GEOPHYSICS

1963

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Abstract

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GEOPHYSICS

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ON THE RECONCILIATION OF THE AVAILABLE EXPERIMENTAL DATA ON THE SPECTRUM AND ASYMMETRY OF LOCALLY ISOTROPIC TURBULENCE

(Presented by Academician A. N. Kolmogorov on June 20, 1963)

In recent years much attention has been attracted by the question of the properties of turbulence at very large wave numbers. The experimentally established fact that there is a strong nonuniformity in the distribution of energy in the region of such wave numbers led Kolmogorov ([1]) to draw attention to the possible nonuniversality of the behavior of turbulence in the region of very large wave numbers. The modification proposed by him of his widely known earlier theory ([2]) of the universal structure of turbulence in the region of small scales of motion led to an increased interest in the theory of small-scale turbulence. At the same time, several groups of researchers have recently published measurement data extending up to wave numbers sufficiently large for them to be used in testing the theory.

In each of these studies it is possible to calculate the rate of energy dissipation ε directly from measurements of the spectrum. With the sole exception of the work ([3]), these measurements agree with one another surprisingly well. Indeed, the measured velocity spectra are difficult to distinguish if they are represented in dimensionless form, namely as $\nu^{-5/4}\varepsilon^{-1/4}F(k\varepsilon^{-1/4}\nu^{3/4})$, where ν is the kinematic viscosity.

Quantitatively, these measurements are easiest to compare with one another by relying on the values of the constant A appearing in the known expression for the spectral density of turbulent energy in the inertial interval:

$$E(k) = A\varepsilon^{2/3}k^{-5/3}. \quad (1)$$

According to Kolmogorov' s original theory ([2]), the quantity A should be a universal constant. Recent investigations of the possible role of the effect of spatial intermittency of dissipation show that some variation of the quantity A may occur.

Two other important parameters of a locally isotropic field are associated with the constant A. The first is the structure constant C, entering the expression

$$D_{ll} = C\varepsilon^{2/3} r^{2/3}, \tag{2}$$

where $D_{ll} = \langle \{u_l(x+r) - u_l(x)\}^2 \rangle$, u_l is the longitudinal component of velocity, and r is the distance between the points of observation. The second parameter is the skewness coefficient

$$S = D_{lll}/(D_{ll})^{3/2}, \qquad D_{lll} = \langle \{u_l(x+r) - u_l(x)\}^3 \rangle. \tag{3}$$

It follows from the theory of locally isotropic turbulence ([2]) that, if $r \gg \nu^{3/4}\varepsilon^{-1/4}$, then C is a universal constant related to the constant S by the expression

$$C = (-{}^5/_4 S)^{2/3} = 0.862(-S)^{-2/3}. \tag{4}$$

The constants A and C, as is not difficult to show ([8]), are related by

$$A = \frac{55}{81\,\Gamma(4/3)}\,C = 0.76\,C. \tag{5}$$

The quantities A, C, and S can be calculated using the results of the measurements carried out in ([3–7]). The results of the calculations are summarized in Table 1. Although the authors of papers ([5]) and ([7]) do not give estimates of the errors of their measurements, it is nevertheless clear that they do not consider their results to differ greatly from the results of paper ([4]).

Table 1

Turbulence field	Re (approx.)	A	C	S
Wind tunnel (behind a grid) ([3])	10^7	2.70	3.55	0.12
Tidal current ([4])	10^8	1.44 ± 0.06	1.90	0.31 ± 0.02
Jet* ([5])	10^7	1.60	2.10	0.26
Hydrodynamic tunnel (behind a grid) ([6])	10^6	1.29 ± 0.06	1.70	0.36 ± 0.02

Turbulence field	Re (approx.)	A	C	S
Surface layer of the atmosphere ([7])	10^5	1.47	1.94	0.32

* The results on the jet axis and off the axis were averaged.

Excluding the measurements in the wind tunnel, all these data agree well with one another, which is strong support for the hypothesis of the universality of the form of the turbulence spectrum in the region of large wave numbers. The measurements in the wind tunnel did not show the presence of isotropy; therefore one may think that in this sense they are anomalous.

A more sensitive characteristic is the asymmetry S. In works ([3–7]) no one attempted to measure the quantity S directly. However, such measurements in the near-ground layer of the atmosphere were made by Gurvich ([9]). It is instructive to compare his results with the above calculations of the values of S.

Gurvich used two acoustic microanemometers, each of which substantially averaged the wind velocity over an interval of 2.5 cm (the size of the sensor). The quantity S was measured with the microanemometers separated by distances $r = 25$ and 50 cm. He obtained, for $r = 25$ cm, $S = -0.45 \pm 0.05$, and for $r = 50$ cm, $S = -0.40 \pm 0.06$. These quantities are significantly larger than those given in Table 1, which were obtained from spectral measurements. Because the size of the sensors was small in comparison with the distance between the anemometers, Gurvich thought it unnecessary to try to introduce a correction allowing for the finite size of the sensors.

The main purpose of this note is to show that, with the aid of a sufficiently reasonable assumption, one can introduce a correction that brings Gurvich' s results into better agreement with the others.

The assumption is that in the expression $S = D_{lll} D_{ll}^{-3/2}$, averaging over the size of the sensors affects only the function D_{ll}. This assumption is based on the fact that D_{lll} is in a certain sense a measure of the deviation of the turbulence regime from a purely Gaussian one and, apparently, depends on motions of sufficiently large scale. This scale must be, at least, comparable with the distance r. However, D_{ll} depends on any small-scale motion that contains energy.

This assumption can also be explained in another way, using spectral expansions. We have the well-known expression for the degree of variation of the energy spectral density in isotropic turbulence ([10]):

$$\frac{\partial}{\partial t}E(k) = T(k) - 2\nu k^2 E(k), \tag{6}$$

where the function $T(k)$ describes the nonlinear transfer of energy over the spectrum. Using the results of the book ([10]), it is not difficult to show that in the inertial interval

$$D_{lll}(r) = \frac{4}{5}\int_0^\infty T(k')\frac{\sin k'r}{k'}\,dk'. \tag{7}$$

It follows from expression (6) that in the inertial interval $T(k') \sim {k'}^{1/3}$. Therefore D_{lll} depends almost completely on wave numbers for which $k' < \pi/r$.

But the size of the sensors leads to a noticeable loss of information only for wave numbers greater than π/L in the spectrum $E(k)$, where L is the size of the sensor.

Since $r \gg L$, our assumption is that the value $T(k')$ depends mainly on those wave numbers in the spectrum $E(k)$ for which k/k' is not very large compared with unity. Such an assumption agrees with various other assumptions about the dependence of the spectrum $T(k)$ on the spectrum $E(k)$.

With his instrument, using as the structure function D_{ll}, Gurvich in fact measured the quantity

$$\frac{1}{L^2}\left\langle\left\{\int_0^L u\,dx - \int_r^{r+L} u\,dx\right\}^2\right\rangle. \tag{8}$$

If we assume homogeneity of the turbulent flow, (8) can be rewritten in the form

$$\frac{2}{L^2}\int_0^L\int_0^L \langle uu'\rangle\,dx\,dx' - \frac{2}{L^2}\int_0^L\int_r^{r+L}\langle uu'\rangle\,dx\,dx'. \tag{9}$$

From expression (2) we have

$$\langle uu'\rangle = \langle u^2\rangle - \frac{C}{2}\,\varepsilon^{2/3}|x-x'|^{2/3}. \tag{10}$$

Expression (9) is readily integrated, after which we obtain

$$-C\varepsilon^{2/3}r^{2/3}\{1 - \tfrac{18}{40}(L/r)^{2/3} - \tfrac{1}{54}(L/r)^2 + O(L/r)^4\}. \tag{11}$$

For Gurvich $L/r = 0.1$ and 0.05. Obviously, the terms in which the power of L/r is greater than or equal to 2 are insignificant. But the term $\frac{18}{40}(L/r)^{2/3}$ is not so small. For $L/r = 0.1$ it is equal to 0.10, and for $L/r = 0.05$ it is equal to 0.06. Equation (3) shows that neglecting this effect overestimates the value of S by 14% for $L/r = 0.1$ and by 9% for $L/r = 0.05$. Taking these corrections into account, we find

$$-S = 0.39 \pm 0.04 \quad \text{(instead of } 0.45 \pm 0.05) \quad \text{for } L/r = 0.1;$$

$$-S = 0.36 \pm 0.05 \quad \text{(instead of } 0.40 \pm 0.06) \quad \text{for } L/r = 0.05.$$

If these numbers are compared with the values given in Table 1, it can be seen that they still remain somewhat larger than the others, but the former striking contrast is no longer present.

In conclusion, it may be said that there is no significant difference among all these results, although the methods of measurement and the nature of the turbulence are quite different, and the Reynolds numbers vary over a wide range. Apparently, any differences that may exist because of the different character of the intermittency of the dissipation regions can become evident only when measurement techniques attain a higher order of accuracy than is available at present.

I am very glad to thank Corresponding Member of the Academy of Sciences of the USSR A. M. Obukhov and other staff members of the Institute of Atmospheric Physics of the Academy of Sciences of the USSR for their friendly attitude and useful discussions during my visit to the USSR. I especially wish to thank G. S. Golitsyn for his help in writing the Russian text of the article.

Received
17 VI 1963

REFERENCES

1. A. N. Kolmogorov, *J. Fluid Mech.*, **13**, No. 1 (1962).
2. A. N. Kolmogorov, DAN, **32**, No. 1 (1941).
3. A. K. Kistler, T. Vrebalovich, *Bull. Am. Phys. Soc.*, Ser. II, **6**, 207 (1961).
4. H. L. Grant, R. W. Stewart, P. Moilliet, *J. Fluid Mech.*, **12**, No. 2 (1962).
5. M. M. Gibson, *J. Fluid Mech.*, **15**, No. 2 (1963).
6. C. H. Gibson, W. H. Schwartz, *J. Fluid Mech.*, 1963—in press.
7. S. Pond, R. W. Stewart, R. W. Burling, *J. Atmosph. Sci.*, 1963—in press.

8. V. I. Tatarskii, *Theory of Fluctuation Phenomena in Wave Propagation in a Turbulent Atmosphere*, Publishing House of the Academy of Sciences of the USSR, 1959.

9. A. S. Gurvich, DAN, **134**, No. 5 (1960).

10. G. K. Batchelor, *The Theory of Homogeneous Turbulence*, IL, 1955, p. 103.

Note: Figure translations are in progress. See original paper for figures.