillations the nonlinear (quadratic) terms in r are essential. A rigorous accounting of these terms also leads to formula (11), if the energy of the particle remains unchanged on average.

Thus, the present work eliminates the existing discrepancy between the classical and quantum calculations of radiation damping of oscillations.

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CITED LITERATURE

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[2] A. A. Kolomenskii, A. N. Lebedev, ZhETF, **30**, 207, 1161 (1956).
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[4] L. D. Landau, E. M. Lifshitz, *The Classical Theory of Fields*, Moscow, 1960.

Note: Figure translations are in progress. See original paper for figures.