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Abstract

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SOLUTION OF THE PROBLEM OF THE MOTION OF CHARGED PARTICLES OF DIFFERENT SIGNS AND MASSES IN THE FIELD OF "MAGNETIC BOTTLES" ON ELECTRONIC COMPUTING MACHINES

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The cases of motion of charged particles of one sign considered in (1) demonstrate their diffusion across and along the magnetic field, the type of diffusion being determined by the field parameters. As follows from the calculations, in fields close to those of practical interest, losses of particles of one sign (ions) begin at densities of 10^5 particles/cm^3 and occur mainly across the magnetic field in the center of the chamber. Increasing the magnetic-field strength leads to an increase of diffusion along it and, already with a twofold increase, to losses only along the field.

In the present work the motion of enlarged charges of different signs and masses is considered with allowance for their scattering, and the influence of certain additional magnetic fields on the behavior of such particles is investigated. The phenomena obtained should be regarded as diffusion of an inhomogeneous plasma, since the possibilities of resolving the "enlarged charges" on the machine are limited by its memory, and it is impossible to model the process with complete accuracy.

The method developed in (1) makes it possible to take into account the behavior of enlarged particles with different masses and signs of charge. The equations in this case take the form:

$$\ddot{r}_{j(k)} = \gamma_{(k)} \left\{ \frac{V^2_{\varphi\, j(k)}}{r_{j(k)}} \frac{1}{\gamma_{(k)}} + H_{z\, j(k)} V_{\varphi\, j(k)} + \varepsilon_{r\, j(k)} \right\},$$

$$\ddot{z}_{j(k)} = \gamma_{(k)} \left\{ -V_{\varphi\, j(k)} H_{r\, j(k)} + \varepsilon_{z\, j(k)} \right\}, \qquad (1)$$

$$\ddot{\varphi}_{j(k)} = \frac{\gamma_{(k)}}{r_{j(k)}} \left\{ \dot{z}_{j(k)} H_{r\, j(k)} - \dot{r}_{j(k)} H_{z\, j(k)} - 2\dot{\varphi}_{j(k)} \dot{r}_{j(k)} + \varepsilon_{\varphi\, j(k)} \right\}, \qquad (2)$$

Figure 2 and Figure 3

Figure 1: Figure 2 and Figure 3

where

$$H_r = \beta I_1(r) \sin z, \qquad H_z = \alpha + \beta I_0(r) \cos z;$$

$\varepsilon_r,\ \varepsilon_z,\ \varepsilon_\varphi$ are interaction terms, defined as projections of the expression

$$\bar{\varepsilon}_{j(k)} = q_{j(k)} \sum_{\substack{i(l) \\ j(k)\neq i(l)}}^{N(M)} \frac{\bar{R}_{i(l),\,j(k)}}{R^3_{i(l),\,j(k)}},$$

$$q_j = \frac{q_+}{N}, \qquad q_k = \frac{q_-}{M},$$

where q_- is the total charge of the electrons, and q_+ is the total charge of the ions, on the corresponding axes. The magnetic fields created by the moving particles, at the densities considered in the problem (less than 10^9 particles/cm^3), are much weaker than the external ones and were not taken into account. The index $j = 0, 1, \ldots, N$ numbers the enlarged ions, and the index $k = 0, 1, \ldots, M$ numbers the enlarged electrons.

Fig. 1. Example of the distribution of particles by velocities. **1** –ion spectrum, **2** –electron spectrum.

The initial values were taken analogously to (1), with j and k arranged uniformly with zero initial velocities v_k. Changing the sequence of alternation of j and k did not affect the mean values of the quantities.

The difference in the coefficients of equation (1) ($\gamma_k \simeq 1840$) at the present stage of development of electronic computing machines does not make it possible to follow the solution of systems with k for sufficiently large τ. However, the joint solution for j and k gave the following results:

Fig. 2. Example of the distribution of particles in the rz plane.
1–$q_j = 0.5$; 2–$q_{j(k)} = -0.5$ (1/3 of the particles have negative charge; 2/3 positive charge); 3–$q_{j(k)} = 0.5$ (1/2 of the particles have negative charge; 1/2 positive charge); 4–$\varepsilon_j = 0$

Fig. 3. Example of dependences:
1–$\delta z(\tau)$; 2–$\bar{r}(\tau)$; 3–$\delta r(\tau)$; 4–$|\bar{z}(\tau)|$; 5–$|\bar{r}(\tau)|$; 6–$\delta|\bar{z}(\tau)|$; 7–$\delta|\bar{r}(\tau)|$ for $c = a$.
$1'$, $2'$, $3'$–analogous dependences for $c = 0$.

1. Due to the "mobile" electrons, the ideality of the plasma is continuously violated.

2. The electrons are accelerated. Their energy spectrum is blurred. The position of the peak is determined by the initial ion concentration* (Fig. 1).

3. As the initial density increases (under the condition $q_+ = q_-$), $\bar{V}_k$ increases.

These three facts can be used to model phenomena in plasma with the aid of an electronic computer. Two schemes were considered.

I. On a system of positively charged enlarged charges (1), a field of negative charges was superimposed, located at the nodes of a uniform spatial lattice with period Δx, Δy, Δz. The number of lattice nodes was equal to the number of positive "enlarged" charges $q_+ = q_-$ (the possibility is not excluded of imparting oscillatory motions to the lattice nodes for different sets of amplitudes and oscillation frequencies). Calculations performed on this model gave a picture close to that obtained in (1): the arising inhomogeneities of the positive charge lead to its diffusion across and along the magnetic field—though at large values of q_j.

II. Taking into account the mechanism accelerating the electrons, i.e., the presence of a volume positive charge, electrons were passed through a uniform spatial lattice with positive charges at the nodes. A different set of initial values was taken. The energy spectrum is close to that obtained in solving (1).

Also of interest, and considerably simpler in analysis ($\gamma_k = 1$), is a system of positive and negative ions:

4. The ideality of such a system is not violated, i.e., the positive and negative ions are distributed uniformly in space and time for a uniform initial distribution.

5. Diffusion occurs across and along the magnetic field (Fig. 2) (in the equilibrium state of charges $q_+ = q_-$, the diffusion is the same as in the case –

* Everywhere in the calculations $N + M < 200$.

than for positively charged particles alone with a density an order of magnitude smaller).

6. $|\bar{r}(\tau)|$ decreases, $|\bar{z}(\tau)|$ increases, which indicates an additional orientation of the particles along the field as a result of collisions ($|\bar{r}(\tau)| = \sum |\dot{r}(\tau)|/N$; $|\bar{z}(\tau)| = \sum |\dot{z}(\tau)|/N$).

7. $\delta\dot{r}(\tau)$ increases; $\delta\dot{z}(\tau) \cong \text{const}$.

8. The rate of change of the dependences $\bar{r}(\tau)$, $\delta r(\tau)$, $\delta z(\tau)$ is proportional to the initial concentration of ions.

Recently attempts have been made to confine particles by means of imposed auxiliary magnetic fields.

Fig. 4. Example of dependences: 1–$\bar{r}(\tau)$; 2–$\delta z(\tau)$; 3–$\delta r(\tau)$ for $C_n = D_n = a$. $1'$, $2'$, $3'$–analogous dependences for $C_n = D_n = 0$

Figure 2: Fig. 4. Example of dependences: 1–$\bar{r}(\tau)$; 2–$\delta z(\tau)$; 3–$\delta r(\tau)$ for $C_n = D_n = a$. $1'$, $2'$, $3'$–analogous dependences for $C_n = D_n = 0$

A conductor with current at the center ($r = 0$)

$$H_\varphi = \frac{C}{r}, \quad \text{where } C \text{ is a constant.} \tag{3}$$

Calculations carried out with the ordinary fields of a magnetic bottle (2) and (3), according to equations (1), for positive and negative ions and various C, showed the following (Fig. 3):

1. It is possible to choose C in such a way that $\bar{r}(\tau)$ lies below the analogous dependence in the case $C = 0$.
2. At the same time $\delta z(\tau)$, $|\bar{z}(\tau)|$, and $\delta|\bar{z}(\tau)|$ increase strongly.
3. $\bar{r}(\tau)$, $|\bar{r}(\tau)|$, and $\delta|\bar{r}(\tau)|$ have the form of decreasing dependences.
4. With increasing C, properties 1, 2, 3 become more pronounced.
5. For $C \sim a$, losses occur only through the end tubes.
6. The time at which losses begin can be somewhat delayed by increasing β.

Fig. 4. Example of dependences: 1–$\bar{r}(\tau)$; 2–$\delta z(\tau)$; 3–$\delta r(\tau)$ for $C_n = D_n = a$. $1'$, $2'$, $3'$–analogous dependences for $C_n = D_n = 0$.

Conductors on the outer wall of the chamber. The fields they create are conveniently written in the form

$$
\begin{aligned}
H_r &= \left(\frac{r}{a}\right)^{n-1} (C_n \sin n\varphi - D_n \cos n\varphi)(-1)^p \cos pz, \\
H_\varphi &= \left(\frac{r}{a}\right)^{n-1} (C_n \cos n\varphi + D_n \sin n\varphi)(-1)^p \cos pz, \\
H_z &= \frac{ap}{n} \left(\frac{r}{a}\right)^{n} (C_n \sin n\varphi - D_n \cos n\varphi)(-1)^p \sin pz.
\end{aligned} \tag{4}
$$

Although analysis of the behavior of a single particle makes it possible to establish stability regions for certain n, C_n, and D_n, calculation of the motion of many positive and negative enlarged charges leads to the following (see, for example, Fig. 4).

1. $\delta z(\tau)$ increases more steeply in comparison with the case $C_n = D_n = 0$; $\delta r(\tau)$ changes hardly at all (in comparison with the case $C_n = D_n = 0$).

2. For $C_n(D_n) \neq 0$, the minimum (maximum) of $\delta r(n)$ ($\delta z(n)$) occurs at $n = 4$ (the initial values are those of the case $C_n = D_n = 0$).
3. With increasing p, $\delta r(\tau)$ and $\delta z(\tau)$ approach the analogous dependences for $C_n = D_n = 0$.

Calculations performed for the combinations (2), (3), and (4) did not yield a noticeable improvement in the conditions for confinement of particles.

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Note: Figure translations are in progress. See original paper for figures.