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Abstract

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Stabilization of the "Universal" Instability of a Weakly Inhomogeneous Plasma with Relativistic Electrons in a Magnetic Field

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1. It is known that in an inhomogeneous rarefied plasma confined by a magnetic field, a local instability arises for short-wavelength nonmagnetic perturbations, not associated with the configuration of the magnetic field ([1]). In ([2]) it was shown that this instability is "universal," i.e., it does not depend on the relations between the temperature gradient and the density gradient. In that work an isothermal plasma was considered, and the finite ion Larmor radius was taken into account by the method developed in ([3]). Below we consider a plasma with nonrelativistic ions and relativistic electrons. In this case the electron Larmor radius is comparable with the ion one, and allowance for its finiteness, as will be shown below, may lead to stabilization of the plasma with respect to the "universal" instability. We shall adopt the following assumptions: 1) the plasma pressure is much smaller than the magnetic pressure ($P \ll H^2/8\pi$); 2) quasineutrality ($n_i = n_e$); 3) potentiality of the electric fields of the perturbation ($\mathbf{E} = -\nabla\varphi$); 4) the collision time is much longer than the characteristic times of the problem.

2. The distribution function near the point $x = 0$ may be written in the form (the magnetic field is directed along z, the inhomogeneity along x)

$$f_\alpha = f_{0\alpha} + \left(x + \operatorname{sign} e_\alpha \frac{p_y}{m_\alpha \Omega_\alpha}\right) \frac{\partial f_{0\alpha}}{\partial x} \qquad (\alpha = i, e), \tag{1}$$

where

$$f_{0e} = \frac{\sigma n}{4\pi (mc)^3 K_2(\sigma)}\, e^{-\sigma\gamma} \xrightarrow[\sigma\to 0]{} \frac{n\sigma^3}{4\pi (mc)^3} e^{-\frac{\sigma}{mc}p};$$

$$\sigma = \frac{mc^2}{T_e}; \qquad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}; \qquad m_e = m, \quad m_i = M; \tag{2}$$

$$f_{0i} = n(2\pi MT_i)^{-3/2} \exp\left\{-\frac{p^2}{2MT_i}\right\}; \qquad \Omega_\alpha = \left|\frac{eH}{m_\alpha c}\right|.$$

The dispersion equation has the form (see, for example, ([3]))

$$F = F_i + F_e = 0; \tag{3}$$

$$F_\alpha = \int (dp) \left(\nabla_p \frac{\partial f_\alpha}{\partial \mathbf{p}}\right). \tag{4}$$

Choosing the perturbations in the form

$$\varphi = \varphi_0(x) \exp i(yk_y + zk_z - \omega t),$$

it is not difficult to obtain

$$F_e = \frac{\sigma n}{mc^2} + \frac{\sigma}{mc^2}\left(\omega - \frac{k_y}{\Omega_e}\frac{c^2}{\sigma}\frac{d}{dx}\right) \times$$

$$\times\, i\pi \sum_l \int dp \cdot \delta_+\left(\omega - k_{zv}z + \frac{l\Omega_e}{\gamma}\right) f_{0e} I_l^2 \left(\frac{k_y p_\perp}{m\Omega_e}\right); \tag{5}$$

$$\delta_+(x) = \frac{i}{\pi} P\,\frac{1}{x} + \delta(x);$$

$$F_i = \frac{n}{T_i} + \frac{1}{T_i}\left(\omega + \frac{k_y}{\Omega_i}\frac{T_i}{M}\frac{d}{dx}\right) \times$$

$$\times i\pi \sum_l \int dp \cdot \delta_+(\omega - k_z v_z + l\Omega_i) f_{0i} l^2 \left(\frac{k_y p_\perp}{M\Omega_i}\right). \tag{6}$$

Assuming

$$\omega \ll \Omega_i; \qquad \Omega_e/\gamma; \qquad k_y \bar{p}_\perp \ll m_\alpha \Omega_\alpha, \tag{7}$$

we obtain, after substituting (4)–(6) into (3):

$$\sum_\alpha \left\{\frac{n}{T_\alpha} + \frac{i\pi}{T_\alpha}\left(\omega + \operatorname{sign} e_\alpha \cdot \frac{k_y T_\alpha}{m_\alpha \Omega_\alpha}\frac{d}{dx}\right) \times\right.$$

$$\times \int dp \cdot \delta_+(\omega - k_z v_z) \left(1 - \frac{k_y^2 p_\perp^2}{m_\alpha^2 \Omega_\alpha^2}\right) f_{0\alpha} \Bigg\} = 0. \tag{8}$$

Solving equation (8) in the general form is difficult, and we shall study it in various limiting cases. In addition, we shall be interested in short-wavelength perturbations, which makes it possible to use the inequality

$$\frac{k_y k_0 T_i}{\omega \Omega_i M} \gg 1, \tag{9}$$

where $1/k_0$ is the characteristic scale length of the inhomogeneity. We shall regard the electrons as strongly relativistic ($\sigma \ll 1$).

3. Consider the case of "large" frequencies:

$$\omega \gg k_z v_e \gg k_z v_i.$$

Neglecting the subtractive terms in (8), we obtain

$$\omega^2 \left[M(nT_i)' - 16(mc)^2 \left(\frac{n}{\sigma^2}\right)'\right] = -\frac{2}{3}\left(Mc\Omega_i \cdot \frac{k_z}{k_y}\right)^2 n', \tag{10}$$

where the prime denotes differentiation with respect to x.

In order to present the result in a more transparent form, let us put

$$\frac{T_e'}{T_e} = -\frac{1}{2} s \frac{T_i'}{T_i}.$$

The stability condition has the form

$$1 - a + (1 - sa)\frac{d \ln T_i}{d \ln n} > 0, \tag{11}$$

where

$$a = 16 \frac{m}{M} \frac{T_e}{T_i} \frac{1}{\sigma} \sim 16 \frac{r_e}{r_i};$$

r is the Larmor radius. For $a \to 0$, condition (11) coincides with that obtained in [2]. The stabilizing effect of the plasma due to the finiteness of the Larmor radius of relativistic electrons begins to be significant for $a \gg 1$. (For example, at $T_e \sim 5$ MeV, $T_i \sim 0.01$ MeV, $\sigma \sim 10^{-1}$, we obtain $a \sim 40$.) We shall also assume $|s| \gtrsim 1$. In this case condition (11) gives

$$\frac{d\ln T_i}{d\ln n} > -\frac{1}{s} \qquad (s>0),$$

$$\frac{d\ln T_i}{d\ln n} < \frac{1}{|s|} \qquad (s<0). \tag{12}$$

4. "Intermediate" frequencies: $k_z v_i \ll \omega \ll k_z v_e$. The dispersion equation has the form

$$i\pi\frac{\omega}{ck_z}b_e + \omega\frac{n}{T_e} + b_i = 0, \tag{13}$$

where

$$b_e = n' - 48\frac{k_y^2c^2}{\Omega_e^2}\left(\frac{n}{\sigma^2}\right)',$$

$$b_i = n' - \frac{k_y^2}{\Omega_i^2 M}(nT_i)'. \tag{14}$$

Writing $\omega = \omega' + i\omega''$, we obtain

$$\omega' = -\frac{1}{\pi}\frac{ck_z}{b_e}\frac{\sigma n\Omega_e}{c^2k_y}\omega'',$$

$$\omega'' = \frac{\pi b_i b_e c|k_z|}{\pi^2 b_e^2 + n^2\sigma^2\Omega_e^2 k_z^2/(ck_y)^2}. \tag{15}$$

From the stability condition $b_i b_e < 0$ we obtain

$$-\frac{1}{k^2r_i^2} \gtrless \frac{d\ln T_i}{d\ln n} \gtrless -\frac{1}{s} \qquad (s>0),$$

$$\frac{d\ln T_i}{d\ln n} < \frac{1}{|s|} \qquad (s<0). \tag{16}$$

5. "Low" frequencies: $\omega \ll k_z v_i$. In this case the subtracted terms from the electron and ion distribution functions play an essential role. The contribution of the electrons to the dispersion equation is small, and one may use the results of work ([2]). The stability condition has the form

$$\frac{d\ln T_i}{d\ln n} \lesssim 1. \tag{17}$$

6. Using the results (12), (16), and (17), we obtain that on the branches of oscillations under consideration the plasma is stabilized for the following values of the parameter $d \ln T_i / d \ln n$:

$$-\frac{1}{s} < \frac{d \ln T_i}{d \ln n} < 1 \qquad (s > 0),$$

$$\frac{d \ln T_i}{d \ln n} < \frac{1}{|s|} \qquad (s < 0). \tag{18}$$

Thus, the presence of a relativistic electron component in an inhomogeneous plasma leads to the result that the instability arising on longitudinal oscillations is not "universal."

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Note: Figure translations are in progress. See original paper for figures.