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A. A. SOKOLOV, I. M. TERNOV

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Abstract

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A. A. SOKOLOV, I. M. TERNOV

ON POLARIZATION AND SPIN EFFECTS IN THE THEORY OF SYNCHROTRON RADIATION

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As is known, synchrotron radiation is strongly polarized. In particular, in the classical approximation $^7/_8$ of the total radiation intensity should be assigned to the σ-component (the electric vector of the radiation field is directed along the radius toward the center of the trajectory) and $^1/_8$ to the π-component (the electric vector of the radiation field is almost perpendicular to the plane of the orbit, see (1)). This conclusion was experimentally confirmed by the experiments of F. A. Korolev et al. (2).

In the present article we wish to investigate the influence of the orientation of the electron spin on the polarization and intensity of the radiation, if the electron moves in a constant and homogeneous magnetic field.

In studying spin effects, it is convenient to divide the solutions of the Dirac equation into two states which characterize the orientation of the spin either a) along the motion or opposite to the motion, or b) along the field and opposite to the field, i.e., in our problem, almost perpendicular to the motion.

The Dirac equation

$$i\hbar\frac{\partial\psi}{\partial t} = \hat{\mathscr{H}}\psi, \qquad \hat{\mathscr{H}} = c(\alpha\mathbf{P}) + \rho_3 m_0 c^2, \tag{1}$$

where

$$\mathbf{P} = \mathbf{p} - \frac{e}{c}\mathbf{A}, \qquad A_x = A_1 = -\frac{1}{2}yH, \qquad A_2 = \frac{1}{2}xH, \qquad A_3 = 0, \tag{2}$$

describing the motion of an electron in a constant and homogeneous magnetic field, has the solution (3)

$$\psi_{1,3} = e^{-i\varepsilon cKt}\frac{e^{ik_3z}}{\sqrt{L}}\frac{e^{i(l-1)\varphi}}{\sqrt{2\pi}}\,f_{1,3}(\rho), \qquad \psi_{2,4} = e^{-i\varepsilon cKt}\frac{e^{ik_3z}}{\sqrt{L}}\frac{e^{il\varphi}}{\sqrt{2\pi}}\,f_{2,4}(\rho); \tag{3}$$

$$f_1 = \sqrt{2\gamma}\, C_1 I_{n-1,s}(\rho), \qquad f_2 = \sqrt{2\gamma}\, iC_2 I_{n,s}(\rho),$$

$$f_3 = \sqrt{2\gamma}\, C_3 I_{n-1,s}(\rho), \qquad f_4 = \sqrt{2\gamma}\, iC_4 I_{n,s}(\rho). \tag{4}$$

In these formulas $\rho = \gamma r^2$, $\gamma = e_0 H/2c\hbar$; $e_0 = -e > 0$ is the elementary charge; $E = \varepsilon c\hbar K = \varepsilon c\hbar\sqrt{k_0^2 + k_3^2 + 4\gamma n}$; $n = l + s = 0, 1, \ldots$ is the principal quantum number; $l = 0, \pm 1, \ldots, -\infty \leq l \leq n$ is the azimuthal quantum number; $s = 0, 1, \ldots$ is the radial quantum number. The quantity $\varepsilon = \pm 1$ characterizes the sign of the energy, and the functions $I_{n,s}(\rho)$ are connected with the Laguerre polynomials $Q_s^l(\rho)$ by the relation

$$I_{n,s}(\rho) = \frac{1}{\sqrt{n!\, s!}}\, e^{-\frac{1}{2}\rho} \rho^{\frac{n-s}{2}} Q_s^{n-s}(\rho). \tag{5}$$

For an unambiguous determination of the coefficients C_μ, in addition to the Dirac equation (1) and the normalization condition, the wave function must be subjected to one further condition characterizing the direction of the spin.

In the case of investigating longitudinal polarization, the best additional condition to take is the condition of conservation of the spin projection on the direction of motion:

$$(\vec{\sigma}\vec{P})\psi = \hbar\tilde{k}\xi\psi. \tag{6}$$

The operator $(\vec{\sigma}\vec{P})$ is the time component $T_{\mu 4}$ of the electron polarization pseudovector, which in the general case has the form

$$T_{\mu 4} = \frac{1}{2}\{P_4\sigma_\mu + \sigma_\mu P_4\}, \tag{7}$$

where $\sigma_\mu = \{\vec{\sigma},\, i\rho_1\}$ is the spin pseudovector, and $P_4 = (\mathcal{H} - e\Phi)\frac{1}{c}$ is the fourth component of the generalized momentum; in our case the scalar potential $\Phi = 0$ (see (4)).

When projecting the spin onto the direction of the field, it is more convenient to subject the wave function to the condition (see (5))

$$\{m_0 c\sigma_3 + \rho_2[\vec{\sigma}\vec{P}]_3\}\psi = \hbar k\zeta\psi. \tag{8}$$

The operator on the left-hand side of the equation is the component F_{124} of the third-rank polarization tensor:

$$F_{\mu\nu\lambda} = \frac{1}{2}\{P_\lambda\alpha_{\mu\nu} + \alpha_{\mu\nu}P_\lambda\}, \tag{9}$$

where $\alpha_{23} = \rho_3\sigma_1$, $\alpha_{14} = i\rho_2\sigma_1$, etc., are components of the tensor of the intrinsic magnetic and electric moments.

Let us note that the operators appearing on the left-hand sides of equations (6) and (8) are integrals of motion, i.e., they commute with the Hamiltonian $\mathcal{H}$. In these equations the quantities ξ and ζ are equal to ± 1 and characterize the corresponding two possible spin directions. The quantities $\tilde{k}$ and k are equal to

$$\tilde{k} = \sqrt{K^2 - k_0^2}, \qquad k = \sqrt{K^2 - k_3^2}. \tag{10}$$

In studying motion when the spin is directed along or opposite to the motion ($\xi = \pm 1$), the coefficients in equation (4) will be equal to:

$$C_1 = \tilde{\xi} a \tilde{A}, \qquad C_2 = \tilde{a}\tilde{B}, \qquad C_3 = \varepsilon \tilde{b}\tilde{A}, \qquad C_4 = \varepsilon\tilde{\xi}\tilde{b}\tilde{B}, \tag{11}$$

where

$$\tilde{a} = \sqrt{\frac{1}{2}\left(1 + \varepsilon\frac{k_0}{K}\right)}, \qquad \tilde{b} = \sqrt{\frac{1}{2}\left(1 - \varepsilon\frac{k_0}{K}\right)},$$

$$\tilde{A} = \sqrt{\frac{1}{2}\left(1 + \xi\frac{k_3}{\tilde{k}}\right)}, \qquad \tilde{B} = \sqrt{\frac{1}{2}\left(1 - \xi\frac{k_3}{\tilde{k}}\right)}. \tag{12}$$

In particular, from the last equalities, for $\varepsilon = 1$ the result of work ([6]) follows.

When projecting the spin onto the direction of the field, we must put

$$C_1 = aA, \qquad C_2 = -\zeta bB, \qquad C_3 = bA, \qquad C_4 = \zeta aB, \tag{13}$$

where

$$A = \sqrt{\frac{1}{2}\left(1 + \zeta\frac{k_0}{K}\right)}, \qquad B = \sqrt{\frac{1}{2}\left(1 - \zeta\frac{k_0}{K}\right)},$$

$$a = \frac{1}{2}\left\{\sqrt{1 + \varepsilon\frac{k_3}{K}} + \varepsilon\zeta\sqrt{1 + \varepsilon\frac{k_3}{K}}\right\},$$

$$b = \frac{1}{2}\left\{\sqrt{1 + \varepsilon\frac{k_3}{K}} - \varepsilon\zeta\sqrt{1 - \varepsilon\frac{k_3}{K}}\right\}. \tag{14}$$

Using in what follows the method developed in work ([1]) (see also ([7])), one can find the intensity of the synchrotron radiation of the π- and σ-components, taking the spin direction into account.

In studying the longitudinal polarization of the electron (see the supplementary condition (6)), the radiation intensity associated with a change in the spin orientation does not depend on how the initial spin is directed (along the motion or against the motion). In the case of polarization of the electron along the field, however, the radiation intensity will already depend on the initial direction of the spin (along the field or against the field):

$$\begin{aligned} W_\sigma^{\uparrow\uparrow} &= W^{\mathrm{cl}}\left\{\frac{7}{8} - \xi\left(\frac{25\sqrt{3}}{12} - \zeta\right) + \xi^2\left[\frac{335}{18} + \frac{245\sqrt{3}}{48}\zeta\right] + \cdots\right\}, \\ W_\sigma^{\uparrow\downarrow} &= W^{\mathrm{cl}}\left\{\xi^2\frac{1}{18}\right\}, \\ W_\pi^{\uparrow\uparrow} &= W^{\mathrm{cl}}\left\{\frac{1}{8} - \xi\frac{5\sqrt{3}}{24} + \xi^2\frac{25}{18} + \cdots\right\}, \\ W_\pi^{\uparrow\downarrow} &= W^{\mathrm{cl}}\xi^2\frac{23}{18}\left\{1 + \xi\frac{105\sqrt{3}}{184}\right\}. \end{aligned} \tag{15}$$

Here

$$W^{\mathrm{cl}} = \frac{2}{3}\frac{e^2 c}{R^2}\left(\frac{E}{m_0 c^2}\right)^4, \qquad \xi = \frac{3}{2}\frac{\hbar}{mcR}\left(\frac{E}{m_0 c^2}\right)^2.$$

The arrows indicate the relative direction of the spin in the initial and final states; moreover, for $\zeta = 1$ the initial spin is directed along the field, and for $\zeta = -1$ against the field (see ([8])).

We determine the transition probabilities in an analogous way and denote by n_1 the number of electrons whose spin is directed against the field, and by $n_2 = n_0 - n_1$ the number of electrons whose spin is directed along the field. For the change of these quantities due to radiation we find:

$$n_{1,2} = \frac{(15 \pm 8\sqrt{3})n_0 \mp \left(15(n_{20} - n_{10}) + 8\sqrt{3}\, n_0\right) e^{-t/\tau}}{30}. \tag{16}$$

Here the upper signs refer to n_1, and the lower signs to n_2; at the initial time $n_1 = n_{10}$ and $n_2 = n_{20}$.

The lifetime τ is equal to

$$\tau = \left[\frac{5\sqrt{3}}{8}\frac{\hbar}{m_0 cR}\left(\frac{E}{m_0 c^2}\right)^5 - \frac{e^2}{m_0 cR^2}\right]^{-1} \tag{17}$$

and for $E \sim 1$ Bev, $H \sim 10^4$ oersted, it is of the order of an hour.

For times $t \gg \tau$ the ratio n_1/n_2 tends to the limiting value

$$\frac{n_1}{n_2} = \frac{15 + 8\sqrt{3}}{15 - 8\sqrt{3}} \tag{18}$$

independently of the initial distribution of the electrons over spin states along the field. From (18) it is seen that in this limiting case, for approximately 95% of the electrons the spin must become turned against the field, if other factors capable of reversing the spins of the electrons are neglected.

Moscow State University
named after M. V. Lomonosov

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CITED LITERATURE

1. A. A. Sokolov, I. M. Ternov, ZhETF, **31**, 473 (1956).
2. F. A. Korolev et al., DAN, **110**, 542 (1956).
3. A. A. Sokolov, I. M. Ternov, ZhETF, **25**, 698 (1953).
4. A. A. Sokolow, J. Phys. USSR, **9**, 363 (1945); A. A. Sokolow, Ann. Phys., **8**, 237 (1961); D. M. Fradkin, R. H. Good, Rev. Mod. Phys., **33**, 343 (1961).
5. A. A. Sokolov, M. M. Kolesnikova, ZhETF, **38**, 1778 (1960); J. Hiegevoerd, S. A. Wouthuysen, Nucl. Phys., **40**, 1 (1963).
6. I. M. Ternov, V. S. Tumanov, DAN, **124**, 1038 (1959).
7. A. A. Sokolov, A. N. Matveev, I. M. Ternov, DAN, **102**, 65 (1956).
8. I. M. Ternov, Yu. M. Loskutov, L. I. Korovina, ZhETF, **41**, 1294 (1961).

Note: Figure translations are in progress. See original paper for figures.