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Abstract

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ON ION-SOUND OSCILLATIONS EXCITED BY AN ELECTRON CURRENT

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1. As is known, if the ion temperature T_i is much lower than the electron temperature T_e ($T_i \lesssim 0.1T_e$), then, in the presence of an electron stream with velocity u, ion sound is amplified in a plasma only if $u > c$, where c is the velocity of ion sound. In the present work it is shown that the growth of the amplitude of ion-sound oscillations is limited by ion Landau-type damping on slow beats of the waves. We note that slow beats arise only for waves propagating at an angle to one another. Therefore, in this problem one cannot confine oneself to a one-dimensional treatment; the interaction of oblique waves must be taken into account. As a result of this interaction the growth of the amplitudes ceases, and the plasma passes into a state of stationary weak turbulence.

In this work the distribution over frequencies ω of the amplitudes of the established random oscillations is found. It turns out that the spectral function of the oscillations of the electric potential is proportional to $1/\omega$ in the interval $1/\tau_i < \omega < \omega_{0i}$ (here $1/\tau_i$ is the collision frequency of ions with neutrals, and ω_{0i} is the ion plasma frequency) and rapidly goes to zero outside these limits. Qualitatively, this spectrum agrees with that experimentally measured in ([1]).

2. To determine the turbulent spectrum we shall use the nonlinear equations of work ([1]). Eliminating p, Q, and q from these equations, we arrive at an integral equation for I, the spectral function of the established oscillations of the electric potential in the plasma (another derivation of this kinetic equation for waves is given in ([3])):

$$k^2\varepsilon(\chi)I(\chi) = 16\pi^2 \sum_{nj} e_n e_j \int \frac{d\chi' I(\chi)I(\chi')}{k''^2\varepsilon(\chi'')} \left\{\int\int \left[\mathbf{k}'\mathbf{g}^n_\chi \mathbf{k}''\mathbf{g}^n_{\chi''} + \mathbf{k}''\mathbf{g}^n_\chi \mathbf{k}'\mathbf{g}^p_{-\chi'}\right] f^n_0\, dv\right\}$$
$$\times \left\{\int\int g^j_{\chi''}\left[\mathbf{k}'(\mathbf{k}\mathbf{g}^j_\chi) + \mathbf{k}(\mathbf{k}'\mathbf{g}^j_{\chi'})\right] f^j_0\, dv\right\}$$
$$+ 4\pi \sum_j e_j \int (\mathbf{k}'\mathbf{g}^j_\chi) g^j_{\chi''}\left[\mathbf{k}'(\mathbf{k}\mathbf{g}^j_\chi) + \mathbf{k}(\mathbf{k}'\mathbf{g}^j_{\chi'})\right] f^j_0 I(\chi)I(\chi')\, d\chi'\, dv$$
$$+ \frac{16\pi^2}{k^2\varepsilon(-\chi)} \sum_{nj} e_n e_j \left\{\int\int \mathbf{k}'\mathbf{g}^n_\chi \mathbf{k}''\mathbf{g}^n_{\chi''} f^n_0\, dv\right\} \times$$
$$\times \left\{\int\int \mathbf{k}'\mathbf{g}^j_{-\chi}(\mathbf{k}''\mathbf{g}^j_{-\chi''}) f^j_0\, dv\right\} I(\chi')I(\chi'+\chi)\, d\chi'. \tag{1}$$

Here χ is a four-vector, $\chi = (\mathbf{k}, \omega)$, $d\chi = d\mathbf{k}\, d\omega$. The operator g^j_χ for our case, when there is no magnetic field, has the form

$$g^j_\chi = \frac{e_j}{m_j}\frac{1}{\mathbf{kv} - \omega}\frac{\partial}{\partial \mathbf{v}}.$$

The left-hand side of (1) contains the usual linear dielectric permittivity $\varepsilon(\chi)$, elec-

the imaginary part of which, because of the presence of a directed electron velocity, is negative in a certain region of χ, so that the solutions of the linearized Vlasov equation give growing ion-acoustic oscillations. The right-hand side of (1) contains nonlinear absorption—the imaginary part, which is equal to the imaginary part on the left. Taking the right-hand side into account, the solutions of (1) must be stationary, with real ω and $\mathbf{k}$.

We solve (1) by successive approximations, setting in the zeroth approximation the large quantity—the real part of the left-hand side of (1), $k^2\,\mathrm{Re}\,\varepsilon I$—equal to zero, whence[4]

$$I(\chi) = I(\mathbf{k})\,\delta(\omega - ck\ \mathrm{sign}\, k_z), \qquad c = \sqrt{T_e/m_i}. \tag{2}$$

In the next approximation we equate the imaginary parts of (1) in order to determine $I(\mathbf{k})$, neglecting the real part of the nonlinear correction (1), which gives only a negligible change in the solution (2) of the dispersion equation. In order that the nonlinear part of the right-hand side of (1) be large, we substitute $I(\chi)$ and $I(\chi')$ from the instability region in the form (2). In this case the beats with frequencies $\chi'' = \chi + \chi'$, because of the nondecay nature of the spectrum, will no longer be proper oscillations, and $\varepsilon(\chi'')$ nowhere vanishes.

The largest contribution to the imaginary part of (1) is then given by the region

$$\frac{\omega + \omega'}{|\mathbf{k} + \mathbf{k}'|} \sim v_i,$$

where there is strong ion Landau damping. Therefore, in integrating over $d\chi'$ one may restrict oneself to this region only, and in the sums (1) all electron terms may be neglected because of the high electron temperature. The electron part of $\varepsilon(\chi'')$ is also proportional to $1/T_e$, and it may be neglected. The third term in (1) takes into account decay interactions of waves, and it may be neglected, since the spectrum of ion-acoustic oscillations is nondecay.

Consider the expression entering the first brace of the first term in (1):

$$V_1 = \frac{4\pi e^2}{m_i} \int \frac{1}{\omega - \mathbf{kv}} \mathbf{k}' \frac{\partial}{\partial \mathbf{v}} \frac{\mathbf{k}'' \, \partial f_i^0 / \partial \mathbf{v}}{\omega'' - \mathbf{k}''\mathbf{v}} \, d\mathbf{v} = \frac{4\pi e^2}{m_i} \int \frac{\mathbf{kk}'}{(\omega - \mathbf{kv})^2} \frac{\mathbf{k}'' \, \partial f_i^0 / \partial \mathbf{v}}{\omega'' - \mathbf{k}''\mathbf{v}} \, d\mathbf{v}. \tag{3}$$

Expanding $1/(\omega - \mathbf{kv})^2$ in a series in powers of $\mathbf{kv}$, and discarding terms containing v_i to powers higher than the third (since $\omega \gg kv_i$), we readily find

$$\dot{V}_1 = -\frac{\mathbf{kk}'}{\omega^2} \left[1 + 2\xi \frac{\omega''}{\omega} + 3\xi^2 \left(\frac{\omega''}{\omega} \right)^2 + 3 \left(\frac{k_\perp v_i}{\omega} \right)^2 + \right.$$
$$\left. + 12\xi \frac{k_\perp^2 v_i^2 \omega''}{\omega^3} + 4\xi^3 \left(\frac{\omega''}{\omega} \right)^3 \right] D_i + \frac{36\pi e^2 N}{m_i} \frac{(\mathbf{kk}')(\mathbf{kk}'')^2}{k^2 \omega^4} +$$
$$+ \frac{16\pi e^2 N}{m_i} \frac{(\mathbf{kk}')(\mathbf{kk}'')^3 \omega''}{k^4 \omega^5}, \tag{4}$$

where $D_i = k''^2 \varepsilon_i(\chi'')$ is the ion part of the dielectric permittivity, $\xi = (\mathbf{kk}'')/k''^2$, $k_\perp = |\mathbf{k} - \xi \mathbf{k}''|$, and N is the ion density. The imaginary part of the expression contained in the second bracket of the second term in (1) is equal to the imaginary part of (3), and for the imaginary part of the last term of (1) we easily find an expansion analogous to (4). Equating the imaginary parts of (1), we find that the principal parts of the first and second terms cancel. In experiment the cross section of the discharge is usually small; therefore the waves propagate almost parallel to the electron flow, and terms containing higher powers of k'' may be discarded.

Discarding also higher powers of T_i, we obtain:

$$k^2 \operatorname{Im} \varepsilon(\varkappa) I \simeq \simeq -\frac{e^2}{m_i^2} \int dk' \, I(\mathbf{k}) I(\mathbf{k}') \frac{(\mathbf{kk}')^2}{\omega^4} \left(\frac{k_\perp v_i}{\omega} \right)^2 \left(4 - 24\xi \frac{\omega''}{\omega} \right) \operatorname{Im} D_i. \tag{5}$$

Since $\omega \simeq ck$, and $c \gg v_i$, one may put

$$\operatorname{Im} D_i \simeq -\frac{4\pi^2 e^2 N}{m_i}\, \delta'\left(\frac{\omega''}{k''}\right); \tag{6}$$

$$\omega'' \operatorname{Im} D_i \simeq \frac{4\pi^2 e^2 N k''}{m_i}\, \delta\left(\frac{\omega''}{k''}\right).$$

Here δ' denotes the derivative of the δ-function with respect to its argument. Then from (5) we obtain:

$$k^2 \operatorname{Im} \varepsilon(\varkappa) I \simeq \simeq \frac{16\pi^2 e^4 T_i}{T_e^4}\, I \int dk'\, I(k')\, \frac{(kk')^2 k''^2}{k^4} \left\{\delta'(k-k') - 6\frac{\xi}{k}\delta(k-k')\right\}, \tag{7}$$

whence

$$k^2 \operatorname{Im} \varepsilon(\varkappa) I \simeq \frac{32\pi^2 e^4 T_i}{T_e^4}\, k^6 I\, \frac{\partial}{\partial k} \int_{t_0}^{1} dt\, t^2(1-t^2) \frac{1}{k^2} I(k,t), \tag{8}$$

where t is the cosine of the angle between $\mathbf{k}'$ and the direction of the electron flow—the z-axis; t_0 characterizes the width of the discharge.

We see that (8) has positive solutions only in the region of k where $\operatorname{Im} \varepsilon(k,t)$ is negative, which coincides with the region of instability. Outside this region we put $I(k,t) = 0$, and (8) is satisfied automatically.

For a hot plasma the ionic part in $\operatorname{Im} \varepsilon(\varkappa)$ may be neglected; however, for a weakly ionized plasma at small frequencies ω it is necessary to take into account collisions of ions with neutrals. Dissipation due to collisions of ions with neutrals leads to a cutoff of I on the low-frequency side. At not small frequencies, if one assumes approximately that the oscillations propagate along z, one may put

$$k^2 \operatorname{Im} \varepsilon(\varkappa) \simeq -\frac{4\pi^2 e^2}{m_e}\, \left.\frac{\partial f_0^e(v_z)}{\partial v_z}\right|_{v_z = c}, \tag{9}$$

but $\partial f_0^e/\partial v_z$ changes strongly because of the interaction with the oscillations, and to determine it one must use the quasilinear equation for the electrons with collisions taken into account ([5]):

$$\frac{e}{m_e} E_0 \frac{\partial f_0^e}{\partial v_z} = \frac{v_e^2}{\tau_e} \frac{\partial}{\partial v_z} \left\{2\frac{\partial f_0^e}{\partial v_z} + \frac{1}{v_e^2}(2v_z - u) f_0^e\right\} + \frac{\pi^2 e^2}{m_e^2} \frac{\partial}{\partial v_z} \int k_z^2 I(\mathbf{k}) \frac{\partial f_0^e}{\partial v_z} \delta(\omega - k_z v_z)\, dk. \tag{10}$$

Here E_0 is the external electric field, and τ_e is the electron mean free time. Hence we obtain:

$$\left.\frac{\partial f_0^e(v_z)}{\partial v_z}\right|_{v_z=c} \simeq \frac{1}{\tau_e} \frac{(u-c) f_0^e(v_z=c)}{\frac{v_e^2}{\tau_e} + \frac{2\pi^3 e^2}{c m_e^2} \int k^3 I(k,t)\, dk}. \tag{11}$$

Taking v_e^2/τ_e in (11) to be small, we substitute (11) into (9), and (9) into (8).

Limiting the integral over dk in (8) by the upper limit—the reciprocal Debye length—and passing to the distribution in ω according to the formula $2\pi(1 - t_0)I(k)k^2 dk \simeq I(\omega)d\omega$, we obtain

$$I(\omega) \sim \frac{1}{\omega} \frac{T_e^2}{e^2} \sqrt{\frac{m_{eT} e}{m_{iT} i} \frac{1}{\omega_0 \tau_e} \frac{u}{c}}, \tag{12}$$

where ω_0 is the electron plasma frequency. For ω less than the ion collision frequency or greater than the ion plasma frequency, $I(\omega)$ rapidly goes to zero.

The spectrum of potential oscillations has the form (12) only for devices in which only waves propagating at a small angle to the electron flow grow. Otherwise it follows from equation (10) that, because of the strong interaction of particles with the waves, the current in the plasma is locked.

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REFERENCES

[1] A. V. Nedospasov, Report No. 217 at the Conference on Plasma Physics in Salzburg, 1961.
[2] B. B. Kadomtsev, V. I. Petviashvili, ZhETF, **43**, 2234 (1962).
[3] V. I. Petviashvili, Dissertation, Tbilisi, 1963.
[4] G. V. Gordeev, ZhETF, **27**, 18 (1954).
[5] B. B. Kadomtsev, ZhETF, **43**, 1688 (1962).

Note: Figure translations are in progress. See original paper for figures.