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**Abstract**

**Full Text**

## **GEOPHYSICS**

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# **HYDRODYNAMIC CALCULATION OF OROGRAPHIC CLOUDINESS IN A STABLE AND UNSTABLE ATMOSPHERE**

*(Presented by Academician A. A. Dorodnitsyn on 29 VI 1962)*

The spatial problem of the flow of an air current around irregularities of the Earth's surface was first investigated in the work of A. A. Dorodnitsyn <sup>(1)</sup>. It was assumed that the motion takes place within a certain layer above which the atmosphere is at rest; the atmosphere was regarded as an ideal compressible and baroclinic fluid moving steadily and adiabatically. The entire flow was considered as a small perturbation (a linear problem) of the "basic" motion, the velocity of the basic motion being assumed independent of height, while the unperturbed temperature was taken to be a linear function of height. In that work a numerical example was given for the case of a stably stratified basic state.

The aim of the present work is to examine the features that arise under unstable stratification and to compare the stable and unstable cases. We shall assume that the velocity of the basic motion  $U(z_1)$  is directed along the horizontal axis  $x_1$ , and at first we shall impose no restrictions on the dependence of the velocity  $U$  and the temperature  $\tilde{T}$  of the basic motion on the height  $z_1$ .

Then, after linearization, simplification of the theory of convection, and neglect of turbulence and the Coriolis force, we obtain for the vertical component of the momentum  $\tilde{\rho}w' = W$  the equation \*

$$\varepsilon^2 \Delta W_{xx} + W_{zzxx} + \delta \Delta W - \beta W_{xx} = 0, \quad (1)$$

where

$$\varepsilon = \frac{H}{L}, \quad \delta(z) = \frac{g}{\tilde{T}} \frac{(\gamma_a - \gamma)}{U^2} H^2, \quad \beta(z) = \frac{1}{U} \frac{d^2 U}{dz^2}, \quad \Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \quad \left( x = \frac{x_1}{L}, \right. \\ \left. y = \frac{y_1}{L}, \quad z = \frac{z_1}{H} \right),$$

$H$  is the characteristic height,  $L$  the characteristic length,  $\tilde{\rho}$  the density in the basic motion,  $d\tilde{T}/dz = -\gamma(z)$ ,  $\gamma_a$  the adiabatic gradient,  $g$  the acceleration due to gravity, and  $x_1, y_1, z_1$  are orthogonal rectilinear coordinates.

As boundary conditions one may take:

$$\text{for } z = 0 \quad W = \Gamma(x, y); \quad \text{for } z = 1 \quad W = 0. \quad (2)$$

We shall regard  $\Gamma(x, y)$  as a known function \*\*.

In order not to encumber the subsequent exposition, we shall consider the solution of (1) under the boundary conditions (2) at the middle level (at  $z = 1/2$ ). To this end, in the upper half-plane  $z = 1/2$  we draw between the lines  $y = 0$  and  $y = 1$  ( $q - 1$ ) equally spaced lines  $y_j = j/q$  ( $j = 1, 2, \dots, q - 1$ ) along  $y$ , and replace  $\frac{\partial^2}{\partial z^2} W_{xx}$  and  $\frac{\partial^2}{\partial y^2} W$  by the approximate expressions

$$\left. \frac{\partial^2}{\partial z^2} w_{xx} \right|_{z=1/2} = 4(\Gamma_{xx} - 2\overline{W}_{xx}); \quad \left. \frac{\partial^2 \overline{W}}{\partial y^2} \right|_{y=y_j} = \frac{q^2}{4} (\overline{W}_{j+2} - 2\overline{W}_j + \overline{W}_{j-2}),$$

\* For  $U = \text{const}$  and  $\tilde{T}$  a linear function of height, i.e. for  $\delta = D^2 = \text{const}$  and  $\beta = 0$ , this equation was considered in (2).

\*\*  $\Gamma(x, y)$  is the value of the vertical component of the momentum at the Earth's surface. In particular, for the flow of an air current around irregularities of the Earth's surface, in the linear case we have  $\Gamma(x, y) = \rho_0 \varepsilon U \partial \zeta / \partial x$  ( $\rho_0$  is the density  $\tilde{\rho}$  at sea level);  $z = \zeta(x, y)$  is the equation of the surface irregularity.

where  $\overline{W} = W_{z=1/2}$  (we shall agree to denote by the index  $j$  quantities on the line  $y = y_j$ ).

If, further,  $\Gamma(x, y)$  is represented by means of a Fourier series

$$\Gamma_j = \sum_{n=0}^q \tilde{\Gamma}_n(x) \cos \frac{n\pi j}{q}, \quad \tilde{\Gamma}_n(x) = \frac{2}{q} \left\{ \sum_{r=1}^{q-1} \Gamma_r \cos \frac{n\pi r}{q} + \frac{\Gamma_0 + (-1)^n \Gamma_1}{2} \right\}$$

and  $\overline{W}_j$  is sought in the form

$$\overline{W}_j = \sum_{n=0}^q a_n(x) \cos \frac{n\pi j}{q}, \quad (4)$$

then for  $a_n(x)$  we obtain the ordinary differential equation

$$a_n^{\text{IV}} + (\bar{\delta} - 8 - M^2 - \bar{\beta}) a_n'' - \bar{\delta} M^2 a_n = -4\tilde{\Gamma}_n'', \quad (5)$$

where

$$M = q \sin \frac{n\pi}{q}, \quad \bar{\delta} = \left( \frac{g}{T} \frac{\gamma_a - \gamma}{U^2} H^2 \right)_{z=1/2}, \quad \bar{\beta} = \left( \frac{1}{U} \frac{d^2 U}{dz^2} \right)_{z=1/2}.$$

The corresponding characteristic equation has 4 roots.

I. Let us first consider the case when the atmosphere is stably stratified; in other words, let us assume that  $\bar{\delta} > 0$ . Then the roots will be  $+\lambda_1$ ,  $-\lambda_1$ ,  $+i\lambda_2$ ,  $-i\lambda_2$ , where

$$\lambda_1 = \sqrt{\frac{1}{2}(Q - \bar{\delta} + M^2 + 8 + \bar{\beta})}, \quad \lambda_2 = \sqrt{\frac{1}{2}(Q + \bar{\delta} - M^2 - 8 - \bar{\beta})}, \quad (6)$$

$$Q = \sqrt{(\bar{\delta} - M^2 - 8 - \bar{\beta})^2 + 4\bar{\delta}M^2}.$$

Now we can write the solution for  $a_n(x)$  that decays as  $x \rightarrow -\infty$ . This will be

$$a_n(x) = \frac{4}{Q} \left\{ \frac{1}{2\lambda_1} \int_{-\infty}^{+\infty} e^{-\lambda_1|x-x'|} (\tilde{\Gamma}_n)_{x'x'} dx' + \frac{1}{\lambda_2^2} \int_{-\infty}^x \sin \lambda_2(x-x') (\tilde{\Gamma}_n)_{x'x'} dx' \right\}. \quad (7)$$

II. We shall assume the atmosphere to be unstably stratified, i.e.  $\bar{\delta} < 0$ . Then the roots will all be real:  $+\nu_1$ ,  $-\nu_1$ ,  $+\nu_2$ ,  $-\nu_2$ , where

$$\nu_1 = \sqrt{\frac{1}{2}(\bar{\beta} + 8 + M^2 - \bar{\delta} + R)}, \quad \nu_2 = \sqrt{\frac{1}{2}(\bar{\beta} + 8 + M^2 - \bar{\delta} - R)},$$

$$R = \sqrt{(\bar{\beta} + 8 + M^2 - \bar{\delta})^2 + 4\bar{\delta}M^2}. \quad (8)$$

The solution for  $a_n(x)$ , decaying as  $x \rightarrow -\infty$ , will be:

$$a_n(x) = \frac{2}{R} \left\{ \frac{1}{\nu_1} \int_{-\infty}^{+\infty} e^{-\nu_1|x-x'|} (\tilde{\Gamma}_n)_{x'x'} dx' - \frac{1}{\nu_2} \int_{-\infty}^{+\infty} e^{-\nu_2|x-x'|} (\tilde{\Gamma}_n)_{x'x'} dx' \right\}. \quad (9)$$

Figure 1

Figure 1: Figure 1

Figure 2

Figure 2: Figure 2

Using expressions (7) and (9), integrating by parts so that  $\zeta$  enters directly into the final result (at the same time we assume that as  $x \rightarrow -\infty$ ,  $\zeta \rightarrow 0$ ,  $\zeta_x \rightarrow 0$ ,  $\zeta_{xx} \rightarrow 0$ ), after simple transformations we obtain the solution for  $\bar{w}'$  (at the middle level) in the form

$$\bar{w}'_j = -4 \frac{\rho_0}{\rho} \frac{\varepsilon \bar{U}}{q} \sum_{n=0}^q \sum_{r=1}^{q-1} \left\{ \Omega_r^{(n)} \cos \frac{n\pi r}{q} + \frac{1}{2} \left[ \Omega_0^{(n)} + (-1)^n \Omega_q^{(n)} \right] \right\} \cos \frac{n\pi j}{q}$$

( $j = 0, 1, 2, 3, \dots, q$ ), (10)

Fig. 1. Calculated distribution of vertical currents over Holland with stable atmospheric stratification. — is the direction of the main flow.  $1-\widetilde{\rho w'}/\bar{U}a\rho_0 < 2$ ; 2–2; 3–3; 4–4 ( $\bar{\delta} > 0$ )

Fig. 2. Calculated distribution of vertical currents over Holland with unstable atmospheric stratification. — is the direction of the main flow.  $1-\widetilde{\rho w'}/\bar{U}a\rho_0 < 0.1$ ; 2–0.1; 3–0.2; 4–0.3 ( $\bar{\delta} < 0$ )

Fig. 3. Precipitation zones over Holland on 26 X 1945. Atmosphere stable. 1– isohyets 10 mm; 2–15 mm; 3–20 mm; 4–30 mm; 5–40 mm; 6–50 mm

Fig. 4. Precipitation zones over Holland on 27 X 1945. Atmosphere unstable. The designations are the same as in Fig. 3

**Fig. 1**

**Fig. 2**

**Fig. 3**

**Fig. 4**

where

Figure 3

Figure 3: Figure 3

Figure 4

Figure 4: Figure 4

$$\Omega_r^{(n)} = \frac{1}{Q} \left[ \lambda_1^2 \int_{-\infty}^x e^{-\lambda_1(x-x')} \xi \left( x', \frac{r}{q} \right) dx' - \lambda_1^2 \int_x^{+\infty} e^{+\lambda_1(x-x')} \xi \left( x', \frac{r}{q} \right) dx' - 2\lambda_2^2 \int_{-\infty}^x \sin \lambda_2(x-x') \xi \left( x', \frac{r}{q} \right) dx' \right] \quad (11)$$

$$\Omega_r^{(n)} = \frac{1}{2R} \left\{ \int_{-\infty}^x [\nu_1^2 e^{-\nu_1(x-x')} - \nu_2^2 e^{-\nu_2(x-x')}] \xi \left( x', \frac{r}{q} \right) dx' - \int_x^{+\infty} [\nu_1^2 e^{+\nu_1(x-x')} - \nu_2^2 e^{+\nu_2(x-x')}] \xi \left( x', \frac{r}{q} \right) dx' \right\} \quad (12)$$

$$(r = 0, 1, 2, 3, \dots, q).$$

As an example, let us consider flow around a paraboloid of revolution (the vertex of the paraboloid is placed at the point  $z = aH$ ).  $\xi(x, y) = a - b(x^2 + y^2) > 0$ , if  $x^2 + y^2 < a/b$ ;  $\xi(x, y) = 0$  at all other points. Let  $H = 8$  km and  $L = 96$  km ( $\varepsilon = 1/12$ ). The height of the paraboloid is taken to be 800 m. In the calculations the coefficient  $a$  was assumed equal to 0.1, and the coefficient  $b = 57.6$ , in accordance with which the diameter of the base of the paraboloid is 8 km. The data of the basic motion at the mean level were taken as follows:  $\bar{U} = 17$  m · sec<sup>-1</sup>,  $g/\bar{T} = 3 \cdot 10^{-2}$  m · sec<sup>-2</sup> · deg<sup>-1</sup>,  $\beta = 0$  ( $q = 24$ ).

The parameter  $\gamma_a - \gamma$  has the following values: for stable stratification of the atmosphere,  $\gamma_a - \gamma = +3 \cdot 10^{-3}$  deg · m<sup>-1</sup>; for unstable stratification of the atmosphere,  $\gamma_a - \gamma = 2 \cdot 10^{-3}$  deg · m<sup>-1</sup>.

For this example, the vertical velocities were computed by formulas (10)–(12) in the stable and unstable cases. By superposition of the solutions obtained, we calculated the distributions of vertical currents over Holland for stable and unstable stratification of the atmosphere (see Figs. 1 and 2). The calculation was carried out as follows: the entire coast of Holland was represented as a sum of paraboloids in such a way as to obtain a continuous barrier\*. After this, at each point of Holland the influence of all the paraboloids representing the coast was computed, and the result was summed each time; in the calculation the direction of the basic motion was chosen as shown in Figs. 1 and 2, so that the results of the calculations could be compared with the actual data at our disposal. Specifically, for comparison, Figs. 3 and 4 from work (3) are given, where precipitation zones over Holland are shown respectively for 26 X 1945 (stable atmosphere) and 27 X 1945 (unstable atmosphere). The calculation and observations show that in the case of an unstably stratified atmosphere the zones of ascending vertical currents are situated not across the flow, as in the

case of a stably stratified atmosphere, but along the flow. The difference in the configuration of the cloudiness zones for the stable and unstable cases obtained by us agrees with the results of work <sup>(4)</sup>.

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\* The coast, separating land from sea, will play the role of an obstacle because of the difference in roughness between land and sea.

*Note: Figure translations are in progress. See original paper for figures.*

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